Capital Market Equilibrium with Competition among Institutional Investors

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We develop a dynamic general equilibrium model to study how competition among institutional investors affects the stock market characteristics—level, expected return, and volatility. We consider an economy in which multiple fund managers strategically interact with each other, as each manager tries to increase her performance relative to the others. We fully characterize an equilibrium in this economy, and find that a more intense competition is associated with a higher level of the market, lower expected market return, while market volatility is not affected by competition. These findings are broadly consistent with the data.

JEL: G12, G29

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I. Introduction

This paper studies the stock market implications of competition among institutional investors such as mutual and hedge funds.[1] The money management industry has been growing at an impressive rate over the past decades, and as a result institutional investors play a major role in modern financial markets. Hence, any mechanism affecting their behavior is likely to manifest itself in the stock market as a whole. In this paper, we focus on the mechanism that has attracted a great deal of attention in the literature—competition for relative performance with respect to other managers. The reason is that by outperforming her peers a manager increases money flows to her fund (Chevalier and Ellison 1997; Sirri and Tufano 1998; and Del Guercio and Tkac 2002), and hence increases her fees which depend positively on assets under management.

How competition affects fund managers' investment decisions has been studied extensively both theoretically and empirically.[2] However, what appears to be a natural next step—examining general equilibrium implications of competition—remains by and large unexplored in theoretical works.

We aim to fill this gap in the literature. We address a simple, yet fundamental question: How competition among institutional investors affects the level of stock market, expected market return and market volatility. To investigate this question

[1]The terms "institutional investor," "fund manager," and "manager" are used interchangeably throughout the paper.

[2]Browne (2000a), Taylor (2003), Palomino (2005), and Basak and Makarov (forthcoming), among others, study theoretically how competition affects investment decisions of fund managers. Empirically, this question is examined in Brown, Harlow, and Stark (1996), Busse (2001), Qiu (2003), Goriaev, Nijman, and Werker (2005), Kempf and Ruenzi (2008), and Chen and Pennacchi (2009).

in the clearest way, we develop a stylized model of competition which abstracts from other factors which potentially affect managers' behavior. We consider a standard dynamic general equilibrium economy populated by multiple risk-averse fund managers. Each fund manager cares about both her own wealth and relative wealth with respect to the other managers, giving rise to a competition for relative performance. The weight that managers attach to relative wealth determines the competition intensity zero weight corresponds to the base case of no competition, and the higher the weight the more intense is the competition. In reality, the competition intensity is likely to be determined by the sensitivity of the flow-performance relation—the higher the sensitivity, the more each manager benefits from improving her relative performance, and hence the higher is the intensity of competition.

We solve for an equilibrium in closed-form, and provide analytical expressions for all variables of interest. Our key finding is that a higher competition intensity is associated with a higher level of the market and a lower expected market return, while market volatility is unaffected.

Similarly to other stylized models, our work does not have an ambition of being able to explain numerous observed regularities. Rather, it should be viewed as a first step towards a more encompassing analysis. That being said, we can still look at whether our predictions are broadly consistent with the data. Fant and O'Neal (2000) and Huang, Wei, and Yan (2007) look at the flow-performance sensitivity at different time periods and find that the sensitivity at later periods is higher than at earlier periods. Given the aforementioned positive link between flow sensitivity and competition intensity, this finding suggests that competition intensity has been increasing over time, and so our model says that we should observe a declining market premium while there should be no trend in market volatility. These predictions are broadly consistent with the observed trends in the equity premium and market volatility (the evidence is discussed at the end of Section 3).

A. Related literature

There is substantial literature studying equilibrium asset prices in the presence of institutional investors. However, the bulk of this literature disregards competition and instead focuses on benchmarking, i.e., a situation where a manager's performance is benchmarked to some index, implying that her compensation depends on the relative performance with respect to the index. Examples are Gomez and Zapatero (2003), Cuoco and Kaniel (2011), Basak and Pavlova (2011), Brennan, Cheng, and Li (2012), and Leippold and Rohner (forthcoming). Carpenter (2000) and van Binsbergen, Brandt, and Koijen (2007) examine portfolio choice implications of benchmarking.

An exception is Kapur and Timmermann (2005) who, like us, study competition among managers but in a static setting with mean-variance preferences. In their model, it is only under certain conditions that an increase in relative performance evaluation leads to a lower equity premium, whereas in our model this effect holds in general. Moreover, due to assuming a static setting, they do not look at how market volatility is affected by relative concerns.

