Liquidity versus Information Efficiency*

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Abstract

I analyse liquidity, information efficiency and welfare in a market with large and small traders. Large traders create noise in the price for small traders, and vice versa, due to private value differences across the two groups. More liquidity induces large traders to trade more aggressively, creating more noise for small traders; less informative prices, in turn, incite small traders to provide more liquidity. Implications of this interaction are twofold: (i) an increase in competition between large traders may make all traders worse-off, (ii) an increase in the quality of private information may reduce information efficiency.

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1 Introduction

Different aspects of market quality are not equally important to different groups of traders. Lack of liquidity, or a market's inability to accommodate large trades without a change in price, affects large traders more than small traders because the former have a greater need to make big trades. Lack of information efficiency—that is, the market's inability to communicate all available information through prices—affects uninformed traders more than informed traders because prices provide more of the incremental information to the former group. Moreover, these different aspects of market quality may interact: liquidity affects how aggressively one trades on information, which in turn affects information efficiency; conversely, information efficiency affects the willingness of traders to provide liquidity. The interplay between these factors shapes strategic interactions between different groups of traders, and the nature of those interactions is of critical importance for markets. There is, in particular, a long-standing interest in the mechanisms underlying strategic complementarity, i.e., a situation in which the actions of different traders reinforce each other. The literature has identified several such mechanisms and has shown that complementarities are associated with both amplification of shocks and fragility, i.e., a situation where a small shock disproportionately affects a market.[1]

In this paper, I show that traders' heterogeneity—in terms of size, private information quality, and trading motives—produces a novel type of complementarity in which one of the key forces is traders' non–price-taking behaviour. More specifically, a conflict between large informed traders (who care about liquidity, but less so about information efficiency) and small uninformed traders (who care about information efficiency, but less so about liquidity) can create a self-reinforcing tension between these two aspects of market quality and a complementarity between the actions of traders in the two groups. I show that, in the presence of this complementarity, conventional wisdoms about information efficiency and welfare can be overturned. In particular: (i) an increase in competition between large traders may make all traders (including the large) worse-off, (ii) an increase in the quality of private information may reduce information efficiency. These results underscore the importance of interactions between different aspects of market quality and highlight a novel role for complementarities: they can not only amplify but also reverse conventional comparative statics results. Hence both the empirical analysis of and policy discussions about market quality should account for the interactions between liquidity and information efficiency described in this paper.

I consider a centralised market in which two groups, *large traders* and *small traders*, trade a risky asset. The traders within each group are identical, but between the two groups traders are heterogeneous in terms of: (i) size, (ii) trading motives, and (iii) information. There is a

[1] I review the related literature in the Section 2.

discrete number of large traders, but small traders form a continuum. Therefore, whereas large traders can affect prices, small traders cannot; this difference captures the first dimension of heterogeneity. I employ a linear-normal setting: traders are risk neutral and have quadratic inventory costs, and the asset value is distributed normally. Asset value is identical within each group but differs across the two groups. Thus I assume that large traders' values v_L and small traders' values v_S are imperfectly correlated. The difference between v_L and v_S is due to private values differing between the two groups, which reflects the second dimension of heterogeneity. Finally, I assume that large traders know v_L perfectly whereas information about v_S is dispersed among small traders; this difference captures the third dimension of heterogeneity. Trading is structured as a uniform-price double auction: traders simultaneously submit their net demand functions, and all trades are executed at a price that clears the market.

An important feature of the model is that one group of traders' value is a source of noise for the other group.[2] In contrast, in classic noise trading formulation, such as Grossman and Stiglitz (1980), the noise is introduced as an exogenous demand shock of unmodeled traders. Such formulation is unsatisfactory because it is silent about how noise traders respond to changes in liquidity (a form of Lucas (1976) critique). Moreover, classic noise trader formulation precludes any welfare analysis. Another important feature of the model is the non–price-taking behaviour of large traders. A formulation with price-taking large traders is subject to Hellwig (1980) critique (large traders *can* move price but, if they are price-takers, they behave as if they *can not* do so) and is therefore unsatisfactory as well.

My setting can be mapped onto various markets. One example is a stock market in which conventional investors trade with algorithmic traders (ATs). The latter can be viewed as the large traders in my model because they dominate order flow at high frequencies and employ algorithms that minimise the price impact of their trades.[3] Furthermore, ATs often have a much shorter investment horizon than do conventional investors, which justifies the difference in values: v_L (v_S) can be interpreted as the short-run (long-run) price of a stock. Another example is a stock market in which large institutional investors trade with small retail investors. There is considerable empirical evidence that institutional investors, unlike retail investors, can affect prices and also take their price impact into account when trading.[4] Institutional and retail investors therefore can be mapped onto (respectively) large and small traders in the model. The differences in values might stem from the differences in trading motives. For instance, institutional investors, unlike retail investors, may buy a stock not only because it offers an attractive stream of cash flows but also because it allows them to perform better

[2]I follow Vives 2011, Rostek and Weretka 2012 and Du and Zhu 2017, among others, in introducing noise in this way.

[3]For evidence on ATs, see for example Hendershott, Jones, and Menkveld (2011).

[4]For evidence related to institutional investors, see for example Griffin, Harris, and Topaloglu (2003).

relative to a benchmark. Alternatively, the trading of retail investors might be more affected by idiosyncratic risk as compared with institutional investors, because the former's portfolios are more concentrated.[5] I discuss other interpretations of the model—including as a commodity spot market à la Sockin and Xiong (2015)—in Section 8.

My main results derive from a strategic complementarity between how aggressively large traders trade and how willing small traders are to provide liquidity. I say that an agent trades *more aggressively* if her net demand is more sensitive to her information and an agent provides *more liquidity* if she is more willing to accommodate increases in price by reducing her demand, i.e., if her net demand is more sensitive to price. The model's mechanism consists of two parts, as described next.

First, more aggressive trading by large traders induces small traders to provide more liquidity. This outcome is a consequence of two distinct effects. The first is that, if large traders trade more aggressively, then price becomes *less* informative for small traders. Heterogeneity in values is important for this effect. Because large and small traders value the asset differently, traders from one group create noise in the price for the other group. Hence the more aggressive are the large traders, the greater is the noise in the price for the small traders and so the less informative to them is the price.[6] The second effect is that small traders provide more liquidity when price is less informative for them. This effect is is similar to that at work in traditional REE models (e.g., Grossman and Stiglitz 1980). If prices transmit less information then, when some party's purchases are driving up the price, small traders are more willing to sell because it is less likely that the higher price reflects stronger fundamentals.

The second part of the model's mechanism is that liquidity provided by small traders induces large traders to trade more aggressively. As small traders provide more liquidity, the overall liquidity of the market improves, leading large traders to trade more aggressively. The reason is that large traders are not price takers; but trade strategically accounting for their own price impact.

In short, this two-part mechanism generates a new type of complementarity. As large traders trade more aggressively, prices become less informative to small traders. Less informative prices, in turn, incite small traders to provide more liquidity, which encourages large traders to trade even more aggressively.

I show that, given this complementarity, some of the conventional wisdoms about informa-

[5]For evidence regarding the portfolio choices of retail investors, see for example Campbell (2006).

[6]This step is what differentiates my model from traditional models with common values. Indeed, in common values models the price reflects traders' information and noise. If investors trade more aggressively on their information, then the price will incorporate more information as noise traders remain unaffected; thus, information efficiency improves. Weller (2017) offers empirical support for this part of the mechanism. He shows that the amount of information in prices declines as trading by ATs becomes more active.

tion efficiency and welfare can be overturned. In particular, I derive two surprising results. First, the equilibrium price may become *less* informative to small traders when the signals about their values are more precise.[7] The conventional understanding is that price informativeness increases in the precision of the private signals it aggregates. In my model, however, the interactions between liquidity and information efficiency result in price informativeness being affected not only directly by small traders' signal quality but also indirectly by liquidity. An increase in small traders' signal precision improves liquidity by mitigating adverse selection; higher liquidity, in turn, induces large traders to trade more aggressively, thereby injecting noise into the price. I find that this noise can be large enough to make the price less informative for small traders.

My second result concerns welfare. I find that competition between large traders may reduce aggregate welfare and even make small traders worse-off. This outcome is possible because competition between large traders make them trade more aggressively. The resulting less informative prices prevent small traders from achieving efficient asset allocations. To see this, I introduce a first-best benchmark in which large traders take prices as given and small traders know their values. I show that competition generates welfare losses relative to that benchmark through two channels. First, a lack of information efficiency biases the average allocation to small traders vis-à-vis the first-best allocation. I establish that this bias is proportional to the wedge between the true realisation of small traders' value, v_S, and the average beliefs of small traders about it, $\bar{v}_S$, with the coefficient that increases with liquidity. Thus, an increase in competition increases the wedge $v_S - \bar{v}_S$ (as increase in competition harms information efficiency) while also increasing the sensitivity of allocation bias to that wedge (as increase in competition also improves liquidity). Second, with more competition allocations to small traders also become more dispersed (as increase in competition harms information efficiency so that small traders put more weight on their signals). I find that these channels tend to dominate when (i) large traders have more risk-bearing capacity than do small traders (i.e., when large traders are more efficient at managing inventories) and (ii) the informational frictions faced by small traders are high (i.e., when the quality of small traders' private information is low).

The interactions between liquidity and information efficiency described here have implications for both policy discussions about and empirical analyses of market quality. First, it is often argued that improving transparency—that is, making it easier for traders to become more informed by acquiring information on fundamentals—increases price informativeness.[8] As

[7] I also show that a stronger result holds: uncertainty about a small trader's value given information in both her signal and price may increase when such trader have more precise signal. In addition, I establish that the price might be less informative about fundamental value (which could be related to yet different from both v_S and v_L) with improvements in the quality of small traders' private information.

[8] For example, according to Conjecture 4 in Grossman and Stiglitz's 1980 seminal paper: "If the quality of

a matter of fact, several financial regulations have targeted transparency in order to improve that informativeness.[9] Yet I show that, when agents are heterogeneous (along the dimensions described previously), an improvement in the precision of small traders' signals can lead to prices that are *less* informative for them. This finding highlights the potential unintended consequences of policies that target transparency. As another example, it is often argued that greater competition improves welfare.[10] My paper reveals that this tenet need not hold: competition can reduce aggregate welfare and can also render all traders worse-off. More broadly, I emphasise that competition can have detrimental effects on price informativeness and hence on less informed traders, who rely more (than do other traders) on information provided by prices.

2 Relation to the literature

This paper is related to two strands of research. The first is the literature on strategic complementarities and information in markets. Studies in this area are typically cast in a competitive REE framework, which implies that all traders take prices as given.[11] In contrast, the large traders in this paper account for their own influence on prices. This difference is not merely technical: strategic behaviour on the part of large traders is an integral component of the mechanism generating the complementarity. In addition, price taking implies that traders behave competitively and regard the market as perfectly liquid; because my paper focuses on liquidity and on how market quality is affected by competition, strategic behaviour is a desirable feature of the model. In what follows, I review the most closely related REE papers.

As in my work, Cespa and Vives (2011) and Cespa and Foucault (2014) feature equilibrium mechanisms with interactions between liquidity and information efficiency. Yet in these papers, liquidity and information efficiency reinforce rather than oppose each other. Han et al. (2016) describe an equilibrium mechanism with a conflict between liquidity and information efficiency. In that paper, discretionary liquidity traders chase liquidity: when market liquidity is high, they suffer smaller trading losses to informed investors and so more of them enter the market, which reduces information efficiency. The mechanism proposed by Han et al. is different, however: liquidity drives the entry and exit of some traders in their setup, whereas liquidity in my model

the informed trader's information increases [then] . . . the price system becomes more informative."

[9]Examples include the 2002 Sarbanes–Oxley Act and the 2010 Dodd–Frank Act. For discussions of regulatory efforts to address price informativeness by targeting transparency, see Banerjee, Davis, and Gondhi (2016) and Dugast and Foucault (2016).

[10]The important exceptions are industries with increasing returns to scale and especially natural monopolies. Although my model features decreasing returns to scale, competition can still reduce welfare. Moreover, I show that *all* traders may be worse-off with more competition, which is not the case under a natural monopoly.

[11]An incomplete list includes Cespa and Vives (2011), Goldstein, Ozdenoren, and Yuan (2011), Goldstein, Ozdenoren, and Yuan (2013b), Goldstein, Li, and Yang (2013a), Cespa and Foucault (2014), Goldstein and Yang (2015), Huang (2015), Han, Tang, and Yang (2016).

affects how aggressively some agents trade.[12] The implications are also different. Han et al. focus on how information efficiency is affected by the disclosure of public information, whereas I examine how that efficiency is affected by changes in the quality of private information. And in contrast to Han et al., I also look at the implications for liquidity and welfare. The papers of Stein (1987), Goldstein et al. (2013a), and Goldstein and Yang (2015) emphasise the importance for welfare and information efficiency of trading motives and information diversity, which I do as well.[13] However, these papers do not explore the implications of competition for market quality and do not address the interactions between liquidity and information efficiency that are central to my work.

Dugast and Foucault (2016) and Banerjee et al. (2016) make predictions similar to those in my paper. These works demonstrate that an increase in the quality of traders' private information need not make prices more informative, although the mechanisms they cite differ from the one described here. In Banerjee et al., traders may acquire information on asset fundamentals or on noise. The authors show that, under some conditions, lowering the cost of information about a stock's fundamentals may induce traders to acquire more information about noise. As a result, information efficiency may decrease. Thus their model's mechanism operates through a crowding-out effect: traders pay more attention to noise, which reduces their capacity to learn about fundamentals. Dugast and Foucault emphasise an *intertemporal* crowding-out effect. Traders acquiring early, low-precision information may become less able to acquire late, high-precision information—a reduction in capacity that may diminish information efficiency in the long run. In my paper, the mechanism is driven not by information acquisition per se but instead by which traders' information is reflected more in the price.

The second stream of research I contribute to is that on strategic trading. Indeed, the non–price-taking behaviour of large traders is an important part of the equilibrium mechanism in this paper. The literature can be divided in two parts as a function of whether traders use demand schedules or rather market orders to trade. Traders that can use only market orders do not provide liquidity. Since liquidity provision by large and small traders is central to this paper, I shall assume that both groups use demand functions to trade. Hence my paper is most closely related to the literature on demand function equilibria, which I review next.[14]

[12] The paper of Lee (2013), which is an extension of Subrahmanyam and Titman (1999), also features interactions between liquidity and information efficiency that are driven by the entry and exit of some traders. In Lee, however, these two aspects of market quality reinforce each other. The paper by Dow (2004) likewise incorporates a mechanism involving tension between information efficiency and liquidity. In that paper, high liquidity (understood as a low bid–ask spread) makes the market more inviting for hedgers, which in turn reduces information efficiency; the market presence of more hedgers feeds back into the low bid–ask spread. However, Dow focuses on implications of the existence of multiple equilibria. In my model, the main results of that paper hold even when there is a unique equilibrium.

