CHILE*

Efstathios Avdis and Sergei Glebkin

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Abstract

We study how wealth inequality affects market quality—specifically, liquidity and information efficiency. To this end, we introduce CHILE, an asset-pricing framework with asymmetric information, general utility functions, and arbitrary payoffs. It features a large economy (LE) with continuous-and-heterogeneous information (CHI). Making the rich richer and the poor poorer reduces information efficiency but improves liquidity. Making the rich more informed and the poor less informed has the same effect. With endogenous information, richer agents acquire more information, reinforcing the above outcomes. Overall, widening wealth inequality is a double-edged sword for market quality, increasing liquidity but harming information efficiency.

Keywords: inefficient markets, information aggregation, rational expectations with non-CARA preferences, wealth effects, competition

JEL Codes: D01, D53, D82, E19, G12, G14.

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1 Introduction

Market participants come in different sizes. Financial institutions differ in terms of the total value of assets they manage, while households and individuals differ in terms of wealth they invest in financial markets. As a growing literature establishes, these differences—hereinafter, "wealth inequality"—affect asset prices crucially, thereby also returns, risk premia, and risk-free rates (see Panageas, 2020, for a recent survey). Given that prices are but one outcome of financial trade, this literature suggests that wealth inequality may also matter for other outcomes, such as liquidity and informational efficiency. Collectively known as "market quality," these outcomes are vital for many economic decisions, including not only those faced by investors but also by policymakers gauging how well markets function. This raises the following question: How does wealth inequality affect market quality?

This question becomes especially relevant in light of research suggesting that many market participants are *granular*, meaning that idiosyncratic shocks to them, rather than averaging out, affect aggregate outcomes (Gabaix, 2011). In this view, granularity is linked to investor size—shocks to larger investors have a bigger effect on aggregate quantities. As a result, the distribution of investor sizes—i.e., wealth inequality—may play an important role in shaping how individual shocks aggregate and, in turn, how they influence market quality.[1]

To study how wealth inequality affects the aggregation of individual shocks and market quality, we need an asset-pricing framework where (i) wealth influences how investors trade, and (ii) individual shocks do not wash out in aggregation. Such models are hard to come by—especially under asymmetric information, which is essential for analyzing market quality. Most theories in this domain assume Constant Absolute Risk Aversion (CARA) preferences and thus exclude wealth effects, failing to meet the first requirement. Models that do incorporate wealth effects—notably Peress (2004)—rely on a continuum of investors for tractability, where individual shocks cancel out by the law of large numbers, as in Hellwig (1980). Consequently,

[1] Granularity is not merely a theoretical notion—it plays an active role in empirical asset pricing. Shocks to large (i.e., granular) investors are used to identify key asset pricing parameters. For instance, Gabaix and Koijen (2021) use sector-level demand shocks to estimate the elasticity of market demand. These types of idiosyncratic shocks form the basis of *granular instrumental variables*; see Gabaix and Koijen (2024) for the methodology and references therein for recent empirical applications.

these models fail to satisfy the second requirement and are not well suited to studying how wealth inequality shapes the aggregation of individual shocks.

To address these issues, we develop a tractable asset-pricing framework with asymmetric information and general, heterogeneous preferences that accommodate wealth effects, thereby fulfilling the first requirement. In our model, idiosyncratic shocks to investors' demands arise from noise in their private signals. Using an information structure described below, we construct a large economy in which this signal noise aggregates to a finite, non-zero random variable rather than washing out, thus satisfying the second requirement.

Beyond meeting these primary requirements, our framework has two additional advantages. First, the aggregated signal noise is the sole source of noise in the model, ensuring that prices remain partially revealing without the need for exogenous noise. The absence of exogenous noise is beneficial for our research question, as we avoid taking a stand on how such noise responds to changes in the wealth distribution. Second, our framework allows for general asset payoff distributions, sidestepping the well-known limited empirical appeal of models with Normal returns, which permit negative wealth—a valuable benefit given our focus on wealth inequality.