It is worth noting that there is an important difference between competition and benchmarking in terms of the underlying economic mechanisms. Under competition, the interaction between managers is of a strategic nature—a manager recognizes that the other managers do not follow some pre-determined investment rules, but rather respond strategically to others' investment behavior. Extant empirical evidence reveals that strategic motives are indeed prevalent among money managers (see subsection 2.2 in Basak and Makarov (forthcoming) for a literature review). Under benchmarking, on the other hand, strategic interactions are not present because the index does not "respond" to managers' actions. Also, it is worth noting that competition is likely to have a larger impact on the stock market than benchmarking because virtually all managers have incentives to outperform their peers so as to increase money flows, whereas the fraction of managers with benchmarking concerns is relatively small. As stated in Cuoco and Kaniel (2011, p. 265), only "9% of all U.S. mutual funds used [as of 2004] performance-based fees," where "performance-based fee" is what we call "benchmarking."

While we focus on relative concerns when modeling professional money managers, other works focus on skill and ability as defining characteristics of money managers (Berk and Green 2004; and Petajisto 2009). We should also note that a central role of relative wealth concerns has been established in areas other than money management. A notable example is the literature on "catching-" and "keeping-up-with-the-Joneses" in which small investors care about how their consumption compares to past or contemporary consumption of their peers (Abel 1990; Gali 1994; Campbell and Cochrane 1999; Chan and Kogan 2002; Lauterbach and Reisman 2004; and

Gomez, Priestley, and Zapatero 2009, among many others). Other examples include works on excessive investments and financial bubbles (DeMarzo, Kaniel, and Kremer 2007, 2008), information acquisition (Garcia and Strobl 2011), consumption and stock volatility (Bakshi and Chen 1996), corporate investment distortions (Goel and Thakor 2005).

II. Economic Setting

To better clarify which features of our model are standard and which are novel, we lay out our economic setting in two parts. Subsection II.A presents standard assumptions often used in related continuous-time models, and subsection II.B focuses on the novel feature of our analysis—competition.

A. Basic Set-up

We consider a standard continuous-time economy with a finite horizon $[0, T]$. The uncertainty is driven by a Brownian motion ω. The investment opportunities are given by a risk free bond and a risky stock representing the stock market. The bond return is normalized to zero without loss of generality. The stock market represents a claim to the terminal dividend D_T to be received at time T. We assume that D_T is determined as time-T value of a dynamic dividend process D_t, where D_t follows a geometric Brownian motion

$$dD_t = D_t \mu_D dt + D_t \sigma_D d\omega_t, \tag{1}$$

where the dividend mean growth rate, $\mu_D > 0$, and volatility, $\sigma_D > 0$, are constant.

The stock market level, S, follows the process

$$dS_t = S_t \mu_{S,t} dt + S_t \sigma_{S,t} d\omega_t, \tag{2}$$

where the expected market return $\mu_{S,t}$ and volatility $\sigma_{S,t}$ are endogenous processes to be determined in equilibrium. Because the riskless return is normalized to zero, the terms "market return" and "market premium" denote the same thing, and so in what follows we use them interchangeably.

There are M fund managers in the economy. Each manager i is endowed with e_i units of the stock. The total supply of the stock is normalized to unity, $\sum_{i=1}^{M} e_i = 1$. Manager i chooses a dynamic investment strategy $\theta_{i,t}$, the fraction of wealth invested in the stock at time t. Manager i's wealth at time t, $W_{i,t}$, follows the process

$$dW_{i,t} = \theta_{i,t} W_{i,t} \mu_{S,t} dt + \theta_{i,t} W_{i,t} \sigma_{S,t} d\omega_t. \tag{3}$$

B. *Modeling Competition among Managers*

To model fund managers' objective function, we take into account two considerations. First, a manager has incentives to maximize the absolute return on their investment because this increases her assets under management, and hence her compensation. Second, it is also rational for a manager to care about her return relative to the peers, because the higher the relative return is, the more money the manager is likely to receive from retail investors who largely use relative performance when choosing fund, as documented empirically (Chevalier and Ellison 1997, Sirri and Tufano 1998). To capture these two features, we postulate that each manager i's utility function u_i is defined over the composite of own wealth and relative wealth:

$$u_i = \frac{1}{1-\gamma}\left(W_{i,T}^{1-\alpha}\left(\frac{W_{i,T}}{\overline{W}_{-i,T}}\right)^{\alpha}\right)^{1-\gamma}, \tag{4}$$

where $\overline{W}_{-i,T}$ is the geometric average of wealth of all managers excluding i:

$$\overline{W}_{-i,T} = \left(\prod_{j\neq i} W_{j,T}\right)^{\frac{1}{M-1}} \tag{5}$$