[13] Yuan (2005) discusses the importance of information diversity in the context of benchmark securities.

[14] Among the strategic trading papers with market orders, Subrahmanyam (1991) also finds a tension between

Research on demand function equilibria can itself be subdivided into models based on common valuations and those based on private valuations.[15] Given common values, the interaction between liquidity and information efficiency yields the opposite outcome. In common values models, the price reflects traders' information and noise. If traders believe that the market is more liquid then they trade more aggressively on their information, whereas noise traders are unaffected; thus information efficiency improves. So in this scenario, the self-reinforcing tension between liquidity and information efficiency does not arise.

The private values models (described in Vives 2011, Rostek and Weretka 2012, 2015b, Du and Zhu 2017, and Kyle et al. 2017) do incorporate heterogeneity in traders' values; however, they focus on symmetric settings and there is no heterogeneity in price impact. As a result, traders' behaviour is affected by liquidity in a symmetric way and so the price reflects the same combination of their signals; therefore, the complementarity uncovered in this paper does not arise. Manzano and Vives (2016) consider a similar setting: they assume two groups of investors with identical within-group valuations but different valuations between the groups. These two investor groups might not have the same market power. The main difference is that, in their setting, the traders within each group receive the same signal. In equilibrium, then, all traders not only receive their signal but also learn about the other group's signal (which is inferred from the price). Hence there is no interaction between liquidity and information efficiency: traders learn the same information regardless of the liquidity.

3 The model

There are two time periods, $t \in \{0, 1\}$. Two trader groups, *large traders* and *small traders*, are trading a risky asset at time $t = 0$. There are $N > 1$ large traders indexed by $i \in \{1, 2, ..., N\}$ as well as a unit continuum of small traders indexed by $j \in [0, 1]$. Hereafter, I shall facilitate the exposition by using male (female) pronouns for large (small) traders. All traders are risk neutral and have quadratic inventory costs. Traders are identical within each group, and their preferences are characterised as follows. If a large trader purchases x units of the asset (and

liquidity and information efficiency in a Kyle (1985) framework with risk-averse speculators; in that model, increased liquidity trading (which translates into increased variance of noise trading) improves liquidity but reduces information efficiency. Yet because liquidity trading is exogenous, the latter tension is not self-reinforcing. The implications of Subrahmanyam also differ from those in my paper. Because speculators use market orders, they do not provide liquidity; hence it is not possible to examine their liquidity-providing role. Unlike this paper, Subrahmanyam examines neither the effects of competition on welfare nor the effects of changing private information quality on information efficiency.

[15] Papers in which models incorporate common valuations include Kyle (1989), Pagano (1989), Vayanos (1999), Rostek and Weretka (2015a), and Malamud and Rostek (2017). Models featuring private valuations include those of Vives (2011), Rostek and Weretka (2012, 2015b), Babus and Kondor (2013), Bernhardt and Taub (2015), Manzano and Vives (2016), and Kyle, Obizhaeva, and Wang (2017).

pays price price p) at time $t = 0$, then his utility at time $t = 1$ is

$$u_L = (v_L - p)x - \frac{w_L x^2}{2}, \tag{1}$$

where v_L denotes asset value for a large trader and the term $w_L x^2/2$ represents the inventory cost of holding x units of asset. This cost may be due to regulatory capital requirements, collateral requirements, or risk management considerations. I call $1/w_L$ the *risk-bearing capacity* of a large trader.

Suppose a small trader similarly purchases x units of the asset (and pays price price p) at time $t = 0$; then her utility at time $t = 1$ is

$$u_S = (v_S - p)x - \frac{w_S x^2}{2}, \tag{2}$$

where v_S and $1/w_S$ are (respectively) the asset value and risk-bearing capacity of small traders. The preference specification just given, where risk-neutral traders have quadratic inventory costs and private values, is the same as in Vives (2011), Rostek and Weretka (2012, 2015b), and Du and Zhu (2017).

Asset values are realised at time $t = 1$ but are uncertain at time $t = 0$. The difference in the values of large and small traders generates trade. I assume that the values v_L and v_S are (jointly) normally distributed and imperfectly correlated. That is, $v_k \sim N(\bar{v}_k, 1/\tau_{v_k})$ for $k \in \{S, L\}$ with $\text{corr}(v_L, v_S) = \rho \in (-1, 1)$.

The information structure is as follows. Large traders know v_L but do not know v_S. Small traders do not know v_L yet have only dispersed information about v_S. In particular, each small trader j receives a signal $s_j = v_S + \varepsilon_j$, where the ε_j are independent and identically distributed (i.i.d.) as $\varepsilon_j \sim N(0, 1/\tau_\varepsilon)$ and are also independent of v_S and v_L; the parameter τ_ε measures the signal's precision. The information structure can be summarised by the information sets $\mathcal{F}_i = \{v_L\}$ for a large trader i and $\mathcal{F}_j = \{s_j\}$ for a small trader j.[16] In equilibrium, traders learn also from prices.

The trading is structured as a uniform-price double auction. Each trader k submits a net demand schedule $x_k(p)$, where $x_k(p) > 0$ ($x_k(p) < 0$) corresponds to a buy order (sell order). The market-clearing price p^* is such that the net aggregate demand is zero:

$$\sum_{i=1}^{N} x_i(p^*) + \int_0^1 x_j(p^*)\, dj = 0. \tag{3}$$

[16] The assumption that large traders know v_L is for simplicity. In the Appendix A.2 I show that my main results continue to hold when large traders have dispersed information about their value.

In equilibrium, a trader k is allocated $x_k^* = x_k(p^*)$.

The notion of equilibrium that I employ is that of a Bayesian Nash equilibrium, as in Kyle (1989) and Vives (2011); thus traders maximise expected utility given their information and accounting for their price impact, and equilibrium demand schedules are such that the market clears. As in most of the literature, I restrict the analysis to symmetric linear equilibria in which a large trader i and a small trader j have the following demand schedules:

$$x_i = \alpha + \beta \cdot v_L - \gamma \cdot p; \qquad x_j = \alpha_S + \beta_S \cdot s_j - \gamma_S \cdot p. \tag{4}$$

The coefficients (α, β, γ) and $(\alpha_S, \beta_S, \gamma_S)$ are identical for traders within the same group. Note that I rule out trivial (no-trade) equilibria by focusing on equilibria for which $(\beta, \gamma, \beta_S, \gamma_S) \neq 0$.

The model allows for several interpretations. To fix ideas, consider a stock market in which small traders are conventional investors and large traders are algorithmic traders. Recall from Section 1 that ATs can be associated with the model's large traders because they dominate order flow at high frequencies (and so possess market power) and because they employ algorithms that minimise the price impact of their trades (and thus they exercise that market power). Because ATs often have a relatively short investment horizon, v_L and v_S can be seen as (respectively) a stock's short-run and long-run price.

4 Equilibrium

In this section I define the equilibrium measures of liquidity and information efficiency, show how these two aspects of market quality interact, and characterise the model's equilibria. I shall restrict the analysis to the case where

$$\rho \geq 0.$$

If there were a *negative* correlation ρ between the values of large and small traders then the model would still be tractable, but in that case the equilibrium mechanism would feature additional strategic complementarities that are not the focus of this paper.[17] The assumption of a positive correlation is realistic for the applications that I consider here.[18]

The aspect of liquidity on which I focus is *market depth*. That is, liquidity $\mathcal{L}$ is the reciprocal

[17]Given a negative correlation, the following complementarity is possible. When other small traders trade more aggressively, a small trader of interest might have incentives to trade more aggressively as well. The reason is that, when the correlation is negative, if other small traders trade more aggressively then the price might become *less* informative to the focal small trader because the information in other traders' signals may be (partly) cancelled out by information in the large traders' value. Hence the focal small trader will weight the price less and weight her signal more, thereby increasing her trading aggressiveness as well.

[18]Indeed, $\rho = 1$ under the traditional pure common values setup. If the departure from pure common values is not too substantial, then the correlation should still be positive.

of price impact λ (Kyle's lambda, or the slope of the inverse residual supply):

$$\mathcal{L} \equiv \frac{1}{\lambda} = (N-1)\gamma + \gamma_S. \tag{5}$$

By definition, $1/\lambda$ is the price sensitivity of the residual supply. Equation (5) holds as there are $(N-1)$ large traders with sensitivity γ and a unit mass of small traders with sensitivity γ_S contributing to the price sensitivity of the residual supply. Equation (5) implies that liquidity is directly related to the price sensitivities γ and γ_S, an implication that enables to define *liquidity provision* as follows: *a trader who increases (decreases) the price sensitivity of his demand thereby provides more (less) liquidity.*

The measure of information efficiency that I consider is *revelatory price efficiency* (RPE) in the sense of Bond, Edmans, and Goldstein (2012). Revelatory price efficiency measures the extent to which prices reveal the amount of information necessary for decision makers to take value-maximising actions. Since large traders know their values perfectly, only the information about small traders' values v_S contributes to RPE. Formally, RPE is defined as follows:

$$\mathcal{I} \equiv \frac{\mathrm{Var}(v_S)}{\mathrm{Var}(v_S|s_j, p)}.$$

This measure captures the reduction in variance of small traders' values that is due to learning. In Section A.1, I demonstrate that my main results on information efficiency continue to hold if one instead measures it via *forecasting price efficiency* in the sense of Bond et al. (2012).

The key to my results is the strategic complementarity between how aggressively large traders trade (i.e., how large is the coefficient β in their demand) and how much liquidity small traders provide (i.e., how great is the price sensitivity γ_S of their demand). To examine the first part of this mechanism, I study how small traders' behaviour is affected by an exogenous increase in β. In particular, I perform the following exercise. First, I fix the demand parameters (α, β, γ) for *large* traders. Given these postulated exogenous demands, *small* traders rationally maximise their utilities. I then analyse in Proposition 1 how a change in β affects information efficiency $(\mathcal{I})$ and the amount of liquidity provided by small traders (γ_S) in this partial equilibrium scenario.

To examine the mechanism's second part, I fix the demand parameters $(\alpha_S, \beta_S, \gamma_S)$ for *small* traders. Given these postulated exogenous demands, *large* traders rationally maximise their utilities. Then, in Proposition 2, I analyse how a change in γ_S affects liquidity $\mathcal{L}$ and how aggressively large traders trade (i.e., how large is the coefficient β) in this second partial equilibrium scenario. The full equilibrium is analysed in Theorem 1.

Proposition 1. *The price is informationally equivalent to a sufficient statistic* $\pi \equiv v_S + \zeta/\sqrt{\tau_\pi}$, *where* $\zeta \sim N(0,1)$ *is independent of* v_S *and where* τ_π *(the sufficient statistic's precision) is*

$$\tau_\pi \equiv \mathrm{Var}[\pi|v_S]^{-1} = \frac{\tau_L}{1-\rho^2}\left(\rho\sqrt{\frac{\tau_S}{\tau_L}} + \frac{\beta_S}{N\beta}\right)^2.$$

Informational efficiency can then be written as

$$\mathcal{I} = \frac{\tau_S + \tau_\varepsilon + \tau_\pi}{\tau_S}, \tag{6}$$

and small trader j's demand price sensitivity is

$$\gamma_S = \underbrace{\frac{1}{w_S}}_{\text{expenditure effect}} - \underbrace{\frac{1}{w_S}\frac{\partial E[v_S|s_j,p]}{\partial p}}_{>0,\ \text{information effect}}.$$

Both τ_π *and* $\mathcal{I}$ *are decreasing in* β*, ceteris paribus. The information effect,* $\frac{\partial E[v_S|s_j,p]}{\partial p}$*, is decreasing in* β *whereas the expenditure effect,* $1/w_S$*, is independent of* β*. So all else equal,* γ_S *is increasing in* β*. Therefore, if large traders trade more aggressively then the price is less informative for small traders—who then provide more liquidity.*

The first important takeaway from this proposition, which corresponds to the first step in the equilibrium loop illustrated by Figure 1, is that the price is less informative for small traders when large traders trade more aggressively. It is intuitive that, since large traders create noise in the price for small traders, more aggressive trading produces more noise and so prices become less informative. Since only small traders contribute to the RPE, this measure of information efficiency also decreases.

The second takeaway, which corresponds to the second step in the figure, is that small traders provide more liquidity when the price is less informative to them. Two opposite effects determine their price sensitivity γ_S. The first is the *expenditure* effect: a trader demands less of a higher-priced asset because she must spend more to purchase it. The second is the *information* effect: a higher price may signal a higher value of v_S, which may induce a trader to buy more. Observe that, whereas the information effect is weaker for less informative prices, the expenditure effect is not influenced by price informativeness. So when the price is less informative, small traders provide more liquidity.

Proposition 2. *The demand of a large trader i is given by* $x_i = (v_L - p)/(w_L + \lambda)$*, and his aggressiveness is given by*

$$\beta = \frac{1}{w_L + \lambda}.$$

Both liquidity $\mathcal{L}$ *and aggressiveness* β *are increasing in* γ_S*, ceteris paribus. As a result, if small*

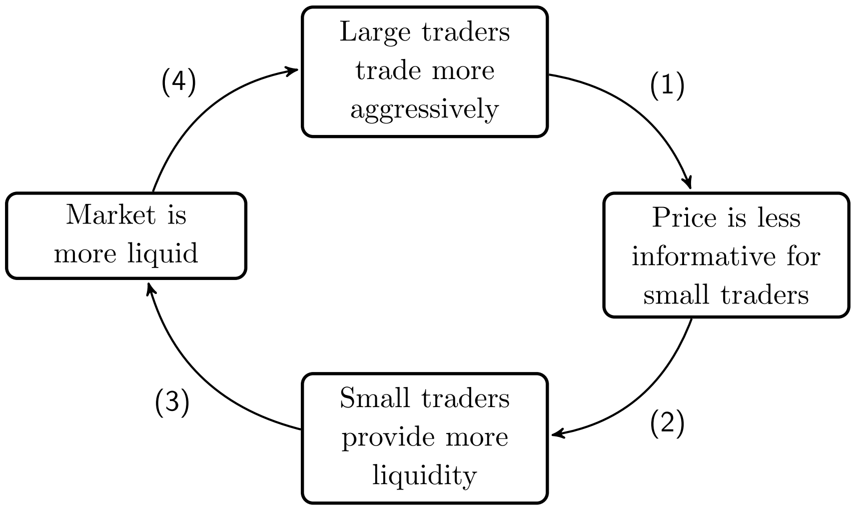


Figure 1: Equilibrium mechanism. The interaction between liquidity and information efficiency facilitates strategic complementarity between how aggressively large agents trade and how willing small agents are to provide liquidity.

traders provide more liquidity then the market becomes more liquid and large traders trade more aggressively.