Our analysis begins by extending the notion of competitive equilibrium from Hellwig (1980) to our setting. The equilibrium objects in our economy are limits of quantities in economies with a finite number of traders (referred to as "discrete" economies). In any discrete economy, traders take prices as given, disregarding both their influence on the price level and on the informational content of prices—thereby sidestepping the "schizophrenia" issue highlighted in Hellwig (1980). We refer to the limit of this process as the *price-taking equilibrium*.[2] We prove that a unique price-taking equilibrium exists, even in the absence of external noise in prices. Despite the generality of our primitives, our end-product is tractable, with all equilibrium objects in closed form.[3]

[2]An alternative resolution, pursued by Kovalenkov and Vives (2014), is to start from the finite-agent Bayesian Nash equilibrium—where agents fully internalize both dimensions of their price impact—and take the large economy limit. We adopt this approach in Appendices G and H, where we derive the limiting BNE and confirm that our main results are not artifacts of the price-taking assumption.

[3]In Avdis, Glebkin, and Peress (2025), we contrast the price-taking equilibrium with the competitive Rational Expectations Equilibrium (REE), where traders ignore their impact on price levels but not informational content, and the Bayesian Nash Equilibrium (BNE), where they account for both. These equilibria differ under our information structure. Here, we focus on wealth inequality and market quality, using the price-taking equilibrium as a natural competitive benchmark that abstracts from market power. Our ongoing work explores market

Turning to our question of how wealth inequality affects market quality, we use our model to study the effect of reducing inequality, changing the population of traders à la "Robin Hood". More specifically, a Robin-Hood variation changes the distribution of wealth across traders, making the rich less rich and the poor less poor, without necessarily affecting aggregate wealth (for institutional investors, the changes are over fund size). Focusing on decreasing-absolute-risk-aversion (DARA) preferences, a plausibly realistic assumption, we show that reducing inequality makes prices more informative. Widening inequality has the opposite effect.

To see why narrowing inequality improves information efficiency, it helps to first develop a baseline for our intuition. We begin by pointing out that prices reveal the weighted average of all private signals, with weights proportional to agents' trading intensities.[4] We then ask the following: is there an "informationally ideal" signal-weighting scheme, however hypothetical, such that the corresponding price reveals maximum information? If so, we intuitively expect that signals of better quality get larger weights. As we show, a weighting scheme confirming this intuition does exist, weighing signals in proportion to their precision alone.

In contrast, the equilibrium weighting scheme is more involved. Accounting for wealth effects, the equilibrium scheme scales signals by risk tolerance on top of precision, placing more weight on the signals of more risk-tolerant traders. As risk tolerance increases in wealth under DARA, the signals of richer traders receive excessive weight, implying that the information content of prices is distorted due to wealth inequality. By transferring wealth from the rich to the poor, a Robin-Hood variation corrects this distortion, moving the equilibrium weighting scheme towards the ideal, making prices more informative.

The mechanism outlined above is empirically relevant at different economic timescales. For long timescales, our result can help interpret the long-term trend towards higher institutionalization, widening wealth inequality, and growing concentration of the asset management industry.[5] At shorter timescales, our model provides novel insights into recent evidence connecting

power by analyzing the limiting BNE. For completeness, key results from Avdis et al. (2025) are summarized in Appendix G for the CARA-normal case. In Appendix H, we extend the analysis to general preferences and payoff structures.

[4]As we discuss below, the trading intensity of an agent is defined as the sensitivity of his demand to changes in private information.

[5]The empirical literature offers recent evidence on these trends. See Ben-David, Franzoni, Moussawi, and Sedunov (2021) on the increasing concentration of the asset-management industry and institutionalization, and

changes in fund size distribution to variations in informational efficiency, as shown by Xiong, Yang, and Zheng (2024).[6]

We have, so far, assumed that our agents are endowed with signals and precisions. Given that, as Grossman and Stiglitz (1980) point out, our understanding of price efficiency rests critically on what we assume about the cost of information, one may wonder if our argument continues to hold if information is costly and endogenous. In fact, since people with different wealth may acquire different amounts of information, wealth variations turn on a new channel by changing signal precisions, one that requires more finessed comparative statics.

We isolate this new channel by studying what happens if we change precisions without changing wealth. A Robin Hood variation on precision—decreasing the precision of larger (as in "richer") agents and increasing that of smaller (as in "poorer") ones—again improves information efficiency. When we reduce the precision of larger agents, they end up trading less aggressively, their signals receive smaller weights, thereby pushing the price towards the informational ideal. The opposite happens on the other end of the population, but with the same end effect.