Utility (4) is similar to that in Browne (2000b) and van Binsbergen, Brandt, and Koijen (2008). These authors look at the case of relative concerns with respect to an exogenous benchmark, and so in their papers utility depends on the ratio of own performance to the performance of an index. To model competition, we replace the index in the denominator of this ratio by the (endogenous) average wealth of competitors. To verify that our results are not driven by the particular choice of utility (4), we have analyzed a version of our model with CARA utility and have formally shown that our main results do not change (the analysis is provided in Appendix A).

In specification (4), $\alpha \in [0,1]$ measures *competition intensity*, the degree to which managers care about outperforming the peers. While (4) can be also justified on behavioral grounds, our leading interpretation for relative performance concerns is fully rational—increasing relative return leads to higher money inflows, as discussed above. Accordingly, the value of α is determined by the sensitivity of the flow-performance relation (as formally shown in Basak and Makarov (forthcoming)). The more sensitive money flows are to performance, the higher is α. Parameter $\gamma > 0$ reflects relative risk aversion.[3] In what follows, we report our results under the condition $\gamma > 1$, consistent with the estimates for mutual fund managers (Koijen 2010) and also with what is typically assumed in calibrations of theoretical models.

The equilibrium in this economy is straightforwardly defined as follows. Taking the stock price characteristics S_t, $\mu_{S,t}$ and $\sigma_{S,t}$ as given, we compute managers' Nash equilibrium strategies: a collection of M trading strategies $(\theta_{1,t}, \ldots , \theta_{M,t})$ such that $\theta_{i,t}$ is manager i's best response to the other managers' strategies, i.e., $\theta_{i,t}$ yields the maximum of the expected utility (4) subject to the budget constraint (3). The equilibrium S_t, $\mu_{S,t}$ and $\sigma_{S,t}$ are such that markets clear after managers play the Nash game.

III. Equilibrium

The main focus of this paper is to examine how competition among fund managers affects the stock market expected return and volatility. However, to better understand the economic mechanisms behind these general equilibrium results, it is helpful to start with a partial equilibrium question: taking as given constant parameters μ_S and σ_S of the stock price dynamics (2), we determine what managers'

[3] In this paper, we want to isolate the effects of competition on economic variables of interest, and for this reason we do not introduce confounding features such as preference heterogeneity. Moreover, if we allowed parameters α and γ to differ across managers, the model would no longer be analytically tractable.

optimal portfolios are. We focus on the case of constant μ_S and σ_S because this is what happens in equilibrium (as established in Proposition 2). Proposition 1 reports managers' optimal portfolios. Here and throughout the paper, a variable with a hat $\hat{}$ denotes an equilibrium quantity in the economy with competition, $\alpha > 0$, while a variable with a superscript ^{B} ("Base case") – an equilibrium quantity in the base case of no competition, $\alpha = 0$.

PROPOSITION 1: *When expected return and volatility of the stock market, μ_S and σ_S, are constant, the optimal portfolios of fund managers are also constant and given by*

$$\hat{\theta}_i = \frac{1}{\gamma - \alpha(\gamma - 1)} \frac{\mu_S}{\sigma_S^2}, \quad i = 1, \ldots, M. \tag{6}$$

In the base case economy with no competition, $\alpha = 0$, the optimal portfolios are

$$\theta_i^B = \frac{1}{\gamma} \frac{\mu_S}{\sigma_S^2}, \; i = 1, \ldots, M. \tag{7}$$

Consequently, competition causes managers to increase the riskiness of their portfolios, $\hat{\theta}_i > \theta_i^B$.

The main result of Proposition 1 is that the presence of competition leads to a higher risk taking in equilibrium. To understand why, note from utility specification (4) that each manager cares about the composite of own wealth and relative wealth, and so, being risk averse, she seeks to minimize the variance of both these components. Importantly, manager i choose the base case portfolios θ_i^B in (7) not only when she has no relative concerns ($\alpha = 0$), but also when she cares about relative wealth but the other managers invest fully in the bond. Indeed, in this case the average wealth $\overline{W}_{-i,T}$ is constant, and so can be dropped from manager i's utility without affecting her behavior, leading her to choose the base case portfolio θ_i^B. If, however, the other managers invest a positive amount in the stock, manager i has incentives to increase her stock investment over the base case level so as to hedge against the increased volatility of the term $\overline{W}_{-i,T}$ in her utility. As a result, in the presence of competition all managers increase their stock investments relative to the no competition case.