It is self-evident that, as small traders provide more liquidity, the market's overall liquidity increases. This corresponds to the third step in Figure 1. Greater liquidity reduces the price impact of large traders. Large traders are strategic and take their price impact into account, from which it follows that they trade more aggressively when their price impact is smaller. This outcome corresponds to the final step in the figure.

In sum, the two parts of my proposed mechanism generate a new type of complementarity. As large traders trade more aggressively, prices become less informative to small traders. Less informative prices induce small traders to provide more liquidity, which then induces large traders to trade even more aggressively. The overall equilibrium is characterised in the following theorem.

Theorem 1. *There exists at least one equilibrium. All equilibrium variables can be expressed in closed form by way of an endogenous variable* $\delta \equiv \sqrt{\tau_\pi/\tau_\varepsilon}$. *In particular, price impact can be expressed as*

$$\lambda(\delta) = \frac{Nw_S}{\sqrt{\tau_L}}(\delta\sqrt{\tau_\varepsilon(1-\rho^2)} - \rho\sqrt{\tau_S})\left(\delta^2 + \frac{\tau_S + \tau_\varepsilon}{\tau_\varepsilon}\right) - w_L.$$

The expressions for other equilibrium variables are presented in Appendix B.3. The equilibrium δ

is the solution to the sixth-order polynomial equation

$$\lambda(\delta)(w_L + Nw_S + \lambda(\delta)) - w_S\left(1 + \delta\left(\delta - \frac{\rho}{\sqrt{1-\rho^2}}\sqrt{\frac{\tau_S}{\tau_\varepsilon}}\right)\right)(w_L + 2\lambda(\delta)) = 0 \tag{7}$$

such that $\lambda(\delta) > -w_L/2$.

The strategic complementarity formalised here allows for multiple equilibria. The number of equilibria is equal to the number of solutions to the polynomial equation (7) that satisfy the inequality $\lambda(\delta) > -w_L/2$ (which is a second-order condition in the optimization problem of large traders). However, I shall assume that the model's parameters are such that the equilibrium is unique up until the Section 7, where I analyse consequences of equilibrium multiplicity. Formally, up until that section I maintain the following assumption.

Assumption 1. *The parameters of the model are such that there exists a unique solution to the polynomial equation* (7) *that satisfies the condition* $\lambda(\delta) > -w_L/2$.

The next proposition gives a sufficient conditions for the existence of a unique equilibrium.

Proposition 3. *The equilibrium is unique for large enough* N. *The equilibrium is unique for large enough* τ_S.

Because multiplicity arises from complementarity, it clearly follows that weakening complementarity is sufficient to guarantee the uniqueness of an equilbrium. There are two ways to effect such weakening, which are related to the two parts of the equilibrium mechanism described previously. The first way would be to limit the informational frictions faced by small investors. If small traders have little uncertainty about v_S ex ante, then they will not rely much on prices for their inference. This approach would break the lower part of the equilibrium loop in Figure 1, in which case more aggressive trading by large traders would not affect small traders' liquidity provision. The second means of weakening complementarity would be to limit the non–price-taking behaviour of large traders. If there are many large traders, then no single trader will have much of an effect on prices and so the market will be almost perfectly liquid. In that event, small traders' provision of additional liquidity would have little effect on the equilibrium level of liquidity. This approach would break the upper part of Figure 1's equilibrium loop and so the additional liquidity provided by small traders would have much less influence on the aggressiveness of large traders.

5 Information efficiency and the quality of private information

Suppose that small traders have signals of better quality (i.e., higher τ_ε). Will prices then be more informative for them? The conventional understanding is that, when signals are more informative, the price that aggregates those signals should likewise be more informative. I will show that this conventional wisdom may not be true.

Equilibrium information efficiency can be written as a function of signal precision τ_ε and equilibrium liquidity $\mathcal{L}$: $\mathcal{I} = \mathcal{I}(\tau_\varepsilon, \mathcal{L})$. Thus changes in the precision τ_ε affect information efficiency both directly and indirectly, where the indirect effect operates through the resulting changes in liquidity. The following proposition establishes that, although the direct effect is in line with common understanding, the indirect one is not.

Proposition 4. *Let equilibrium information efficiency be expressed as $\mathcal{I} = \mathcal{I}(\tau_\varepsilon, \mathcal{L})$. Then the effect of a change in precision τ_ε on information efficiency can be decomposed as follows:*

$$\frac{d\mathcal{I}(\tau_\varepsilon, \mathcal{L})}{d\tau_\varepsilon} = \underbrace{\overbrace{\frac{\partial \mathcal{I}(\tau_\varepsilon, \mathcal{L})}{\partial \tau_\varepsilon}}^{>0}}_{\textit{direct effect}} + \underbrace{\overbrace{\frac{\partial \mathcal{I}(\tau_\varepsilon, \mathcal{L})}{\partial \mathcal{L}}}^{<0} \frac{d\mathcal{L}}{d\tau_\varepsilon}}_{\textit{indirect effect}}.$$

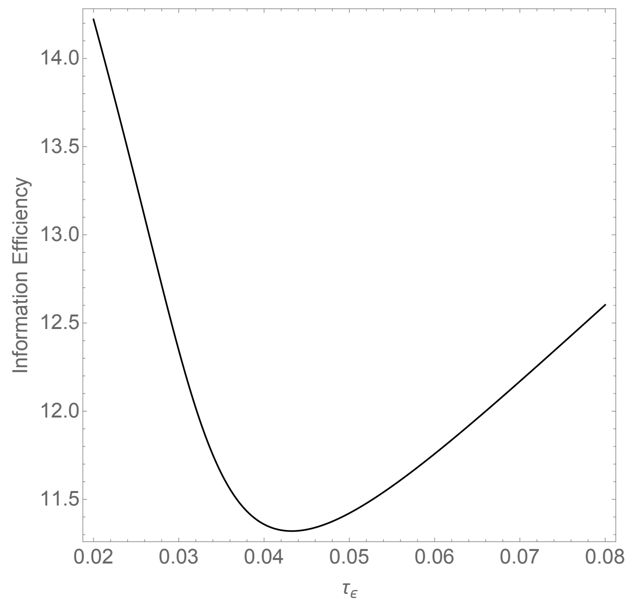
Furthermore, $\frac{\partial \mathcal{I}(\tau_\varepsilon, \mathcal{L})}{\partial \tau_\varepsilon} > 0$ and $\frac{\partial \mathcal{I}(\tau_\varepsilon, \mathcal{L})}{\partial \mathcal{L}} < 0$, and if $\rho > 1/\sqrt{2}$ then $\frac{d\mathcal{L}}{d\tau_\varepsilon} > 0$. Hence the direct effect is always positive whereas the indirect effect is negative—provided that the sufficient condition $\rho > 1/\sqrt{2}$ holds.

The *direct* effect $\frac{\partial \mathcal{I}(\tau_\varepsilon, \mathcal{L})}{\partial \tau_\varepsilon}$ is positive, which accords with conventional understanding: when signals are more precise, the price that aggregates them is more informative. The *indirect* effect can be decomposed into two parts. The first part is the effect of a change in liquidity on information efficiency, $\frac{\partial \mathcal{I}}{\partial \mathcal{L}}$. This effect is negative because of the tension between information efficiency and liquidity discussed previously: higher liquidity induces large traders to trade more aggressively, thus injecting more noise and making price less informative for small traders. The second part is the effect of a change in signal precision τ_ε on liquidity, $d\mathcal{L}/d\tau_\varepsilon$. One anticipates that this effect will be positive given that, when the signal is more precise, a small trader is less concerned about adverse selection and is therefore more willing to provide liquidity. However, there is a countervailing force: greater signal precision τ_ε implies that *all* the small traders are more informed and so the focal small trader is then less willing to provide liquidity. If the correlation ρ is high enough, then the initial force dominates and we have $d\mathcal{L}/d\tau_\varepsilon > 0$.

The foregoing discussion reveals that the indirect effect can be negative. Moreover, com-

Figure 2: Effect of precision τ_ε on information efficiency $\mathcal{I}$.

The graph plots information efficiency as a function of τ_ε. Parameter values: $N = 9$, $\bar{v}_L = \bar{v}_S = 0$, $\tau_S = 0.1$, $\tau_L = 7$, $\rho = 0.9$, $w_L = 4.5$, and $w_S = 1$.



plementarity enables this indirect effect to exceed the direct one. Indeed, the loop in Figure 1 implies that a small change in the precision τ_ε can be amplified and have a profound effect on liquidity—to the extent that $d\mathcal{L}/d\tau_\varepsilon$ can be large. Thus an increase in τ_ε improves liquidity, which makes large traders trade more aggressively and thereby implies less informative prices for small traders, who then provide more liquidity and so further improve liquidity, et cetera. If complementarity is strong then an increase in precision τ_ε can be expected to result in *reduced* information efficiency $\mathcal{I}$. Complementarity is driven by the informational frictions encountered by small traders. In line with this intuition, Figure 2 shows that the unconventional result arises when τ_ε is small and so informational frictions are high. This intuition is further confirmed by the next proposition.

Proposition 5. *Suppose that $(N/w_L)/(1/w_S) > 1/2$. Then there exist $\underline{\tau}_\varepsilon$, $\underline{\tau}_S$, and $\underline{\tau}_L$ such that, for all $\tau_\varepsilon < \underline{\tau}_\varepsilon$, $\tau_S < \underline{\tau}_S$ and $\tau_L < \underline{\tau}_L$: information efficiency $\mathcal{I}$ decreases as precision τ_ε increases.*

The condition $(N/w_L)/(1/w_S) > 1/2$ in this proposition can be understood as follows. Large traders represent a significant part of the market; that is, their aggregate risk-bearing capacity N/w_L is at least half of the small traders' capacity $1/w_S$. This condition is intuitively needed to reinforce steps (1) and (2) in Figure 1. I have already discussed how the unconventional result arises from high informational frictions. Proposition 5 clarifies what is needed for informational frictions to be high: the ex ante precision of the values of both trader types and also the signal's precision must be sufficiently low. These conditions are needed to reinforce steps (3) and (4) in the figure.

Remark: *On endogenous information acquisition.* Suppose that the quality τ_ε of private information is not exogenously given; instead, let small traders choose the precision τ_ε subject to an increasing cost $C(\tau_\varepsilon)$. Now, provided the conditions of Proposition 5 are satisfied, there is a complementarity in information acquisition. If other small traders acquire more information then, by Proposition 5, information efficiency will decrease and thereby incentivise the focal small trader to acquire more information as well. This complementarity implies that the main result of this section is not only robust but also even stronger when information is endogenous, since the complementarity means that a small drop in the cost of acquiring information can be amplified and thus lead to an even greater reduction in information efficiency.

6 Welfare and competition

Suppose that large traders have more market power, as may occur following a merger or the exit of some large traders. Will small traders then be worse-off—and will aggregate welfare be lower? The conventional understanding is that, when large traders have more market power, they will reduce their demand and supply less liquidity (at inferior prices) to small traders. Hence small traders should be worse-off. I now show that this conventional wisdom might not be true: it is possible for all traders to be better-off when the market power of large traders increases.

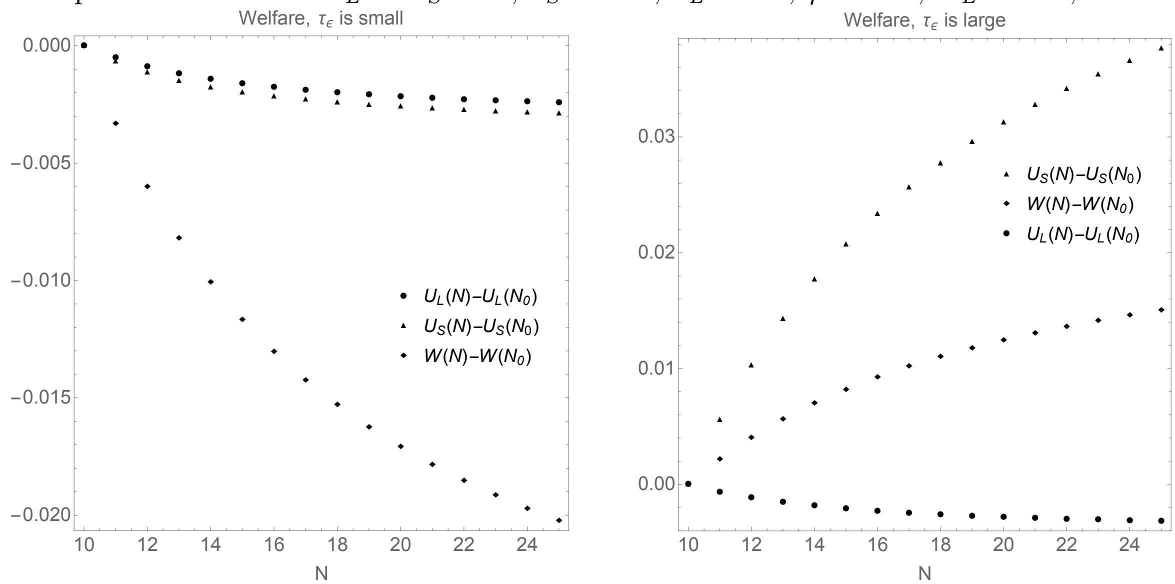
I consider two ways of varying the degree of competition among large traders: (i) large traders can *enter or exit* the market; and (ii) large traders can *merge or split*, so that the number of large traders N changes even as the total risk-bearing capacity of large traders (N/w_L) remains unchanged. The first way corresponds to a change in the number N of large traders without changing any other parameters of the model; the second implies that, when the number of large traders changes, the cost parameter w_L changes with N as follows: $w_L = w_L(N) = N/c_L$, where the constant c_L is equal to the aggregate risk-bearing capacity of large traders.[19] My results hold for both means of varying the degree of competition, so hereafter I shall not explicitly indicate why N changes (except where confusion could otherwise result). I start by analysing the effect of competition on liquidity and information efficiency.