This result has a surprising corollary: prices can become less informative even if everyone receives weakly more information. More concretely, increasing the precision of sufficiently large traders without changing that of others exacerbates the distortion discussed above, because it pushes the signal-weighting scheme further away from the informational ideal. Coming off as an information-aggregation paradox, this property is, in fact, a hallmark of imperfect aggregation with heterogeneous wealth effects. In short, more can be less, because more badly-aggregated information is less information.

Returning to whether wealth effects change if information is endogenous, we revisit our comparative statics in an extension where traders acquire information in the spirit of Verrecchia (1982), with information costs convex in precision (see Appendix D). We show that large traders acquire more information than smaller ones. This is an intuitive result: due to trading more aggressively than others, large traders have stronger incentives to acquire more information

Saez and Zucman (2016) on widening wealth inequality.

[6]These shifts in fund size distribution may result, for example, from fund flows moving from smaller to larger funds or from mergers between funds.

because they make more use of it. Consequently, the overall response to a Robin Hood variation in wealth combines two effects. One coming from wealth changes alone, improving efficiency directly; and another, coming from how precision changes in response to how wealth changes, improving efficiency indirectly by amplifying the direct effect.

A central mechanism in our analysis concerns how idiosyncratic shocks—arising from noise in traders' private signals—are aggregated into prices, and how this aggregation depends on wealth inequality. Earlier studies on the link between wealth and information efficiency adopt the standard large-economy framework introduced by Hellwig (1980), in which noise from individual signals washes out in the aggregate.[7] As a result, these models do not capture the effects we highlight. Their predictions also contrast with ours. Under Hellwig's information structure, redistributing wealth from rich to poor reduces the trading intensity of wealthy investors more than it increases that of poorer ones, ultimately lowering information efficiency.[8] Recent empirical evidence from Xiong et al. (2024) supports our predictions rather than those of prior models.

Turning to other aspects of market quality, we ask how wealth inequality affects liquidity. We show that narrowing wealth inequality induces two conflicting effects on the willingness of agents to provide liquidity, both of which are knock-on effects of prices becoming more efficient as discussed above. On the one hand, as efficiency improves, each trader puts more weight on commonly-observed price information, aligning his expectations closer to those of others. This effect decreases the agents' willingness to trade, reducing liquidity. On the other hand, as efficiency improves, prices deviate less from fundamentals and are thus de facto less volatile, reducing the risk that traders must absorb when they trade. This effect increases liquidity. As both effects are active in our model, wealth inequality affects liquidity non-monotonically. Nevertheless, we can separate out the risk component if we scale liquidity by return volatility,

[7] In addition to Peress (2004), see Makarov and Schornick (2010), Kurlat and Veldkamp (2015) and Mihet (2022).

[8] Because signal noise cancels out in these models, they require the introduction of exogenous noise to prevent prices from becoming fully revealing. Consequently, information efficiency depends on the relative trading intensity of informed traders—which decreases after wealth redistribution—versus that of noise traders, which remains unchanged. Incorporating noise traders into our model would capture this additional effect, but it would not alter our main results, as the mechanism we highlight in the baseline model would remain the dominant force (see Appendix F for a detailed analysis incorporating noise). We exclude noise in the main model for parsimony, focusing on the novel mechanism in its simplest form while ensuring consistency with empirical findings.

obtaining a globally monotone comparative static: as wealth inequality decreases, so does risk-adjusted liquidity.

Our framework has tractability rarely seen beyond the case with homogeneous agents and CARA preferences.[9] What enables it in our case is the way we model information. In contrast to the traditional methodology for large markets (Hellwig, 1980; Admati, 1985, and consequent literature), we do not assume that traders have signals of finite precision, because that would imply that as the number of traders becomes large, so does the total amount of information.[10] What is more, with signals of finite precision, traders would make finite speculative trades, with the unfortunate consequence that aggregate demand would explode for large numbers of traders.

We instead use an assumption similar to that in Section 9 of Kyle (1989), whereby a finite amount of information is distributed among all traders. By definition, then, the total amount of information remains finite regardless of the number of traders. Moreover, as individual demands are based on signals with precision inversely related to the size of the economy, aggregate demand is finite, even for infinitely many traders. As we show, we can formally treat information structures with the aforementioned properties as diffusion processes running through a heterogeneous continuum, giving us structures we call continuous-and-heterogeneous information (CHI). Recognizing that our model also requires a large economy (LE), we adopt the name CHILE for the class of models described in this paper.