We now turn to the general equilibrium implications of managers' competition.

PROPOSITION 2: *The equilibrium stock market level $\hat{S}_t$, expected market return $\hat{\mu}_S$, and market volatility $\hat{\sigma}_S$ are*

$$\hat{S}_t = D_t e^{\left(\mu_D + (\alpha(\gamma-1)-\gamma)\sigma_D^2\right)(T-t)},\ \hat{\mu}_S = (\gamma - \alpha(\gamma-1))\sigma_D^2,\ \hat{\sigma}_S = \sigma_D. \tag{8}$$

The corresponding base case values with no competition, $\alpha = 0$, are:

$$S_t^B = D_t e^{\left(\mu_D - \gamma\sigma_D^2\right)(T-t)},\ \mu_S^B = \gamma\sigma_D^2,\ \sigma_S^B = \sigma_D. \tag{9}$$

Consequently, a higher competition intensity is associated with a higher stock market level, a lower expected market return, and constant market volatility.

Proposition 2 reveals that the stock market level S_t increases with the competition intensity α. The reason is that a higher α increases the demand for the stock market, as discussed above, and so the market level increases. More interesting economic mechanisms are at play behind are the two other results of Proposition 2 concerning the market premium $\hat{\mu}_S$ and volatility $\hat{\sigma}_S$. As is well known, the market premium reflects the compensation for risk associated with holding the market. However, as discussed after Proposition 1, investing in the market allows managers to control the volatility of their relative wealth component of utility function, and in this respect the market plays an important role from the viewpoint of risk averse managers who value the ability to minimize the volatility of this component. The more managers care about relative wealth, the more valuable this ability is. Hence, a higher α is associated with a lower compensation for holding the market, implying a lower $\hat{\mu}_S$. That α has no effect on volatility $\hat{\sigma}_S$ follows the result that the market level S_t is proportional to the contemporaneous dividend D_t, where the coefficient of proportionality is deterministic, as seen from the the first equation in (8). Intuitively, the market is a claim to the future dividend payment D_T, and so the market level S_t is given by (appropriately discounted) time-t expectation of D_T, and so is proportional to D_t because the dividends follow a geometric Brownian motion. Hence, the market volatility equals the dividend volatility, regardless of how intense the competition is.

Another way to look at the results of Propositions 1 and 2 is as follows. From equation (6) for the stock weight, observe that a higher α increases the first fraction on the right-hand side of (6). Given that in equilibrium the stock weight has to, by market clearing, remain the same, the second fraction μ_S/σ_S^2 must decrease. Obviously, there are many ways in general to alter μ_S and σ_S so that to decrease μ_S/σ_S^2, and what Proposition 2 finds is that the actual way this happens in equilibrium is rather special—it is only the expected market return μ_S that changes in response to a change in competition intensity, while the market volatility σ_S does not change. Finally, we note that the equilibrium does not depend on the number of competing managers M.

A. Empirical Evidence

The presented model is fairly stylized, and as such it does not lend itself to comprehensive empirical testing. Despite this, we can still relate the model to the data by looking at whether the model's predictions are broadly consistent with some salient properties of the stock market. As established above, our model predicts that a higher competition intensity is associated with a lower expected market return, while the market volatility is unaffected. A natural empirical implication is as follows. Consider a financial market at two time periods—*past* and *present*—and suppose that the competition in the present is more intense that in the past. Then, our model predicts that the market premium in the present should be lower than in the past, while there should be no notable difference in the volatilities.[4]

This prediction is borne out in the data. First, to see that the premise of the above argument—that the competition is becoming more intense with time—is realistic, recall our earlier discussion in subsection II.B that competition intensity α reflects the sensitivity of the flow-performance relation. Fant and O'Neal (2000) and Huang,

[4]When talking about how the two variables are expected to change, we have in mind secular trends rather than short-term fluctuations.