Proposition 6. *Liquidity $\mathcal{L}$ is increasing in N whereas information efficiency $\mathcal{I}$ is decreasing in N.*

[19] This setup is related to a reduced-form approach to modeling the wealth effect in a setting with CARA utility (see Makarov and Schornick 2010).

Figure 3: Effect of the extent of competition on welfare.

The graphs plot aggregate welfare $(\mathcal{W}(N) - \mathcal{W}(N_0))$, small traders' welfare $(\mathcal{U}_S(N) - \mathcal{U}_S(N_0))$, and large traders' welfare $(\mathcal{U}_L(N) - \mathcal{U}_L(N_0))$ as functions of N when $\tau_\varepsilon = 0.1$ (Panel (a)) and $\tau_\varepsilon = 1$ (Panel (b)). The welfare measures are normalised to zero at $N = N_0 = 10$. Other parameter values: $\bar{v}_L = \bar{v}_S = 0$, $\tau_S = 0.1$, $\tau_L = 25$, $\rho = 0.8$, $w_L = 0.1$, and $w_S = 1$.



(a) (b)

If competition increases then large traders exhibit a smaller reduction in demand and provide more liquidity, which is a standard result (see e.g. Ausubel et al. 2014); hence liquidity increases with competition. At the same time, increased competition leads large traders to trade more aggressively and thus to inject more noise into the price for small traders; hence information efficiency declines as competition increases. I shall demonstrate that both of these effects—the increase in liquidity and the decrease in information efficiency—can have adverse effects on welfare.

My measure of welfare for each trader type is ex ante utility. I use $\mathcal{U}_L$ and $\mathcal{U}_S$ to denote the ex-ante expected utility of a large and a small trader, respectively. Then total welfare is $\mathcal{W} \equiv N\mathcal{U}_L + \mathcal{U}_S$.

Panel (a) of Figure 3 displays this section's main result: as competition between large traders increases, all traders can be worse-off.[20] To gain more intuition for this result, I start by analysing the *first-best* (FB) case. In this benchmark, all traders know their values and take prices as given. Traders bid according to their marginal utilities, so that $x_j = (v_S - p)/w_S$ for

[20]This figure plots this outcome for the case of N changing in response to the entry/exit of large traders. A qualitatively similar figure is obtained when the changes in N are due to a merger/split; however, in order to conserve space, I do not reproduce that figure here.

all $j \in [0,1]$ and $x_i = (v_L - p)/w_L$ for all $i \in \{1, 2, ..., N\}$. It is then easy to show that the equilibrium allocations to small and large traders are given by

$$x_S^{\text{FB}} = \frac{v_S - v_L}{w_S + w_L/N} \quad \text{and} \quad x_L^{\text{FB}} = -\frac{x_S}{N},$$

respectively. Welfare is given by

$$\mathcal{W}^{\text{FB}} = \frac{E[(v_L - v_S)^2]}{2(w_S + w_L/N)}.$$

I proceed by considering, in my next proposition, the *welfare loss* (WL)—that is, the difference between the first-best and the equilibrium welfare. This approach allows me to identify the main sources of inefficiency.

Proposition 7. *The welfare loss* $\text{WL} \equiv \mathcal{W}^{\text{FB}} - \mathcal{W}$ *can be expressed as*

$$\text{WL} = \frac{w_S + w_L/N}{2} E[(x_S^{\text{FB}} - \bar{x}_S)^2] + \frac{1}{2} w_S E[(x_j - \bar{x}_S)^2]. \tag{8}$$

Here $\bar{x}_S \equiv \int_0^1 x_j \, dj$*, the average allocation to small traders, is given by*

$$\bar{x}_S = \psi \cdot (x_S^{\text{FB}} + b), \tag{9}$$

where

$$\psi = \frac{w_S + w_L/N}{w_S + (w_L + 1/\mathcal{L})/N}, \quad b = \frac{\bar{v}_S - v_S}{w_S + w_L/N}, \quad \bar{v}_S = \int_0^1 E[v_S | p, s_j] \, dj.$$

The allocation to a small trader j *is given by*

$$x_j = \bar{x}_S + \beta_S \varepsilon_j. \tag{10}$$

The expression (8) for welfare loss, as derived in Vives (2017), incorporates two sources of inefficiency. First, whereas all small traders receive the same allocation in the first-best case, the equilibrium allocations x_j are dispersed around the average allocation $\bar{x}_S$. The corresponding welfare loss is captured by the second term in (8). The allocations to small traders are dispersed because, by putting some weight on their signals, small traders also put some weight on the idiosyncratic noise in those signals; hence the deviation of individual allocation x_j from the average allocation $\bar{x}_S$ is $\beta_S \varepsilon_j$ (cf. (10)). Second, the average allocation $\bar{x}_S$ is likely to be different from the first-best allocation x_S^{FB}; the corresponding welfare loss is captured by the second term in (8).

Equation (9), which is specific to this model, shows how lack og liquidity and information efficiency cause the average allocation $\bar{x}_S$ to deviate from the first-best allocation x_S^{FB}. First of all, a lack of information efficiency causes the average small trader's forecast $\bar{v}_S$ to differ from the true value v_S, and that difference contributes to a *bias* b in (9). Second, a lack of liquidity leads large traders to reduce their demand and so the allocation is scaled down by a factor $\psi < 1$, which increases with liquidity. Because ψ increases with $\mathcal{L}$ and approaches zero (unity) as $\mathcal{L}$ approaches zero (infinity), I refer to ψ as *scaled liquidity*.

To explicate why an increase in competition can harm welfare, I decompose the welfare loss into four components and then identify which of these components contribute the most to this result.

Proposition 8. *The welfare loss can be decomposed into four terms,* $\mathrm{WL} = \mathrm{WL}_1 + \mathrm{WL}_2 + \mathrm{WL}_3 + \mathrm{WL}_4$, *where*

$$\mathrm{WL}_1 \equiv (1-\psi)^2 \mathcal{W}^{\mathrm{FB}}, \quad \mathrm{WL}_2 \equiv \frac{\psi^2 E[(v_S - \bar{v}_S)^2]}{2(w_S + w_L/N)},$$
$$\mathrm{WL}_3 \equiv \frac{w_S}{2} E[(x_j - \bar{x}_S)^2], \quad \mathrm{WL}_4 \equiv -\left(w_S + \frac{w_L}{N}\right)\psi(1-\psi)\,\mathrm{Cov}(b, x_S^{\mathrm{FB}}).$$

Observe that WL_1 *is decreasing in* N *whereas* WL_3 *is increasing in* N*. If* $\mathrm{Var}(v_S|s_j,p)^{-1} > 2\tau_\varepsilon$, *for which it is sufficient that* $\tau_\varepsilon < \tau_S$, *then* WL_2 *decreases with increasing* N*.*

To see how this decomposition is derived, consider the deviation of $\bar{x}_S$ from the first-best allocation x_S^{FB}. According to (9), that deviation can be expressed as

$$x_S^{\mathrm{FB}} - \bar{x}_S = (1-\psi)x_S^{\mathrm{FB}} - \psi \cdot b. \tag{11}$$

The first term displayed in Proposition 8, WL_1, is proportional to $E[(1-\psi)^2(x_S^{\mathrm{FB}})^2]$ and corresponds to the first term in equation (11). This term decreases with N, which captures the standard industrial organization result (see e.g. Tirole 1988; Ausubel et al. 2014) underpinning the common wisdom: with more competition (i.e., less market power), large traders exhibit less reduction in demand and provide more liquidity at better prices to small traders, which ameliorates the welfare loss. Note that this term vanishes as the scaled liquidity ψ approaches unity.

The proposition's second displayed term, WL_2, is proportional to $E[\psi^2 b^2]$ and corresponds to the second term in equation (11). Reduced information efficiency *and* improved liquidity increase this component of welfare loss as competition increases. Indeed, a lack of information efficiency generates bias b whereas improved liquidity increases a loading ψ on that bias in (11). Note that WL_2 does *not* vanish as the scaled liquidity ψ approaches unity.

The third term, WL_3, is proportional to $E[(x_j - \bar{x}_S)^2]$ and arises from the dispersion of individual allocations x_j around the average allocation $\bar{x}_S$. This term is decreasing in information efficiency: the higher the information efficiency, the less small traders load on their signals and on the noise in those signals. Since competition diminishes information efficiency, it follows that WL_3 increases with N. This term, too, does not vanish as scaled liquidity ψ approaches unity.

The fourth term, WL_4, is proportional to $\mathrm{Cov}((1-\psi)x_S^{\mathrm{FB}}, \psi \cdot b)$ and is due to the interaction between the two terms in (11). It is intuitive that, if bias b is (on average) positive when x_S^{FB} is positive (i.e., if $\mathrm{Cov}(b, x_S^{\mathrm{FB}}) > 0$), then the bias can partly compensate for the "scaling down" effect due to illiquidity and thereby reduce the corresponding loss in welfare. Numerical simulations reveal that WL_4 can either decrease or increase with N depending on the parameters chosen. However, this term always vanishes as scaled liquidity ψ approaches unity.

As $\psi \to 1$, the terms WL_2 and WL_3 become the dominant components of welfare loss; since each of these terms is increasing in N, it is intuitive that the welfare loss increases with competition. I establish this result numerically when w_L/w_S is small (i.e., when large traders are more efficient at managing inventories) and when $\tau_\varepsilon < \tau_S$ and τ_S is small (i.e., when small investors face high levels of informational friction).[21] Panel (b) in Figure 3 illustrates the importance of informational frictions. When τ_ε is large, the effect of competition on welfare is in line with common wisdom: although large traders can be

I can summarise these findings as follows. The adverse effects of competition on welfare are due to the second and third components of welfare loss stipulated in Proposition 8. Competition reduces information efficiency, which renders the small traders' allocations more dispersed. Furthermore, information inefficiency biases the average allocation to small traders in comparison with the first-best allocation while any increased liquidity (as would follow from greater competition) increases the loading on that bias. This intuition also explains why all traders—even small ones—can be worse-off under more competition.

7 Multiple equilibria and market fragility

Until this point I have assumed that Assumption 1 holds, so that there exists a unique equilibrium. However, the complementarity revealed in Figure 1 shows that the market can be *fragile*; in other words, there could be multiple equilibria driven by self-fulfilling expectations about liquidity or information efficiency. Large traders who think that liquidity is high would trade

[21] The first condition (small w_L/w_S) ensures that the value of ψ is close to 1 and does not change much as N changes. The second condition (small τ_S exceeding $\tau_v arepsilon$) ensures that, despite scaled liquidity not changing much, the liquidity itself and information efficiency do change with competition and so any change in WL_2 and WL_3 remains significant.

more aggressively, making prices less informative for small traders—who would then provide more liquidity and make the market more liquid indeed. Similarly, small traders who think that the price is uninformative would provide more liquidity and thus make the market more liquid, which in turn would induce large traders to trade more aggressively and inject more noise into the price and so make it uninformative indeed. In this section I give a sufficient condition for multiplicity and show how liquidity and information efficiency can be ranked across multiple equilibria. I start with the following claims.

Proposition 9. *There exist constants $\bar{w}$, $\underline{\tau}_2$, and $\bar{\tau}_2 > \underline{\tau}_2$ such that, if*

$$w_L < \bar{w}, \quad N > 4, \quad \textit{and} \quad \underline{\tau}_2 < \tau_L < \bar{\tau}_2,$$

then there exist at least three distinct equilibria.

The above proposition provides a sufficient condition for fragility. The intuition is as follows. The condition $\tau_L < \bar{\tau}_2$ ensures that the price is not too noisy for small traders, so that they rely on it for purposes of inference. This reliance ensures that changes in price informativeness affect how much liquidity the small traders provide. The condition $\underline{\tau}_2 < \tau_L$ ensures that price informativeness is not too high, so changes in the aggressiveness of large traders can still affect it. Finally, the conditions $w_L < \bar{w}$ and $N > 4$ ensure that large traders constitute a substantial fraction of the market; that is, their aggregate risk-bearing capacity N/w_L is large and they have a significant effect on price informativeness. Note also that it is the *combination* of market power and informational frictions that generate fragility. Recall from Proposition 3 that, if either of these forces is weakened, then the equilibrium is unique.

Proposition 10. *Suppose the model's parameters are such that there exist multiple equilibria. Consider two equilibria, A and B, and suppose that information efficiency in greater in equilibrium A than in equilibrium B. Then the liquidity is lower in equilibrium A than in equilibrium B.*

This proposition establishes that the equilibria can be *ranked* with respect to both liquidity and information efficiency. Moreover, the rankings in terms of these two aspects of market quality are the opposite of each other. So in the equilibrium with greater liquidity, large traders are more aggressive and inject more noise in the price; that behaviour makes this equilibrium also the one with lower information efficiency. This outcome contrasts with results reported in Cespa and Vives (2011) and Cespa and Foucault (2014), where multiple equilibria can also arise yet there is no tension between liquidity and information efficiency across equilibria.

8 Additional interpretations of the model

I now present two additional interpretations of the model. Both are cast in the production economy setting.

8.1 A commodity market

In this interpretation, large and small traders are engaged in trading a commodity (e.g., crude oil or aluminium). The large traders are commodity *producers*. The small traders are commodity *consumers*—that is, firms buying the commodity to produce the final good.

The production technology employed by commodity producers is characterised by the *convex* cost function

$$v_L \cdot y + \frac{w_L}{2} y^2, \tag{12}$$

where $v_L \sim N(\bar{v}_L, 1/\tau_{v_L})$ is a cost shock that is known to producers but not to consumers. Thus producers are better informed about their own production technology than are consumers. Producers are risk neutral and maximise their profit,

$$p \cdot y - \left(v_L y + \frac{w_L}{2} y^2 \right).$$

The term y in this expression is the amount of the commodity sold, or the net supply. The net demand of producers is $x = -y$, and substituting this equation into the preceding display yields

$$(v_L - p)x - \frac{w_L}{2} x^2. \tag{13}$$

This profit expression conforms with the utility equation (1).