Working with CHILE yields several advantages. First, it unlocks the full arsenal of stochastic calculus, improving tractability similarly to when one switches from discrete-time to continuous-time formulations. Second, the noise in traders' signals aggregates into an Itô integral—a finite, non-zero random variable—rather than vanishing via the Law of Large Numbers, as in

[9]Within asymmetric-information asset pricing, papers that go beyond the CARA-Normal framework (imposing, however, other assumptions for tractability) include Peress (2004), Peress (2014), Breon-Drish (2015), Malamud (2015) and Chabakauri, Yuan, and Zachariadis (2022). Allowing for non-Normal distributions, Breon-Drish (2015) and Chabakauri et al. (2022) require CARA, and thus do not incorporate wealth effects. Malamud (2015) requires complete markets. Peress (2004) and Peress (2014) achieve tractability by requiring the risk of assets to be small. See Section 9 for details.

[10]As the total amount of information is the precision of the sufficient statistic of private signals, it equals the sum of signal precisions held by all traders. Thus, in economies where the precision of each signal is finite (as in "neither infinite nor infinitesimal," i.e., neither infinitely large nor infinitely small), the total amount of information becomes infinite with an infinite number of traders.

standard large-economy models. This type of aggregation yields prices that are noisy, ensuring that our price-taking equilibrium is well-defined even without external noise. Third, unlike with a discrete economy, working with a continuous economy implies that our main primitives are functions. Consequently, we can carry out comparative statics through the calculus of variations, opening up questions that are otherwise hard to address. Finally, we note that the traditional approach to modeling large economies, which relies on aggregation via the Law of Large Numbers, falls short in fully capturing the effects of heterogeneity. As demonstrated in Appendix E, in such economies, it is possible to derive an aggregation result whereby an economy with investors differing in wealth, precision, and preferences appears observationally equivalent to one with homogeneous investors.[11]

2 Discrete economies

We introduce a sequence of economies indexed by the number of agents n, or equivalently by $\mu = 1/n$, the mass of each agent. All equilibrium objects—demands, posterior beliefs, and prices—depend on μ, which identifies the economy within the sequence. Our primary interest is the continuous economy obtained in the limit $\mu \to 0$ ($n \to \infty$), which we characterize in Section 3.

We now describe a generic economy in this sequence; that is, we fix μ (and hence $n = 1/\mu$). The economy is defined on a fixed probability space $(\Omega, \mathcal{F}, \mathbb{P})$, with trading at $t = 1$ and consumption at $t = 2$.[12] There are two assets, one risky and one risk-free. As we use the risk-free asset as a numeraire, we normalize its gross return to 1. The risky asset pays off $V(v)$ in the second period, where $v \sim N(0, \tau_v^{-1})$ is the *fundamental*, and $V(v)$, a weakly increasing function of v, is the *payoff function*.[13]

The risky asset trades at price P, determined in the first period. Allowing for both price-

[11] Consequently, in economies with Law of Large Numbers aggregation, Robin Hood variations in wealth that leave aggregate trading intensity unchanged have *no* effect on market quality, unlike in CHILE.

[12] In Appendix D, we study information acquisition, for which we add a period at $t = 0$.

[13] The normalization $\mathbb{E}[v] = 0$ is made without loss of generality, as any mean can be incorporated into the general form of $V(\cdot)$.

inelastic and price-elastic supply components, we write the total supply of the risky asset as

$$\Theta(P) = \bar{\theta} + \theta(P),$$

where $\bar{\theta}$ is a constant and $\theta(P)$ is a continuous function.

The economy is populated by n agents. Agent i, $i = 1, \dots, n$, is associated with point $a_i = (i-1)\mu$ on the unit interval $[0, 1)$; informally, agent i lives on the interval $[a_i, a_i + \mu)$. The population of traders is characterized by three profiles: initial wealth $W_0(a)$, preferences over terminal wealth $u(W, a)$, and signal precisions $t(a)$ (signals are characterized below). Agent i's primitives are $W_0(a_i)$, $u(W, a_i)$, and $t(a_i)$; thus, discrete primitives are obtained by "sampling" the profiles at the leftmost points of agent subintervals.