Wei, and Yan (2007) divide their full sample period into sub-periods, and uncover that the sensitivity of flow-performance relation at later sub-periods is higher than at earlier ones, and so it is indeed reasonable to assume that α has been increasing with time. Consistent with our model, empirical evidence shows that the market premium has been decreasing (Blanchard, Shiller and Siegel 1993, Jagannathan, McGrattan and Scherbina 2000, Welch 2000, Claus and Thomas 2001, Fama and French 2002,), while market volatility does not seem to have a trend (Campbell, Lettau, Malkiel, and Xu 2001).

IV. Conclusion

We develop a stylized dynamic general equilibrium model to study how competition among fund managers affects the fundamental characteristics of the stock market: expected return, volatility, and level. We find that the higher the intensity of competition, the higher is the level of the market, the lower is expected market return, while market volatility is unaffected. These predictions are broadly consistent with the data.

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V. Mathematical Appendix

PROOF OF PROPOSITION 1:

Given that markets are complete, there exists a state price density process ξ_t given by

$$d\xi_t = -\xi_t \kappa d\omega_t, \tag{10}$$

where

$$\kappa \equiv \mu_S/\sigma_S \tag{11}$$

is the market price of risk. As is well-known (see, e.g., Duffie (2001)), the dynamic budget constraint (3) is equivalent to

$$E_t[\xi_T W_{i,T}] = \xi_t W_{i,t}. \tag{12}$$

The first-order condition for maximizing the expected utility function (4) subject to (12)is

$$0 = \hat{W}_{i,T}^{-\gamma} \overline{W}_{-i,T}^{\alpha(\gamma-1)} - y_i \xi_T,$$
$$\hat{W}_{i,T} = (y_i \xi_T)^{-1/\gamma} \overline{W}_{-i,T}^{\alpha(\gamma-1)/\gamma}, \tag{13}$$

where y_i is the Lagrange multiplier attached to the budget constraint (12). Considering M equations (13) for each manager $i = 1, \ldots, M$, we obtain a system of M equations with M unknowns defining Nash equilibrium wealth profiles ($\hat{W}_{1,T}$,..., $\hat{W}_{M,T}$). To solve it, let us consider the first two equations of this system, and substitute (5) in them. This gives

$$\hat{W}_{1,T} = (y_1 \xi_T)^{-1/\gamma} (\hat{W}_{2,T} * \ldots * \hat{W}_{M,T})^{\frac{\alpha(\gamma-1)}{\gamma(M-1)}}, \tag{14}$$
$$\hat{W}_{2,T} = (y_2 \xi_T)^{-1/\gamma} (\hat{W}_{1,T} * \hat{W}_{3,T} * \ldots * \hat{W}_{M,T})^{\frac{\alpha(\gamma-1)}{\gamma(M-1)}}. \tag{15}$$

Dividing (14) by (15), we get

$$\frac{\hat{W}_{1,T}}{\hat{W}_{2,T}} = \left(\frac{y_1}{y_2}\right)^{-\frac{1}{\gamma}} \left(\frac{\hat{W}_{2,T}}{\hat{W}_{1,T}}\right)^{\frac{\alpha(\gamma-1)}{\gamma(M-1)}} \Rightarrow \hat{W}_{2,T} = \left(\frac{y_1}{y_2}\right)^{\frac{1}{\gamma\left(\frac{\alpha(\gamma-1)}{\gamma(M-1)}+1\right)}} \hat{W}_{1,T}. \tag{16}$$

Replacing subscript 2 in (16) by $j = 3, \ldots, M$, we obtain the relations between Nash equilibrium wealth of manager j, $\hat{W}_{j,T}$, and manager 1, $\hat{W}_{1,T}$, and substituting all of them into (14) yields

$$\begin{aligned}\hat{W}_{1,T} =& (y_1\xi_T)^{-\frac{1}{\gamma}} \left(\frac{y_1}{y_2}\right)^{\frac{1}{\gamma\left(1+\frac{\gamma(M-1)}{\alpha(\gamma-1)}\right)}} \hat{W}_{1,T}^{\frac{\alpha(\gamma-1)}{\gamma(M-1)}} * \ldots * \left(\frac{y_1}{y_M}\right)^{\frac{1}{\gamma\left(1+\frac{\gamma(M-1)}{\alpha(\gamma-1)}\right)}} \hat{W}_{1,T}^{\frac{\alpha(\gamma-1)}{\gamma(M-1)}} \\ =& y_1^{-\frac{1}{\gamma}+\frac{M-1}{\gamma\left(1+\frac{\gamma(M-1)}{\alpha(\gamma-1)}\right)}} (y_2 * \ldots * y_M)^{-\frac{M-1}{\gamma\left(1+\frac{\gamma(M-1)}{\alpha(\gamma-1)}\right)}} \hat{W}_{1,T}^{\frac{\alpha(\gamma-1)}{\gamma}} \xi_T^{-\frac{1}{\gamma}}, \\ \hat{W}_{1,T} =& K_1 \xi_T^{-\frac{1}{\gamma-\alpha(\gamma-1)}},\end{aligned}$$