Consumers $j \in [0, 1]$ have a production technology characterised by the *concave* production function

$$Y(x) \equiv v_S \cdot x - \frac{w_S}{2} x^2; \tag{14}$$

here $v_S \sim N(\bar{v}_S, 1/\tau_{v_S})$ is a productivity shock common to all consumer firms. This shock will drive the aggregate output of the economy's final good and can be interpreted as the economy's *strength*. The firms have dispersed information concerning that strength. In particular, each firm j is endowed with a signal

$$s_j = v_S + \varepsilon_j;$$

here $\varepsilon_j \sim N(0, 1/\tau_\varepsilon)$ and is independent of all other random variables in the model. Firms are risk neutral and maximise their respective profits,

$$p_g\left(v_S \cdot x - \frac{w_S}{2}x^2\right) - p \cdot x, \tag{15}$$

where $p_g = 1$ is the price of the final good (endogenised in what follows) and p is the commodity's price. The expression (15) conforms with the utility equation (2).

I close the model by assuming that the final good is sold to consumers $l \in [0, 1]$, who have a linear Marshallian utility function over the amount z of the final good consumed and over the money $m = m_0 - p_g z$ left after purchasing the final good :

$$u_l(z, m) = z + m_0 - p_g z,$$

where m_0 represents each consumer's endowment of money. The existence of a *continuum* of consumers implies that they are price takers and that the final good's price is equal to their marginal utility; thus, indeed, $p_g = 1$.

The setting considered here is a natural framework for the study of commodities markets. The linear-quadratic specification of the cost and of the production functions is common in the commodities literature.[22] The information structure—with a cost shock known to producers but not to firms and where firms have dispersed information regarding the economy's strength—is the same as in Sockin and Xiong (2015). The setting of this interpretation can be viewed as a generalisation of that paper in which producers are allowed to have market power.[23]

8.2 A foreign exchange market

In this interpretation, foreign currency is the good being traded. Suppose that the home currency is the pound and the foreign currency is the dollar; here the price p is how many pounds one dollar is worth. The large traders are exporters, and the small traders are importers. Exporters receive dollars from selling their goods, while importers must buy dollars so they can purchase raw materials abroad. The supply and demand attributable to those two groups determine the exchange rate.

The price of the good that the large traders produce and export is denominated in dollars, and exporters cannot affect that price (which I normalise to 1). Assume that the cost of producing y units of the export good is given by (12); then the revenue from selling y units of

[22] See, for example, Grossman (1977), Kyle (1984), Stein (1987), and Goldstein and Yang (2017).

[23] The market power of producers is clearly relevant in commodities markets. In the crude oil market, for example, OPEC accounts for more than 40% of world production (Fantini (2015)); in the aluminum market, the six largest producers account for more than 40% of world production (Nappi 2013).

the good is y dollars or $p \cdot y$ pounds. Therefore, the profit from a sale of y units (corresponding to the net demand of $x = -y$) is given by (13).

The small traders need to import raw materials whose price is denominated in dollars (and normalised to 1). The cost of buying x units of raw materials is therefore x dollars or $p{\cdot}x$ pounds. With x units of raw materials, an importer can produce $Y(x)$ units of the good; the production function $Y(x)$ is given by (14). The price of the good produced by importers is denominated in pounds (and normalised to 1). The profit from selling x units of the good is therefore given by (15), just as in Section 8.1 interpretation. It follows that this foreign exchange interpretation conforms to the model presented in Section 3.

9 Discussion

The key step in this paper's equilibrium mechanism is that large traders make prices less informative when trading more aggressively. I establish this connection both for revelatory price efficiency (in Proposition 1) and forecasting price efficiency (in Proposition 1.A.2 of the Appendix). In line with the latter demonstration, the papers of Weller (2017) and Gider, Schmickler, and Westheide (2016) find a negative relation between price informativeness and the activity of algorithmic traders. That result constitutes empirical support for the mechanism posited in my paper.

The result that price informativeness can decline when traders become more informed (Propositions 5 and 5.A.1) helps explain the evidence presented by Farboodi, Matray, and Veldkamp (2018) and Bai, Philippon, and Savov (2016); these authors show that, despite the prices of stocks in the S&P 500 index becoming more informative in recent decades, the price informativeness of stocks that are *not* in that index has fallen. This evidence is puzzling when one considers that technological progress has made information about all stocks more easily available (and so the quality τ_ε of private information should have increased, on average), which suggests that price informativeness should likewise have increased for all stocks. One implication of my model is that the opposite may be true for stocks that are less transparent—namely, those with respect to which there is a lower quality of private information (τ_ε) and of public information (τ_S). Stocks of that type are likely to be smaller, less liquid, and less "glamorous" than those covered by the S&P 500 index. This result has implications also for the design of empirical tests that rely on exogenous changes in information efficiency. For example, many scholars follow Kelly and Ljungqvist (2012) in treating the reduced number of sell-side analysts (due to the closure of brokers' research departments) as an exogenous negative shock to information efficiency. Although that reduction can be interpreted as a negative shock to information quality, my paper indicates that it could be as well a positive shock to price informativeness. This issue is more likely to arise in the case of stocks that are smaller and/or less transparent.

My paper bears implications also for policies that target transparency and competition. Recall from Section 1 that legislative attempts have been made to increase market transparency in order to improve price informativeness. I have established that improved transparency—which translates (in the model) into an increase in the precision of traders' signals—can actually lead to prices that are *less* informative for them. This outcome underscores the potential unintended consequences of policies targeting transparency.

The results in Section 7 show that the combination of market power and informational frictions can lead to market fragility.[24] I argued in Section 8 that the model applies to diverse markets, including stocks and commodities markets. It thus provides a unified explanation for the episodes of fragility observed in those markets and thereby confirms the role of market power in generating such fragility. Notwithstanding the literature devoted to the role played by informational frictions in generating fragility, the role played by market power, as highlighted in this paper, seems to be new.

It is widely argued that greater competition is beneficial for welfare, and this tenet underlies antitrust policies worldwide.[25] However, my paper shows that this presumption need not be valid in markets characterised by market power and informational frictions. In such circumstances, increased competition can reduce the welfare not only of large traders with market power but also of small traders—that is, by making prices less informative for them. More broadly, this paper shows how competition can have detrimental effects on price informativeness and hence on less informed traders, who are the ones most reliant on information provided by prices.

10 Conclusion

There are many markets that involve traders who are heterogeneous in terms of market power. In such markets, there are large traders who can affect prices and small traders who cannot. I demonstrate that, when large and small traders have different values, they create noise in the price for each other and so the following complementarity arises. When large traders trade more aggressively, they create noise in the price for small traders. The resulting less informative prices induce small traders to provide more liquidity, which feeds back into more aggressive trading by large traders. I show that this complementarity has the effect of overturning two conventional

[24]See also Proposition 3, which shows that there is a unique equilibrium when N is sufficiently large. In other words, fragility is reduced under increased competition

[25]See, for example, the "Guide to Antitrust Laws" available on the US Federal Trade Commission website (https://www.ftc.gov/tips-advice/competition-guidance/guide-antitrust-laws).

wisdoms. In particular: increased competition among large traders can make all traders *worse*-off; and an increase in the quality of private information may *reduce* information efficiency.

The model can be extended in several directions. Incorporating multiple assets would allow one to examine cross-asset effects in the presence of interactions between market liquidity and information efficiency. It would also be of interest to explore a dynamic extension. Consider a large trader capable of trading in more than one period. If he trades more aggressively in the first period then the price will be less informative for small traders, who will then provide more liquidity. In that event, it will be less costly for the large trader to trade even more aggressively in subsequent periods. So in light of the complementarity on which my paper focuses, large investors may trade more rapidly than when it is absent. These extensions are left for future research.

A Robustness and extensions

A.1 Forecasting price efficiency and quality of private information

In the main text, information efficiency was defined from the perspective of agents who want to learn about their values. These values might be different from the asset's fundamental value because of private values associated with holding the asset. In this section, I define information efficiency from the perspective of an econometrician who is given prices and wants to forecast an asset's fundamental value. That is, rather than the *revelatory price efficiency* considered before, I now consider *forecasting price efficiency* as a measure of information efficiency.

I assume that fundamental value of asset can be written as

$$f = k_S v_S + k_L v_L + \eta,$$

where $k_S > 0$ and $k_L > 0$ are constants and $\eta \sim N(\bar{\eta}, \tau_\eta^{-1})$ is independent of v_S and v_L. I define the forecasting price efficiciency as

$$\mathcal{I}_\mathcal{F} = \frac{\mathrm{Var}(f)}{\mathrm{Var}(f|p)}.$$

I will demonstrate that, provided the loading k_S of fundamental value f on the value of small traders v_S is different from zero, there exist model parameters such that my key results concerning information efficiency (i.e., Propositions 1 and 5) continue to hold. Key to this demonstration is the following lemma.

Lemma 1. *For a given price, the conditional variance of fundamental value is*

$$\mathrm{Var}(f|p) = \frac{1}{\tau_\eta} + k_L^2 \frac{1-\rho^2}{\tau_L}\left(\frac{1}{\sqrt{\tau_\pi}}\left(\frac{k_S}{k_L}\sqrt{\frac{\tau_L}{1-\rho^2}} + \frac{\rho\sqrt{\tau_S}}{\sqrt{1-\rho^2}}\right) - 1\right)^2, \tag{16}$$

where $\tau_\pi = \mathrm{Var}(v_S|p)^{-1} - \tau_S$ signifies the precision of information about v_S that is contained in the price. Moreover, if

$$\sqrt{\tau_\pi} < \frac{k_S}{k_L}\sqrt{\frac{\tau_L}{1-\rho^2}} + \frac{\rho\sqrt{\tau_S}}{\sqrt{1-\rho^2}}, \tag{17}$$

then the forecasting price efficiency $\mathcal{I}_\mathcal{F}$ is increasing in τ_π and depends on τ_ε only through τ_π. A sufficient condition for (17) to hold is

$$\frac{w_L}{N w_S} + \frac{w_L}{N(w_L + 2N w_S)} < \frac{k_S}{k_L}. \tag{18}$$

It is immediate that, if (18) holds, then τ_π and $\mathcal{I}_\mathcal{F}$ move in the same direction. So when large traders trade more aggressively, the revelatory price efficiency decreases. It is therefore possible to formulate the following version of Proposition 1.

Proposition 1.A.1. *Suppose* (18) *holds. Then, when large traders trade more aggressively, the revelatory price efficiency decreases.*

Proposition 5 stipulates conditions under which $\mathcal{I}$ decreases with τ_ε. If $\mathcal{I}$ decreases with τ_ε, then τ_π decreases as well. Therefore, when (18) holds and the conditions of Proposition 5 are satisfied (which is possible when $(N/w_L)/(1/w_S)$ is large enough and τ_L, τ_S, and τ_ε are low enough), Proposition 5 holds.

Proposition 5.A.1. *There exist M as well as $\underline{\tau}_\varepsilon$, $\underline{\tau}_S$, and $\underline{\tau}_L$ such that, for all $(N/w_L)/(1/w_S) > M$,, $\tau_\varepsilon < \underline{\tau}_\varepsilon$, $\tau_S < \underline{\tau}_S$, $\tau_L < \underline{\tau}_L$, we have that the forecasting price efficiency $\mathcal{I}_\mathcal{F}$ is decreasing in τ_ε.*

A.2 Model with uninformed large traders

Here I consider a model that differs from the one in Section 3 only in that large traders do not know their values perfectly. Instead, a large trader i is endowed with a signal $s_i = v_L + n_i$, where the n_i are i.i.d. as $n_i \sim N(0, 1/\tau_n)$ and are independent of v_S and v_L.

I consider symmetric linear equilibria in which a large trader i and a small trader j have the demand schedules

$$x_i = \alpha + \beta \cdot s_i - \gamma \cdot p \quad \text{and} \quad x_j = \alpha_S + \beta_S \cdot s_j - \gamma_S \cdot p, \tag{19}$$

respectively. The coefficients (α, β, γ) and $(\alpha_S, \beta_S, \gamma_S)$ are identical for traders within the same group.

Since both groups of traders learn in the extended model, I introduce two measures of revelatory price efficiency, one each defined from the perspective of small and large traders:

$$\mathcal{I}^S = \frac{\text{Var}(v_S)}{\text{Var}(v_S|s_j, p)}, \qquad \mathcal{I}^L = \frac{\text{Var}(v_L)}{\text{Var}(v_L|s_j, p)}.$$

The main results of this section are that (a) the complementarity described in Section 4 continues to hold in this extended setting and (b) an increase in the precision of small traders' signals can reduce informational efficiency both for large and small traders.

As in Section 4, I examine the mechanism's first part by fixing the demand parameters (α, β, γ) for large traders. Given these exogenously postulated demands for large traders, small traders rationally maximise their utilities. I then analyse (in Proposition 1.A.2) how a change in β affects $\mathcal{I}^S$—and the amount of liquidity provided by small traders, γ_S—while keeping everything else fixed.

To examine the second part of the mechanism, I fix the demand parameters $(\alpha_S, \beta_S, \gamma_S)$ for small traders. Given these exogenously postulated demands for small traders, large traders rationally maximise their utilities. I then analyse (in Proposition 2.A.2) how a change in γ_S affects liquidity ($\mathcal{L}$) and how aggressively large traders trade (β) while keeping everything else fixed. The full equilibrium is analysed in Theorem 1.A.2.

Proposition 1.A.2. *The equilibrium price is informationally equivalent to a sufficient statistic $\pi \equiv v_S + (1/\sqrt{\tau_\pi})\zeta_u$, where $\zeta_u \sim N(0,1)$ is independent of v_S and where τ_π is the sufficient statistic's precision:*

$$\tau_\pi \equiv \mathrm{Var}[\pi|v_S]^{-1} = \left(\left(\frac{\tau_L}{1-\rho^2}\left(\rho\sqrt{\frac{\tau_S}{\tau_L}} + \frac{\beta_S}{N\beta}\right)^2\right)^{-1} + \frac{N\beta^2}{\tau_n}\right)^{-1}.$$

The revelatory price efficiency for small traders can be written as

$$\mathcal{I}^S = \frac{\tau_S + \tau_\varepsilon + \tau_\pi}{\tau_S}. \tag{20}$$

Small trader j's demand is given by $x_j = (E[v_j|s_j,p] - p)/w_S$, and her price sensitivity can be written as

$$\gamma_S = \underbrace{\frac{1}{w_S}}_{\text{expenditure effect}} - \underbrace{\frac{1}{w_S}\frac{\partial E[v_S|s_j,p]}{\partial p}}_{>0,\ \text{information effect}}.$$

Both τ_π and $\mathcal{I}$ are decreasing in β. The information effect, $\frac{\partial E[v_S|s_j,p]}{\partial p}$, is decreasing in β whereas the expenditure effect, $1/w_S$, is independent of β; as a result, γ_S is increasing in β. Therefore, if large traders trade more aggressively then the price is less informative for small traders and they provide more liquidity.