The signal of agent i, who lives in segment $[a_i, a_i + \mu)$, is

$$\Delta s_i = v \cdot \Delta a_i + \frac{1}{\sqrt{t\left(a_i\right)}} \Delta B(a_i). \tag{1}$$

Here, $B(a)$ is a standard Brownian motion on $[0, 1)$, independent of v, $\Delta a_i = a_{i+1} - a_i = \mu$, and $\Delta B(a_i) = B(a_i + \mu) - B(a_i) \sim \mathcal{N}(0, \mu)$. The scaled signal $\Delta s_i/\mu$ has a standard "fundamental plus noise" structure with precision $t(a_i)\mu$.[14] We refer to $t(a)$ as the *precision* of agent a.

Agent i's optimal demand in the economy with size parameter μ, given a signal realization s, solves

$$x^*(s, P, a_i, \mu) = \arg\max_x \int_{\mathbb{R}} u\Big(W_0(a_i) + x\left(V(v) - P\right), a_i\Big) f(v \mid s, P, \mu)\, dv, \tag{2}$$

where $f(v \mid s, P, \mu)$ denotes the posterior density of v given the signal realization $\Delta s_i = s$ and the price P. The explicit dependence on μ reflects the fact that beliefs—and hence demands—vary across economies in the sequence.

[14] Dividing both sides of (1) by μ gives $\Delta s_i/\mu = v + \Delta B(a_i)/(\mu\sqrt{t(a_i)})$. Since $\Delta B(a_i) \sim \mathcal{N}(0, \mu)$, the noise term has variance $\mu/(\mu^2 t(a_i)) = 1/(t(a_i)\mu)$, i.e., precision $t(a_i)\mu$.

2.1 Price taking and optimal demands

Our goal is to study a price-taking equilibrium—a natural benchmark that abstracts from market power. In such an equilibrium, traders take the price as given, disregarding their influence on both the price level and the informational content of prices.

These two dimensions of price taking have distinct implications for the optimization problem (2). Disregarding one's influence on the price level means that the agent treats the densities in (2) as unaffected by his choice of x—the standard price-taking assumption in competitive markets. Disregarding one's influence on the informational content of the price means that the agent assumes that, conditional on v, the price and his signal are independent—that is, he does not perceive his own noise reflected in the price, a concern originally raised by Hellwig (1980).[15]

Under the conditional independence assumption, the posterior density factors as

$$f(v \mid s, P, \mu) \propto \underbrace{\exp\left(t(a_i)\left(sv - \frac{\mu}{2}v^2\right)\right)}_{\propto \text{ density of } \Delta s_i|v} \underbrace{g(v, P, \mu)}_{\text{density of } (v,P)}, \tag{3}$$

where $g(v, P, \mu)$ is the joint density of v and P as conjectured by the agent, which we call the *price belief*. In equilibrium, the price belief is required to be consistent with the actual joint distribution of v and P in the limiting economy ($\mu \to 0$).

Substituting (3) into (2) and dropping terms unaffected by the maximization over x, the optimal demand solves

$$x^*(s, P, a_i, \mu) = \arg\max_x \int_{\mathbb{R}} u\Big(W_0(a_i) + x\left(V(v) - P\right), a_i\Big) \exp\left(t(a_i)\left(sv - \frac{\mu}{2}v^2\right)\right) g(v, P, \mu)\, dv. \tag{4}$$

To summarize, our agents are heterogeneous along three dimensions: initial wealth $W_0(a)$, preferences $u(W, a)$, and precisions $t(a)$. The profiles $W_0(a), u^{(l)}\left(W_0(a), a\right), l = 0, 1, 2, 3,$ and $t(a)$ are arbitrary functions of a, continuous on $[0, 1)$. We assume that $u(W, a)$ is increasing, strictly concave, and thrice continuously differentiable in W on an open neighborhood of $W_0(a)$.

[15] The price and the signal cannot be independent unconditionally, as they both reflect the fundamental v. Conditioning on v isolates the noise in each, and independence amounts to saying that the agent's idiosyncratic noise is unrelated to the noise in the price.

In addition, we assume that the technical conditions introduced and discussed in Section C.1 hold.

Remark 1. Our agents can be interpreted either as individual traders or as fund managers. In the latter case, $W(a)$ represents the value of the assets managed by fund a, with a portion of $W(a)$ serving as the fund manager's compensation. To lighten notation, we absorb all proportionality coefficients into the parameter a of the utility profile.