from which we obtain

$$\hat{W}_{1,T} = K_1 \xi_T^{-\frac{1}{\gamma-\alpha(\gamma-1)}}, \tag{17}$$

where

$$K_1 = \left(y_1^{-\frac{1}{\gamma}+\frac{M-1}{\gamma\left(1+\frac{\gamma(M-1)}{\alpha(\gamma-1)}\right)}} (y_2 * \ldots * y_M)^{-\frac{M-1}{\gamma\left(1+\frac{\gamma(M-1)}{\alpha(\gamma-1)}\right)}} \right)^{\frac{\gamma}{\gamma-\alpha(\gamma-1)}}. \tag{18}$$

Analogously to (17), we can obtain Nash equilibrium wealth of manager i, $i = 1, \ldots, M$:

$$\hat{W}_{i,T} = K_i \xi_T^{-\frac{1}{\gamma-\alpha(\gamma-1)}}, \tag{19}$$

where K_i is obtained from K_1 in 18 by switching subscripts 1 and i. To derive manager 1's equilibrium portfolio, we substitute (17) into a no-arbitrage condition

$\xi_t \hat{W}_{1,t} = E_t[\xi_T \hat{W}_{1,T}]$:

$$\xi_t \hat{W}_{1,t} = K_1 E_t[{\xi_T}^{1-\frac{1}{\gamma-\alpha(\gamma-1)}}] = C_t \xi_t^{1-\frac{1}{\gamma-\alpha(\gamma-1)}},$$

$$\hat{W}_{1,t} = C_t \xi_t^{-\frac{1}{\gamma-\alpha(\gamma-1)}}. \tag{20}$$

In (20), for brevity we use C_t to denote a certain deterministic function of time which, as will be seen momentarily, does not affect managers' Nash equilibrium investment strategies. Applying Ito's Lemma to (20) and using (10), we get that the diffusion term of $d\hat{W}_{1,t}$ is equal to $\frac{\kappa}{\gamma-\alpha(\gamma-1)}\hat{W}_{1,t}$. Equating this term to the diffusion term $\hat{\theta}_{1,t}\hat{\sigma}_S\hat{W}_{1,t}$ in (3), and using (11), we get

$$\hat{\theta}_{1,t} = \frac{1}{\gamma-\alpha(\gamma-1)}\frac{\mu_S}{\sigma_S^2} \tag{21}$$

For other managers, the derivations are analogous, and so (6) obtains. Plugging $\alpha = 0$ in (6) yields (7).

PROOF OF PROPOSITION 2:

In the above proof of Proposition 1, we relied on μ_S and σ_S being constants only when deriving manager 1's investment strategy (21), and all the analysis before equally holds when these parameters are stochastic. The analysis below does not rely on managers' investment policies, and so in what follows we do *not* assume μ_S and σ_S are constant, and hence we do not assume that the market price of risk κ in (10) is constant. Rather, we establish that *in equilibrium* these parameters are constant. Substituting (19) in time-T market clearing condition yields

$$D_T = \sum_{i=1}^{M} \hat{W}_{i,T} = \left(\sum_{i=1}^{M} K_i\right) \xi_T^{-\frac{1}{\gamma-\alpha(\gamma-1)}}, \tag{22}$$

and so time-T value of the state price density is

$$\xi_T = \left(\sum_{i=1}^{M} K_i\right)^{\gamma-\alpha(\gamma-1)} D_T^{\alpha(\gamma-1)-\gamma}. \tag{23}$$

From (10), ξ_t is a martingale, and so using (23), we get

$$\begin{aligned} \xi_t = E_t[\xi_T] &= \left(\sum_{i=1}^{M} K_i\right)^{\gamma-\alpha(\gamma-1)} E_t\left[D_T^{\alpha(\gamma-1)-\gamma}\right] \\ &= \left(\sum_{i=1}^{M} K_i\right)^{\gamma-\alpha(\gamma-1)} E_t\left[D_T^{\alpha(\gamma-1)-\gamma}\right]. \end{aligned} \tag{24}$$