This proposition reveals that steps (1) and (2) of the equilibrium loop in Figure 1 continue to hold. The intuition for the first step is similar to that given in Section 4: since large traders create noise in the price for small traders, it follows that large traders trading more aggressively injects more noise into the price for small traders, which makes the the less informative to them. Step (2) in Figure 1 is also addressed by Proposition 1.A.2: small traders provide more liquidity when the price is less informative to them. The information effect is weaker the less informative is the price, whereas the expenditure effect is unaffected by price informativeness. So when price is less informative, small traders provide more liquidity.

Proposition 2.A.2. *Both liquidity $\mathcal{L}$ and aggressiveness β are increasing in γ_S, ceteris paribus. Therefore, if small traders provide more liquidity then the market becomes more liquid and large traders trade more aggressively.*

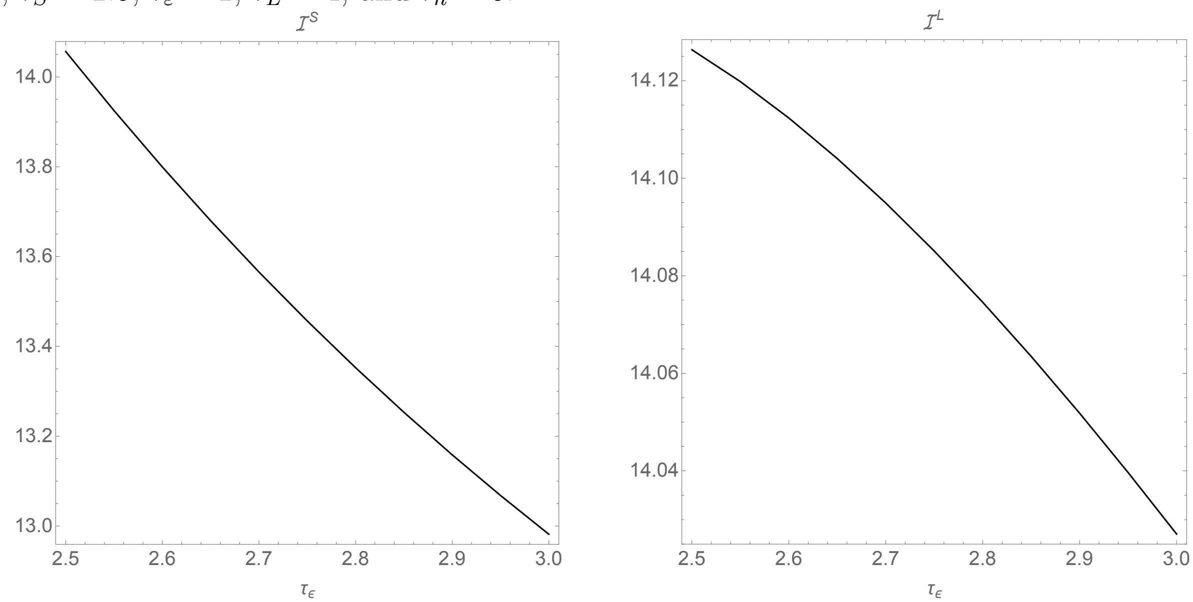
As small traders provide more liquidity, the overall liquidity of the market improves. This corresponds to step (3) in Figure 1. An improvement in liquidity reduces the price impact of large traders. Since large traders are strategic and take their own price impact into account, if that impact is lower then they trade more aggressively. This behaviour corresponds to step (4) in the figure. Thus the preceding two propositions confirm that complementarity is present also in the extended model.The full equilibrium is characterised in the following theorem.

Theorem 1.A.2. *All equilibrium variables can be expressed in closed form through two endogenous variables: $x \equiv \beta_S/\beta$ and λ. The equilibrium is a solution to a system of two nonlinear algebraic equations presented in the Appendix 1.A.2.*

A central result in Section 5 is that price can be less informative for small traders (i.e., $\mathcal{I}^S$ can decrease) as the quality of their private information increases (i.e. as τ_ε increases). This outcome is possible because, with more informative signals, small traders provide more liquidity and thus make the market more liquid for large traders, who then trade more aggressively and thereby inject more noise into the price. Is it possible that price becomes less informative for large traders as well? The answer is Yes. The reason is that, when large traders trade more aggressively, they load more not only on their value v_L but also on the noise n_i in their signals. Since there are a few large traders, that noise does not vanish. This result is illustrated in Figure 4.

Figure 4: Effect of precision τ_ε on information efficiency.

The graphs plot small investors' information efficiency $\mathcal{I}^S$ (Panel (a)) and large investors' information efficiency $\mathcal{I}^L$ (Panel (b)) as a function of τ_ε. Parameter values: $N = 13$, $\bar{v}_L = \bar{v}_S = 0$, $\tau_S = 1.5$, $\tau_\varepsilon = 1$, $\tau_L = 4$, and $\tau_n = 5$.



(a) (b)

B Proofs

I start with the following useful lemma.

Lemma 2. *Large traders' values v_L can be decomposed as follows:*

$$v_L = A + Bv_S + C\zeta,$$

where $B = \rho\sqrt{\tau_S/\tau_L}$, $C = \sqrt{(1-\rho^2)/\tau_L}$, and $A = \bar{v}_L - B\bar{v}_S$. Also, $\zeta \sim N(0,1)$ is independent of v_S.

Proof of Lemma 2. One can check by direct calculation that $\zeta = v_L - A - Bv_S$ has a mean of 0, a variance of 1, and a covariance (with v_S) of 0. The combination of zero covariance and joint normality implies independence. ■

B.1 Proof of Proposition 1

Proof of Proposition 1. The price is informationally equivalent to $\beta_S v_S + N\beta v_L$. After substituting v_L from Lemma 2 and undertaking some re-arrangement, we obtain that the price is informationally equivalent to $\pi \equiv v_S + (1/\sqrt{\tau_\pi})\zeta$, where

$$\tau_\pi = \frac{\tau_L}{1-\rho^2}\left(\rho\sqrt{\frac{\tau_S}{\tau_L}} + \frac{\beta_S}{N\beta}\right)^2. \tag{21}$$

The formula for informational efficiency now follows directly from the projection theorem. It can be seen from the formulas that both τ_π and $\mathcal{I}$ are decreasing in β.

The optimal demand of a small trader j can be written as $x_j = (E[v_s|s_j,p]-p)/w_S$. It then follows that $\gamma_S = \frac{1}{w_S} - \frac{\partial E[v_s|s_j,p]}{\partial p}$. One can write $E[v_s|s_j,p] = \frac{\tau_\pi}{\tau_S+\tau_\varepsilon+\tau_\pi}\pi + ...$, where "..." stands for terms that do not depend on p. One can also write $\pi = \frac{\gamma_S+N\gamma}{\beta_S+N\beta\rho\sqrt{\tau_S/\tau_L}}p + ...$, from which it follows (after some re-arrangement) that

$$\gamma_S = \frac{\frac{1}{w_S} - \frac{\tau_\pi}{\tau_S+\tau_\varepsilon+\tau_\pi}\frac{N\gamma}{\beta_S+N\beta\rho\sqrt{\tau_S/\tau_L}}}{1+\frac{\tau_\pi}{\tau_S+\tau_\varepsilon+\tau_\pi}\frac{1}{\beta_S+N\beta\rho\sqrt{\tau_S/\tau_L}}}.$$

Thus we can see that γ_S is decreasing in β. ■

B.2 Proof of Proposition 2

Proof of Proposition 2. The first-order condition for a large trader i yields (see e.g. Kyle 1989; Vives 2011) $x_i = (v_L - p)/(w_L+\lambda)$; here $1/\lambda$ is the slope of the residual supply, $1/\lambda = \gamma_S + (N-1)\gamma$. The second-order condition is satisfied if and only if $\lambda > -w_L/2$. Hence $\beta = \gamma = 1/(w_L+\lambda)$, and λ is determined by

$$\frac{1}{\lambda} = \frac{N-1}{w_L+\lambda} + \gamma_S.$$

It is easy to show that this equation's solution that satisfies $\lambda > -w_L/2$ is decreasing in γ_S. We can therefore conclude that also β is decreasing in γ_S. ■

B.3 Proof of Theorem 1

Proof of Theorem 1. The first-order conditions from Propositions 1 and 2 can be summarised as follows:

$$x_j = \frac{E[v_s|s_j,p]-p}{w_S}; \qquad x_i = \frac{v_L-p}{w_L+\lambda}.$$

The second-order condition for large traders, $\lambda > -w_L/2$, must also hold.

According to Proposition 1,

$$\gamma_S = \frac{\frac{1}{w_S} - \frac{\tau_\pi}{\tau_S+\tau_\varepsilon+\tau_\pi}\frac{N\gamma}{\beta_S+N\beta\rho\sqrt{\tau_S/\tau_L}}}{1+\frac{\tau_\pi}{\tau_S+\tau_\varepsilon+\tau_\pi}\frac{1}{\beta_S+N\beta\rho\sqrt{\tau_S/\tau_L}}}.$$

Given that $E[v_s|s_j,p] = \frac{\tau_\varepsilon}{\tau_S+\tau_\varepsilon+\tau_\pi}s_j +$ (terms that do not depend on s_j), we can also derive the equality $\beta_S = \frac{1}{w_S}\frac{\tau_\varepsilon}{\tau_S+\tau_\varepsilon+\tau_\pi}$. The first-order conditions for large traders imply that $\beta = \gamma = 1/(w_L+\lambda)$.

Next we express the coefficients β_S, γ_S, β, and γ through the endogenous variable $\delta = \sqrt{\tau_\pi/\tau_\varepsilon}$. It is immediate that

$$\beta_S(\delta) = \frac{1}{w_S}\frac{\tau_\varepsilon}{\tau_S+\tau_\varepsilon(1+\delta^2)}.$$

Theorem 1's expression for $\lambda(\delta)$ follows if we substitute $\beta_S = \beta_S(\delta)$ and $\beta = 1/(w_L+\lambda)$ into (21) and express λ. The terms $\beta(\delta)$ and $\gamma(\delta)$ are related to δ as

$$\beta(\delta) = \gamma(\delta) = \frac{1}{w_L+\lambda(\delta)};$$

from that expression it follows, with regard to $\gamma_S(\delta)$, that

$$\gamma_S = \frac{\frac{1}{w_S} - \frac{\tau_\varepsilon\delta^2}{\tau_S+\tau_\varepsilon(1+\delta^2)}\frac{N\gamma(\delta)}{\beta_S(\delta)+N\beta(\delta)\rho\sqrt{\tau_S/\tau_L}}}{1+\frac{\tau_\varepsilon\delta^2}{\tau_S+\tau_\varepsilon(1+\delta^2)}\frac{1}{\beta_S(\delta)+N\beta(\delta)\rho\sqrt{\tau_S/\tau_L}}}. \tag{22}$$

It remains to derive expressions for $\alpha(\delta)$ and $\alpha_S(\delta)$. The first-order condition for a large trader implies that $\alpha(\delta) = 0$. Given that $E[v_s|s_j,p] = \frac{\tau_S}{\tau_S+\tau_\varepsilon+\tau_\pi}\bar{v}_S+$(terms that depend on s_j and p), we have

$$\alpha_S(\delta) = \frac{1}{w_S}\frac{\tau_S}{\tau_S+\tau_\varepsilon(1+\delta^2)}\bar{v}_S.$$

The polynomial equation (7) for δ can be obtained by re-arranging $\frac{1}{\lambda(\delta)} = \gamma_S(\delta) + (N-1)\gamma(\delta)$.

I now prove that there is at least one solution to (7) such that $\lambda > -w_L/2$. Consider a unique δ^* satisfying $\lambda(\delta^*) = -w_L/2$. We can show that the polynomial (7) evaluated at $\delta = \delta^*$ is negative; at the same time, the polynomial's leading coefficient is positive. Hence the polynomial becomes positive for large enough δ. By the intermediate value theorem, there exists a $\delta^{**} > \delta^*$ such that the polynomial is zero. Since $\lambda(\delta)$ is increasing for $\delta > \delta^*$, we have that $\lambda(\delta^{**}) > -w_L/2$. ■

B.4 Proof of Proposition 3

Proof of Proposition 3. The equilibrium δ solves the following system of equations:

$$\lambda = \frac{Nw_S}{\sqrt{\tau_L}}\sqrt{\tau_\varepsilon(1-\rho^2)}\left(\delta - \frac{\rho}{\sqrt{1-\rho^2}}\sqrt{\frac{\tau_S}{\tau_\varepsilon}}\right)\left(\delta^2 + \frac{\tau_S+\tau_\varepsilon}{\tau_\varepsilon}\right) - w_L, \tag{23}$$

$$\lambda(w_L + Nw_S + \lambda) - w_S\left(1 + \delta\left(\delta - \frac{\rho}{\sqrt{1-\rho^2}}\sqrt{\frac{\tau_S}{\tau_\varepsilon}}\right)\right)(w_L + 2\lambda) = 0, \tag{24}$$

$$\lambda > -\frac{w_L}{2}. \tag{25}$$

The idea behind the proof is as follows. Equation (23) gives the explicit expression for λ as a function of δ. I then combine (23) and (24) to obtain an explicit expression for $\delta(\lambda)$. Finally, I determine how many times the two functions intersect.

I derive an explicit expression for δ through λ by using (24) to write $\delta(\delta - (\rho/\sqrt{1-\rho^2})\sqrt{\tau_S/\tau_\varepsilon})$ as a function of λ and δ, which I can substitute into (23); I then derive δ from the resulting expression. Following these steps yields

$$\delta = \frac{(2\lambda + w_L)(N\rho w_S(\tau_S+\tau_\varepsilon)\sqrt{\tau_S/\tau_\varepsilon} + \sqrt{\tau_L}\sqrt{\tau_\varepsilon}(\lambda + w_L))}{N\sqrt{1-\rho^2}(\lambda\tau_\varepsilon(\lambda + Nw_S + w_L) + \tau_S w_S(2\lambda + w_L))}. \tag{26}$$

One can show that this function is decreasing for large enough N or τ_S. The function $\lambda(\delta)$ given by (23) increases with δ for $\delta > \rho/\sqrt{1-\rho^2})\sqrt{\tau_S/\tau_\varepsilon}$. So for such δ, the functions $\lambda(\delta)$ and $\delta(\lambda)$ intersect at most once. There is no solution to the system with $\delta \le (\rho/\sqrt{1-\rho^2})\sqrt{\tau_S/\tau_\varepsilon}$ because, in that case, $\lambda(\delta) < -w_L$ and so (25) does not hold. ∎

B.5 Proof of Proposition 4

Proof of Proposition 4. Recall that information efficiency can be written as $\mathcal{I} = \frac{\tau_S+\tau_\varepsilon(1+\delta^2)}{\tau_S}$. Equation (23) expresses λ as an increasing function of δ for $\lambda(\delta) > -w_L/2$. One can therefore express δ either as an increasing function of λ or a as a decreasing function of $\mathcal{L}$ from (23). Call this function $\delta(\mathcal{L})$. Then $\mathcal{I} = \frac{\tau_S+\tau_\varepsilon(1+\delta(\mathcal{L})^2)}{\tau_S}$. It follows immediately that $\frac{\partial\mathcal{I}}{\partial\tau_\varepsilon} > 0$ and $\frac{\partial\mathcal{I}}{\partial\mathcal{L}} > 0$.

It remains to prove that $d\mathcal{L}/d\tau_\varepsilon > 0$ for $\rho > 1/\sqrt{2}$. Apply Lemma 3 (to follow) and write

$$\frac{1}{\lambda} = (N-1)\gamma(\lambda) + \gamma_S(\lambda;\tau_\varepsilon).$$

Let

$$z(\lambda;\tau_\varepsilon) \equiv \frac{1}{\tau_\varepsilon} t(\lambda;\tau_\varepsilon)\left(t(\lambda;\tau_\varepsilon) - \frac{B}{C}\right)$$
$$= \frac{1}{\tau_\varepsilon} t(\lambda;\tau_\varepsilon)\frac{\beta_S(\lambda;\tau_\varepsilon)}{N\gamma(\lambda)C},$$

where the equality follows from re-arranging (29). Using this notation, we can write

$$\frac{1}{\lambda} = \frac{\gamma(\lambda)Nw_S + 1}{w_S z(\lambda;\tau_\varepsilon) + w_S} - \gamma(\lambda). \tag{27}$$

Now use $1/\Lambda(\lambda,\tau_\varepsilon)$ to denote the right-hand side of equation (27). I am interested in $\frac{\partial\Lambda}{\partial\tau_\varepsilon}$. The only term that depends directly on τ_ε is $z(\lambda;\tau_\varepsilon)$. Substituting (29) into (27) and then (implicitly) differentiating (28), we obtain

$$\frac{\partial}{\partial\tau_\varepsilon}\left(\frac{t(\lambda;\tau_\varepsilon)\beta_S(\lambda;\tau_\varepsilon)}{\tau_\varepsilon}\right)$$
$$= \frac{-\beta_S(\lambda;\tau_\varepsilon)(\gamma(\lambda)^2N^2\beta_S(\lambda;\tau_\varepsilon)(B^2+C^2(\tau_\varepsilon-\tau_S)) + 2BN\gamma(\lambda)\beta_S(\lambda;\tau_\varepsilon)^2 + BC^2\gamma(\lambda)^3N^3\tau_\varepsilon + \beta_S(\lambda;\tau_\varepsilon)^3)}{CN\gamma(\lambda)\tau_\varepsilon^2(\gamma(\lambda)^2N^2(B^2+C^2(\tau_S+\tau_\varepsilon)) + 4BN\gamma(\lambda)\beta_S(\tau_\varepsilon) + 3\beta_S(\tau_\varepsilon)^2)}.$$

This expression is negative provided that $B^2 + C^2(\tau_\varepsilon - \tau_S) > 0$, which is equivalent to

$$\tau_\varepsilon > \frac{1-2\rho^2}{1-\rho^2}\tau_S.$$

So if this inequality holds, then $\frac{\partial\Lambda}{\partial\tau_\varepsilon} < 0$. Hence an increase in τ_ε shifts the function $\Lambda(\lambda,\tau_\varepsilon)$ upward, and its new intersection with a 45-degree line will be shifted upward as well. Note that $\Lambda(\lambda)$ intersects the 45-degree line from above because $\Lambda(0,\tau_\varepsilon) > 0$ and we assume a unique equilibrium and so the unique intersection cannot be from below. ■

Lemma 3. *Let* $B = \rho\sqrt{\tau_S/\tau_L}$ *and* $C = \sqrt{(1-\rho^2)/\tau_L}$, *and let* $\beta_S(\lambda)$ *be the unique* β_S *that solves*

$$\frac{1}{\beta_S} = w_S\left(\frac{\tau_S+\tau_\varepsilon}{\tau_\varepsilon} + \frac{1}{\tau_\varepsilon C^2}\left(B + \frac{\beta_S}{N(1/(w_L+\lambda))}\right)^2\right). \tag{28}$$

Then equilibrium precision τ_π *can be expressed as*

$$\sqrt{\tau_\pi} \equiv t(\lambda) = \frac{1}{C}\left(B + \frac{\beta_S(\lambda)}{N(1/(w_L+\lambda))}\right) \tag{29}$$

and the price elasticity γ_S *can be written as*

$$\gamma_S(\lambda) = \frac{\tau_\varepsilon - w_S N t(\lambda)(t(\lambda) - B/C)(w_L+\lambda)^{-1}}{\tau_\varepsilon w_S + w_S\cdot t(\lambda)(t(\lambda)-B/C)}. \tag{30}$$

The equilibrium λ solves

$$\frac{1}{\lambda} = \gamma_S(\lambda) + \frac{N-1}{w_L+\lambda}. \tag{31}$$

Proof of Lemma 3. Equation (28) follows if we combine

$$\frac{1}{\beta_S} = w_S\frac{\tau_S+\tau_\pi+\tau_\varepsilon}{\tau_\varepsilon}, \quad \sqrt{\tau_\pi} = \frac{1}{C}\left(B + \frac{\beta_S(\lambda)}{N\beta}\right), \quad \text{and} \quad \beta = \frac{1}{w_L+\lambda}.$$

After some re-arrangement, equation (29) follows directly from (21). Finally, some algebra allows one to derive equation (30) from (22). ■

B.6 Proof of Proposition 5

Proof of Proposition 5. The idea behind this proof is to consider the limiting equilibrium when $\tau_S = 0$ and $\tau_\varepsilon \to 0$.

Let $x \equiv \sqrt{\tau_\pi}$ and write $x = x_0 + x_1\tau_\varepsilon + o(\tau_\varepsilon)$. Let $y \equiv \lambda\tau_\varepsilon$ and write $y = (Nw_S/\sqrt{\tau_L})(x\sqrt{1-\rho^2} - \rho\sqrt{\tau_S})(x^2+\tau_S+\tau_\varepsilon) - w_L\tau_\varepsilon$. Substituting $\tau_S = 0$ and these expressions for x and y into (7) and then collecting zero- and first-order terms in τ_ε, we have

$$x_0 = \frac{2}{N}\sqrt{\frac{\tau_L}{1-\rho^2}} \quad \text{and} \quad x_1 = \sqrt{\frac{1-\rho^2}{\tau_L}}N\frac{w_L - 2Nw_S}{8w_S}.$$

Because $\tau_\pi = x^2 = x_0^2 + 2x_0x_1\tau_\varepsilon + o(\tau_\varepsilon)$, the desired result holds if $2x_0x_1 < -1$. It is easy to check that this inequality holds for $(N/w_L)/(1/w_S) > 1/2$ and sufficiently low τ_L. Thus, I have shown that $\mathcal{I}$ is decreasing in τ_ε at both $\tau_\varepsilon = 0$ and $\tau_S = 0$. From the continuity of $\tau_\pi'(\tau_\varepsilon, \tau_S)$ at $(0,0)$, it follows that $\mathcal{I}$ is decreasing in τ_ε for sufficiently small τ_S and τ_ε. ■

B.7 Proof of Proposition 6

Proof of Proposition 6. The equilibrium is a solution to the system (23)–(25), which can be written as follows:

$$\lambda = L(\delta; N) \equiv \frac{Nw_S}{\varkappa}(\delta-\phi)(\theta+\delta^2) - w_L; \tag{32}$$

$$\delta = D(\lambda; N) \equiv h\left(\frac{\lambda(w_L+Nw_S+\lambda)}{w_S(w_L+2\lambda)}\right). \tag{33}$$

Here

$$\varkappa \equiv \sqrt{\frac{\tau_L/\tau_\varepsilon}{1-\rho^2}}, \quad \phi \equiv \frac{\rho}{\sqrt{1-\rho^2}}\sqrt{\frac{\tau_S}{\tau_\varepsilon}}, \quad \theta \equiv \frac{\tau_S+\tau_\varepsilon}{\tau_\varepsilon},$$

and $\delta = h(x)$ is the inverse of $1 + \delta(\delta - \phi)$.

Lemma 4 (to follow) implies that $\lambda > 0$ in equilibrium. Yet because that inequality is not possible when $\delta < \phi$, we may look for the curves ($L(\delta; N)$ and $D(\lambda; N)$) to intersect in the region where $\delta > \phi$ and $\lambda > 0$.

Since the function $1+\delta(\delta-\phi)$ is strictly increasing for $\delta > \phi$, it follows that the function $h(x)$ is both well-defined and strictly increasing. The equilibrium is therefore the intersection of the curves $\lambda = L(\delta; N)$ and $\delta = D(\lambda; N)$. Moreover, it is easy to see that $\frac{\partial L}{\partial \delta} > 0$ and $\frac{\partial D}{\partial \lambda} > 0$ for $\delta > \phi$, so both curves are strictly upward sloping for a given N. We next compute

$$\frac{\partial L}{\partial N} = \frac{w_S(\delta^2 + \theta)(\delta - \phi)}{\varkappa} - w_L'(N),$$

which is positive if w_L does not depend on N or if $w_L = w_1 N$ (in the second case, $\frac{\partial L}{\partial N} = \frac{\lambda}{N} > 0$).

Analogously, we compute

$$\frac{\partial D}{\partial N} = h'(\cdot) \times \begin{cases} \frac{\lambda}{2\lambda + w_L} & \text{if } w_L \text{ does not depend on } N, \\ \frac{\lambda^2(w_1 + 2w_S)}{w_S(2\lambda + N w_1)^2} & \text{if } w_L = w_1 N. \end{cases}$$

This expression is positive.

Hence an infinitesimal increase in N shifts the curve $L(\delta; N)$ upward and the curve $D(\lambda; N)$ rightward. Their new intersection will therefore be below and to the left of the old one.[26] Thus we have

$$\frac{d\lambda}{dN} < 0 \quad \text{and} \quad \frac{d\delta}{dN} < 0.$$

Since $\mathcal{I} = \frac{\tau_S + \tau_\varepsilon(1+\delta^2)}{\tau_S}$ is increasing in δ and does not depend directly on N, and since $\mathcal{L}$ is inversely related to λ, it follows that

$$\frac{d\mathcal{I}}{dN} < 0 \quad \text{and} \quad \frac{d\mathcal{L}}{dN} > 0.$$

■

Lemma 4. *The equilibrium price impact λ is positive.*

Proof. Rewrite (7) as

$$\lambda = \frac{w_S(1 + \delta(\delta - \phi))(w_L + 2\lambda)}{w_L + N w_S + \lambda}.$$

Then $\delta > \phi$, because otherwise $\lambda < -w_L$ and the second-order condition $2\lambda + w_L > 0$ would not hold. Therefore, $1 + \delta(\delta - \phi) > 0$. Other terms in the equality just displayed are positive

[26] The curve $\lambda = L(\delta; N)$ must intersect the curve $\delta = D(\lambda; N)$ from below because, for $\lambda = 0$, the curve $\lambda = L(\delta; N)$ is to the right of the curve $\delta = D(\lambda; N)$.

owing to the second-order condition $w_L + 2\lambda > 0$. ■

B.8 Proof of Proposition 7

Proof of Proposition 7. First, we write realised total welfare as

$$\begin{aligned}
\mathrm{RTW} &\equiv v_S \int_0^1 x_j\, dj - \frac{w_S}{2}\int_0^1 (x_j)^2\, dj + v_L \sum_{i=1}^N x_i - \frac{w_L}{2}\sum_{i=1}^N x_i^2 \\
&= (v_S - v_L)\bar{x}_S - \frac{w_S}{2}\int_0^1 (x_j)^2\, dj - \frac{w_L}{2N}\sum_{i=1}^N (\bar{x}_S)^2.
\end{aligned}$$

Note that $\int_0^1 (x_j)^2\, dj = \int_0^1 (x_j - X)^2\, dj + X^2$ and $v_S - v_L = x_S^{\mathrm{FB}}(w_S + w_L/N)$; hence, after applying expectations and some re-arranging, the preceding expression for RTW transforms to (8).