Applying Ito's lemma to $D_t^{\alpha(\gamma-1)-\gamma}$ and using (1), it is easy to get that $D_t^{\alpha(\gamma-1)-\gamma}$ follows a geometric Brownian motion with drift $(\alpha(\gamma-1)-\gamma)\mu_D + \frac{1}{2}(\alpha(\gamma-1)-\gamma)(\alpha(\gamma-1)-\gamma-1)\sigma_D^2$, substituting which into (24) yields

$$\xi_t = \left(\frac{\sum_{i=1}^{M} K_i}{D_t}\right)^{\gamma-\alpha(\gamma-1)} e^{\left((\alpha(\gamma-1)-\gamma)\mu_D + \frac{1}{2}(\alpha(\gamma-1)-\gamma)(\alpha(\gamma-1)-\gamma-1)\sigma_D^2\right)(T-t)}. \tag{25}$$

The equilibrium time-t stock price $\hat{S}_t$ is given by a no-artbitrage condition

$$\hat{S}_t = E_t[\xi_T D_T]/\xi_t,$$

and plugging in it (23) and (25) and canceling $\left(\sum_{i=1}^{M} K_i\right)^{\gamma-\alpha(\gamma-1)}$ in the numerator and denominator, we get

$$\hat{S}_t = \frac{E_t[D_T^{\alpha(\gamma-1)-\gamma+1}]}{D_t^{\alpha(\gamma-1)-\gamma} e^{\left((\alpha(\gamma-1)-\gamma)\mu_D+\frac{1}{2}(\alpha(\gamma-1)-\gamma)(\alpha(\gamma-1)-\gamma-1)\sigma_D^2\right)(T-t)}}$$

$$= D_t e^{\left(\mu_D+(\alpha(\gamma-1)-\gamma)\sigma_D^2\right)(T-t)}. \tag{26}$$

Applying Ito's lemma to (26), we get that the stock price dynamics in equilibrium is

$$d\hat{S}_t = (\gamma - \alpha(\gamma-1))\sigma_D^2 \hat{S}_t dt + \sigma_D \hat{S}_t d\omega_t. \tag{27}$$

Equilibrium characterization (8) follows from (26) and (27). Substituting $\alpha = 0$ into (8) yields (9).

Appendix A. CARA version of the model.

In this appendix, we verify that our main implications remain valid when fund managers have CARA utility. We now describe how we modify the economic setting presented in Section 2 in the main text to produce a tractable model with CARA managers. Unless stated otherwise below, the notation and assumptions are as in Section 2.

As is well-known, tractability in CARA settings often requires normally distributed random variables, and this is also the case in our setting. To achieve normally distributed terminal dividend D_T, we assume that D_t follows an arithmetic Brownian motion

$$dD_t = \mu dt + \sigma d\omega_t,$$

where $\mu > 0, \sigma > 0$ are constant. The risk free rate is normalized to zero.

Each investor i has CARA utility function defined over relative wealth:

$$u_i = -\frac{1}{\gamma} \exp(-\gamma(W_{i,T} - \alpha \overline{W}_{-i,T})),$$

where γ captures the absolute risk aversion, and $\overline{W}_{-i,T}$ is the arithmetic average of wealth of all managers excluding i, $\overline{W}_{-i,T} = \frac{1}{M-1} \sum_{j \neq i} W_j$. In addition to different utility, note two changes relative to the setting in Section 2, made for tractability: (i) relative wealth is defined as own wealth minus average wealth of others, and not as the ratio, (ii) average wealth is defined as the arithmetic average, not the geometric. Proposition 3 (corresponding to Proposition 2 in Section 3) describes the stock market equilibrium in this modified economic setting.

PROPOSITION 3: *The equilibrium level of the stock market $\hat{S}$ follows an arithmetic Brownian motion*

$$d\hat{S}_t = \hat{\mu}_S dt + \hat{\sigma}_S d\omega_t.$$

The equilibrium value $\hat{S}_t$, $\hat{\mu}_S$ and $\hat{\sigma}_S$ are

$$\hat{S}_t = D_t + \mu(T-t) - \frac{\sigma^2(T-t)}{\delta}, \quad \hat{\mu}_S = \frac{\sigma^2}{\delta}, \quad \hat{\sigma}_S = \sigma,$$

where

$$\delta = \frac{(M-1)/\alpha + M/(1-\alpha)}{(M-1)/\alpha + 1}\frac{M}{\gamma}.$$

δ increases in competition intensity α, and so a higher competition intensity is associated with a higher level of the stock market, lower market premium, and unchanged market volatility.

This Proposition reveals that using CARA preferences instead of CRRA does not affect our qualitative predictions.

PROOF:

Because markets are complete, the state price density and the budget constraint do not change relative to the main model, and are given by equations, (10) and (12), respectively. The first order condition for investor i is, however, different and given by

$$\exp(-\gamma(W_{i,T} - \alpha \overline{W}_{-i,T})) = y_i \xi_T, \tag{A1}$$

where y_i is Lagrange multiplier associated with investor i's budget constraint. Rearranging (A1)yields

$$W_{i,T} = -\frac{1}{\gamma} \ln(y_i \xi_T) + \alpha \overline{W}_{-i,T}. \tag{A2}$$

Denoting the aggregate wealth as $W \equiv \sum_{i=1}^{M} W_{i,T}$ and summing (A2) across all managers, we get

$$\begin{aligned} W &= -\frac{1}{\gamma} \sum_{j=1}^{M} \ln y_j - \frac{M \ln \xi_T}{\gamma} + \frac{\alpha}{M-1}(MW - W), \\ W &= -\frac{\sum_j \ln y_j + M \ln \xi_T}{(1-\alpha)\gamma}. \end{aligned} \tag{A3}$$

Substituting $\overline{W}_{-i,T} = \frac{1}{M-1}(W - W_{i,T})$ and (A3) into (A2), and expressing from the resulting equation $W_{i,T}$, we have

$$W_{i,T} = -K_i - \delta_i \ln \xi_T, \tag{A4}$$

where

$$K_i \equiv \frac{1}{\gamma + \alpha\gamma/(M-1)} \left(\ln y_i + \frac{\alpha/(M-1)}{1-\alpha} \sum_{j=1}^{M} \ln y_j \right),$$
$$\delta_i \equiv \frac{1}{\gamma + \alpha\gamma/(M-1)} \left(1 + \frac{\alpha M}{(1-\alpha)(M-1)} \right).$$

The market clearing condition at time T is

$$\sum_{i=1}^{N} W_{i,T} = D_T,$$

plugging into which (A4) yields

$$\xi_T = \exp\left(-\frac{D_T + K}{\delta} \right), \tag{A5}$$

where

$$K = \sum_i K_i, \quad \delta = \sum_i \delta_i = \frac{M - 1 + M\alpha/(1-\alpha)}{M - 1 + \alpha} \frac{M}{\gamma}, \tag{A6}$$

By no-arbitrage, the equilibrium stock price is

$$S_t = \frac{E_t[\xi_T D_T]}{\xi_t}. \tag{A7}$$

To compute (A7), we use a well-known fact that the moment generating function of normal random variable $X \sim \mathcal{N}(m, s^2)$ is $E[e^{\beta X}] = e^{\beta m + \frac{1}{2} s^2 \beta^2}$, and moreover it is known that

$$E[Xe^{\beta X}] = \frac{\partial}{\partial \beta} E[e^{\beta X}] = (m + s^2\beta)e^{\beta m + \frac{1}{2}s^2\beta^2} = (m + s^2\beta)E[e^{\beta X}].$$

In our case, sitting at time t, D_T is normally distributed as $\mathcal{N}(D_t + \mu(T-t), \sigma^2(T-t))$, and moreover ξ_t is a martingale, $\xi_t = E_t[\xi_T]$. Applying these properties to (A7) and using (A5), we obtain

$$\text{(A8)} \qquad \hat{S}_t = \frac{E_t[\xi_T D_T]}{\xi_t} = \frac{E_t[D_T \exp(-D_T/\delta)]}{E_t[\exp(-D_T/\delta)]} = D_t + \mu(T-t) - \frac{\sigma^2(T-t)}{\delta}.$$

Apply Ito's lemma to (A8) yields

$$d\hat{S}_t = \frac{\sigma^2}{\delta} dt + \sigma d\omega_t.$$

Finally, to see that δ increases in α, we differentiate the second equation in (A6), which gives after some algebra

$$\frac{\partial \delta}{\partial \alpha} = \frac{1}{(1-\alpha)^2} \frac{M}{\gamma} > 0$$