Given the aggregate demands of large and small traders, the equilibrium price can be expressed as $p = \bar{v}_S - w_S\bar{x}_S = v_L + w_L\bar{x}_S/N$. From this equality it follows that $\bar{x}_S = (\bar{v}_S - v_L)/(w_S + w_L/N)$, which (after some re-arrangement) becomes equation (9). Equation (10) now follows directly from (19). ■

B.9 Proof of Proposition 8

Proof of Proposition 8. The decomposition follows by substituting (9) into (10). The comparative statics of WL_1 and WL_3 follow because ψ increases with N whereas β_S is decreasing in N (as follows from Proposition 6). For the comparative statics of WL_2, note that $E[(\bar{v}_S - v_S)^2] = 1/\tau - \tau_\varepsilon/\tau^2$; this equality is a decreasing function of τ (which, in turn, decreases with N) for $1/\tau > 1/2\tau_\varepsilon$. Thus $\mathrm{WL}_2 = \frac{\psi^2 E[(v_S - \bar{v}_S)^2]}{2(w_S + w_L/N)}$ increases with N for $\mathrm{Var}(v_S|s_j, p)^{-1} = \tau_S + \tau_\varepsilon + \tau_\pi > 2\tau_\varepsilon$. Clearly, the last inequality holds if $\tau_\varepsilon < \tau_S$. ■

B.10 Proof of Proposition 9

Proof of Proposition 9. Let

$$\theta \equiv \frac{\tau_S + \tau_\varepsilon}{\tau_\varepsilon} > 1, \quad \xi \equiv \rho\sqrt{\frac{\tau_S}{\tau_L}}, \quad \varkappa \equiv \sqrt{\frac{\tau_L/\tau_\varepsilon}{1-\rho^2}} > 0,$$

$$\psi \equiv \frac{w_L}{N w_S} > 0, \quad \phi \equiv \varkappa\xi = \frac{\rho}{\sqrt{1-\rho^2}}\sqrt{\frac{\tau_S}{\tau_\varepsilon}},$$

$$Q \equiv -4N\xi + 8\xi + 4\psi, \quad T \equiv 16N^2\xi\psi\left(\xi - \frac{2}{N}\right)(\psi+2),$$

$$l^{\pm} \equiv \frac{-G \pm \sqrt{G^2+F}}{2}, \quad G \equiv 1 + \frac{2(\theta-2)}{N} > 1, \quad F \equiv 2\psi + \psi^2 > 0.$$

Assume that the following inequalities hold:

$$Q < 0, \quad \xi < \frac{1}{N}, \quad Q^2 + T > 0, \quad \psi < 1, \quad N > 4. \tag{34}$$

Also, let $l \equiv \frac{2\lambda + w_L}{2N w_S} > 0$. Then (26) can be rewritten as

$$\delta = \delta(l) \equiv \frac{2\varkappa}{N}\frac{l(l+\theta\xi+\psi/2)}{(l-l^+)(l-l^-)}, \tag{35}$$

and the equilibrium is the solution to the system consisting of (35) and

$$l = l(\delta) \equiv \frac{(\delta^2+\theta)(\delta-\phi)}{\varkappa} - \frac{\psi}{2}.$$

Consider all solutions to the equation

$$\delta(l) = \phi. \tag{36}$$

If the conditions (34) hold then there exist two solutions to (36), which are given by

$$L^{\pm} = \frac{-Q \pm \sqrt{Q^2+T}}{8N(2/N-\xi)}.$$

Furthermore, both solutions $L^{\pm} > l^{+}$.[27] The existence of two solutions to (26) implies that the function $\delta(l)$ attains a local minimum in the region $l > l^+$ and that this minimum is less than ϕ.

[27] It is easy to see that both solutions are positive. However, $\delta(L) = \phi > 0$ is positive only if $L > l^+$.

Also consider all solutions to

$$\delta(l) = \frac{\varkappa}{N}.$$

There are two solutions to this equation, as well—provided that (34) holds. Let L_m denote the maximal solution. Then

$$L_m = \frac{1}{2}(Q_m + \sqrt{Q_m^2 + T_m}) > L^+,$$

where

$$Q_m \equiv \frac{2(\theta-1)}{N} + 1 - \frac{2\theta\phi}{\varkappa} - \psi \quad \text{and} \quad T_m \equiv -(\psi^2 + 2\psi).$$

If

$$L_m < l\left(\frac{\varkappa}{N}\right) = \frac{(\varkappa^2 + \theta N^2)(\varkappa - N\phi)}{\varkappa N^3} - \frac{\psi}{2} \equiv l_m, \tag{37}$$

then there are at least three equilibria.

The condition $Q < 0$ is equivalent to

$$\xi > \frac{\psi}{N-2}. \tag{38}$$

The condition $Q^2 + T > 0$ holds as long as[28]

$$\xi > \frac{2\psi(N(\psi+3)-2)}{N(N(\psi+1)^2-4)+4} \quad \text{and} \quad N(N(\psi+1)^2-4)+4 > 0. \tag{39}$$

Given (34), the second inequality in (39) holds. Note that

$$\frac{2\psi(N(\psi+3)-2)}{N\left(N(\psi+1)^2-4\right)+4} < \frac{8\psi}{N-4} > \frac{\psi}{N-2}.$$

Therefore, both (38) and (39) hold if also the following weaker condition does:

$$\xi > \underline{\xi}_1 \equiv \frac{8\psi}{N-4}.$$

The preceding expression can be written as

$$\tau_L < \frac{\rho^2 \tau_S}{\underline{\xi}_1^2} \equiv \bar{\tau}_2. \tag{40}$$

[28]Indeed,

$$Q^2 + T = 16\xi^2(N(N(\psi+1)^2-4)+4) - 32\xi\psi(N(\psi+3)-2) + 16\psi^2.$$

Condition (39) ensures that the first two terms are positive.

Now suppose that

$$l_m - Q_m > 0.$$

Then (37) holds.[29] The inequality just displayed can be written as

$$\left(\frac{\varkappa^2}{N^2} - \theta\right)\left(\frac{1}{N} - \xi\right) > 1 - \frac{2}{N} - \frac{\psi}{2}.$$

Assume that

$$\xi < \frac{1}{2N}.$$

Then ξ is greater than $(\varkappa^2/N^2 - \theta)(1/2N)$, and the constraint holds provided that

$$\frac{\varkappa^2}{N^2} - \theta > 2N - 4 - N\psi;$$

this inequality is equivalent to

$$\tau_L > (1-\rho^2)\tau_\varepsilon N^2(2N - 4 - N\psi + \theta).$$

The preceding expression holds if also the following stricter inequality holds:

$$\tau_L > (1-\rho^2)\tau_\varepsilon N^2(2N - 4 + \theta).$$

The constraint $\xi < 1/2N$ implies that

$$\tau_L > 4N^2\rho^2\tau_S.$$

In turn, those two constraints hold provided that

$$\tau_L > \underline{\tau}_2 \equiv \max\{4N^2\rho^2\tau_S, (1-\rho^2)\tau_\varepsilon N^2(2N - 4 + \theta)\}. \tag{41}$$

It is clear that

$$\underline{\tau}_2 > 4N^2\rho^2\tau_S > \bar{\tau}_1.$$

[29] The expression (37) is equivalent to

$$Q_m^2 + T_m - (2l_m - Q_m)^2 = 2l_m(2Q_m - 2l_m) + T_m < 0,$$

which is true.

The final step is to derive the conditions under which $\underline{\tau}_2 < \bar{\tau}_2$. We have

$$\sqrt{\underline{\tau}_2} < \frac{\rho\sqrt{\tau_S}}{\underline{\xi}_1} = \frac{\rho\sqrt{\tau_S}}{8\psi}(N-4),$$

which is equivalent to

$$w_L < \bar{w} \equiv w_S \rho \frac{N(N-4)}{8}\sqrt{\frac{\tau_S}{\underline{\tau}_2}}. \tag{42}$$

■

B.11 Proof of Proposition 10

Proof of Proposition 10. The proposition follows by noting that both $\mathcal{I} = \frac{\tau_S+\tau_\varepsilon(1+\delta^2)}{\tau_S}$ and λ (as given by (32)) are increasing functions of δ, which is a decreasing function of N (see the proof of Proposition 6). ■

B.12 Proof of Lemma 1

Proof of Lemma 1. Lemma 2 allows us to write $v_L = A + Bv_S + C\zeta$, where $\zeta \sim N(0,1)$ is independent of v_S. Moreover, from Proposition 1 it follows that $\zeta = (\pi - v_s)\sqrt{\tau_\pi}$. Hence

$$\mathrm{Var}(f|p) = \mathrm{Var}(\eta + k_L A + v_S(k_S + k_L(B - C\sqrt{\tau_\pi})) + k_L C a\sqrt{\tau_\pi}\pi|\pi),$$

which (after some algebra) can be re-arranged to yield (16). The monotonicity of $\mathcal{I}_\mathcal{F}$ in τ_π then follows immediately from (16). Condition (18) ensures that upper bound on τ_π (established in Lemma 5) is below the right-hand side of (17). ■

Lemma 5. *The inequality $\delta < \bar{\delta}$ holds in equilibrium, where $\bar{\delta}$ is given by (43).*

Proof. From the definition of δ it follows that

$$\delta \equiv \varkappa\left(\frac{\beta_S}{N\beta} + \xi\right) = \varkappa\left(\frac{\beta_S}{N}(w_L + \lambda) + \xi\right) < \varkappa\left(\frac{1}{Nw_S}(w_L + \lambda) + \xi\right),$$

where I use the same notation as in the proof of Proposition 9. We next find an upper bound for λ. Start by writing

$$\frac{1}{\lambda} = \Gamma - \gamma < \Gamma < \frac{1}{w_S} + \frac{N}{w_L + \lambda}.$$

Since $\lambda > -w_L/2$, it follows that $N/(w_L + \lambda) < 2N/w_L$ and

$$\lambda < \left(\frac{1}{w_S} + \frac{2N}{w_L}\right)^{-1} = \frac{w_L w_S}{w_L + 2Nw_S}.$$

Thus we obtain the following expression for $\bar{\delta}$:

$$\bar{\delta} = \frac{\varkappa}{Nw_S}\left(w_L + \frac{w_L w_S}{w_L + 2Nw_S}\right) + \phi. \tag{43}$$

■

B.13 Proof of Proposition 1.A.2

Proof of Proposition 1.A.2. The price is informationally equivalent to $\beta_S v_S + N\beta v_L + \sum_{i=1}^{N} n_i$. After substituting v_L from Lemma 2 and then re-arranging, we find that the price is informationally equivalent to $\pi \equiv v_S + (1/\sqrt{\tau_\pi})\zeta_u$, where

$$\tau_\pi \equiv \mathrm{Var}[\pi|v_S]^{-1} = \left(\left(\frac{\tau_L}{1-\rho^2}\left(\rho\sqrt{\frac{\tau_S}{\tau_L}} + \frac{\beta_S}{N\beta}\right)^2\right)^{-1} + \frac{N\beta^2}{\tau_n}\right)^{-1}. \tag{44}$$

The formula for informational efficiency now follows directly from the projection theorem. We can see from the displayed formula that τ_π and hence $\mathcal{I}$ decrease as β increases.

The optimal demand of a small trader j can be written as $x_j = \frac{E[v_s|s_j,p]-p}{w_S}$. Then $\gamma_S = \frac{1}{w_S} - \frac{\partial E[v_s|s_j,p]}{\partial p}$. Now we write $E[v_s|s_j,p] = \frac{\tau_\pi}{\tau_S+\tau_\varepsilon+\tau_\pi}\pi + ...$; here, as before, "..." stands for terms that do not depend on p. One can write $\pi = \frac{\gamma_S+N\gamma}{\beta_S+N\beta\rho\sqrt{\tau_S/\tau_L}}p + ...$, from which (after some re-arrangement) it follows that

$$\gamma_S = \frac{\frac{1}{w_S} - \frac{\tau_\pi}{\tau_S+\tau_\varepsilon+\tau_\pi}\frac{N\gamma}{\beta_S+N\beta\rho\sqrt{\tau_S/\tau_L}}}{1 + \frac{\tau_\pi}{\tau_S+\tau_\varepsilon+\tau_\pi}\frac{1}{\beta_S+N\beta\rho\sqrt{\tau_S/\tau_L}}}.$$

It can be seen from this expression that γ_S decreases in β. ■

B.14 Proof of Proposition 2.A.2

Proof of Proposition 2.A.2. Let $x = \beta_S/\beta$ and $k = \beta_S(\rho\sqrt{\tau_L/\tau_S} + (N-1)/x)$. We can then write

$$\beta = \frac{\frac{\tau_n}{\tau_\iota+\tau_L+\tau_n}}{(\tau_\iota+\tau_L+\tau_n)(\frac{\tau_\iota}{k(\tau_\iota+\tau_L+\tau_n)} + \lambda + w_L)},$$

where τ_ι is the precision of the price from the perspective of large traders; this precision is independent of γ_S. Therefore β depends on γ_S only through λ. For the price sensitivity of large traders' demands, we can write

$$\gamma = \frac{1 - \frac{\tau_\iota}{\lambda k(\tau_\iota+\tau_L+\tau_n)}}{\frac{\tau_\iota}{k(\tau_\iota+\tau_L+\tau_n)} + \lambda + w_L}.$$

Together with $1/\lambda = (N-1)\gamma + \gamma_S$ the displayed equality implies that

$$1 - \gamma_S \lambda = (N-1) \frac{\lambda - \frac{\tau_\iota}{k(\tau_\iota + \tau_L + \tau_n)}}{\frac{\tau_\iota}{k(\tau_\iota + \tau_L + \tau_n)} + \lambda + w_L},$$

from which we can see that an increase in γ_S leads to a decrease in λ. ∎

B.15 Proof of Theorem 1.A.2

Proof of Theorem 1.A.2. Following the steps of Propositions 1.A.2 and 2.A.2, we can write

$$\tau_\pi = \left(\left(\frac{\tau_L}{1-\rho^2} \left(\rho \sqrt{\frac{\tau_S}{\tau_L}} + \frac{x}{N} \right)^2 \right)^{-1} + \frac{N x^2 \beta_S^2}{\tau_n} \right)^{-1}$$

and

$$\beta_S = \frac{\tau_\varepsilon}{w_S(\tau_\pi + \tau_S + \tau_\varepsilon)}.$$

These two equalities allow one to express β_S through x in closed form. The elasticity can be written as

$$\gamma_S = \frac{1}{w_S} \left(1 - \frac{\tau_\pi}{\tau_S + \tau_\varepsilon + \tau_\pi} \frac{\gamma + 1/\lambda}{\beta_S(x)(1 + (N/x)\rho\sqrt{\tau_S/\tau_L})} \right),$$

which depends on x, λ, and γ. I now provide the following closed-form expression for γ as a function of λ and x:

$$\gamma = \frac{1 - \frac{\tau_\iota}{\lambda k(\tau_\iota + \tau_L + \tau_n)}}{\frac{\tau_\iota}{k(\tau_\iota + \tau_L + \tau_n)} + \lambda + w_L}.$$

Then β is given by

$$\beta = \frac{\frac{\tau_n}{\tau_\iota + \tau_L + \tau_n}}{(\tau_\iota + \tau_L + \tau_n)(\frac{\tau_\iota}{k(\tau_\iota + \tau_L + \tau_n)} + \lambda + w_L)}$$

and τ_ι can be expressed as

$$\tau_\iota = \left((N-1) \left(\frac{1}{N - 1 + x\rho\sqrt{\tau_L/\tau_S}} \right)^2 \frac{1}{\tau_n} + \frac{\beta_S(x)^2(1-\rho^2)}{\tau_S k^2} \right)^{-1}.$$

Finally, the equilibrium values of x and λ solve

$$x = \frac{\beta_S}{\beta} \quad \text{and} \quad \frac{1}{\lambda} = \gamma_S + (N-1)\gamma,$$

respectively. ∎

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