Remark 2 (Price-taking assumption). In a large economy, each trader's influence on the price is vanishingly small, so one might expect price-taking behavior to be innocuous. We show that this is not the case: by disregarding their price impact, traders make errors that, while individually small, aggregate to a non-trivial effect. As a result, the price-taking equilibrium differs from both the competitive Rational Expectations Equilibrium (REE) and the Bayesian Nash Equilibrium (BNE); see Appendices G and H. On the other hand, it is precisely these "errors"—concerning traders' influence on the informational content of the price—that sustain a well-defined equilibrium without exogenous noise under price-taking (see Remark 5). Importantly, our key results are not artifacts of the equilibrium concept: they obtain under both competitive REE and BNE.

3 CHILE

We now pass to the limit $\mu \to 0$ ($n \to \infty$). In this limit, the population of agents becomes a continuum indexed by $a \in [0, 1)$—a *large economy* (LE). The discrete signal (1) converges to the continuous stochastic process

$$ds(a) = v\,da + \frac{1}{\sqrt{t(a)}}dB(a). \tag{5}$$

We call the increment $ds(a)$ the *signal of agent a*. Because signals vary across agents, the limiting information structure features *continuous-and-heterogeneous information* (CHI). Combined with the large economy, this defines the class of models we call CHILE.

To represent the CHI structure formally, we define the information available to agents in segment $[b,c)$ as the σ-algebra $\mathcal{F}_{b,c} = \sigma(\{s(z) - s(b)\}_{b \leq z < c})$, and assume that $\mathcal{F}_{b,c}$ is precisely the information set obtained by combining the information of all agents within $[b,c)$. The information available in the entire economy is $\mathcal{F}_{0,1}$, denoted hereafter by $\mathcal{F}_1$.

There are two equilibrium objects of primary interest, from which all others are derived: a cumulative demand function, $X_*(a,P)$, and a market-clearing price function, $\mathbf{P}_*$. We denote their out-of-equilibrium counterparts by dropping the asterisk. We discuss the former in Section 3.1 and the latter in Section 3.2; the equilibrium concept is introduced in Section 3.3.

3.1 Cumulative demand

We define cumulative demand in the continuous economy as the limit of its discrete counterpart. In the economy with size parameter μ, the optimal cumulative demand up to agent a is $\sum_{i:\, a_i<a} x^*(\Delta s_i, P, a_i, \mu)$, where x^* solves (4). The *optimal cumulative demand* in CHILE is then

$$X_*(a,P) = \operatorname*{plim}_{\mu \to 0} \sum_{i:\, a_i < a} x^*(\Delta s_i, P, a_i, \mu). \tag{6}$$

We first establish when this limit exists.

Lemma 1 (Efficiency condition)**.** *The limit in* (6) *exists if, and only if*

$$\int_{\mathbb{R}} (V(v) - P)\, g(v,P,0)\, dv = 0. \tag{7}$$

3.1.1 Efficiency condition

The condition (7) can be rewritten as

$$\mathbb{E}[V(v) - P \mid P] = 0, \tag{8}$$

where the expectation is taken under the conditional distribution of $v \mid P$ implied by the limiting price belief, i.e., the density proportional to $g(v,P,0)$. Therefore, the optimal cumulative demand is well-defined only if the market is weak-form efficient in the sense of (8). The intuition is that a large market with finite asset supply cannot accommodate non-infinitesimal trades from infinitely many agents: if, for example, $\mathbb{E}[V(v)|P] > P$, every agent would buy a finite amount, and the cumulative demand would explode. Consequently, price functions that violate (8) are not viable candidates for equilibrium.[16] Hereafter, we restrict attention to price beliefs $g(v, P, \mu)$ for which the efficiency condition holds in the limit.

3.1.2 Itô representation

A key source of tractability in CHILE is that cumulative demands admit the following representation.

Lemma 2 (Representation lemma)**.** *There exist deterministic functions* $\beta(a, P) : [0, 1)\times\mathbb{R} \to \mathbb{R}$ *and* $\delta(a, P) : [0, 1) \times \mathbb{R} \to \mathbb{R}$ *such that*

$$X_*(a, P) = \int_0^a \beta(b, P)\, ds(b) + \int_0^a \delta(b, P)\, db, \tag{9}$$

Two properties of (9) merit emphasis. First, cumulative demand is an Itô integral, mirroring the structure of the cumulative signal (5). The noise in individual demands does not wash out in aggregation; instead, it accumulates into a finite random variable. Second, the demand of each agent is linear in his signal: the increment $dX(a, P) = \beta(a, P)\, ds(a) + \delta(a, P)\, da$ depends on $ds(a)$ only through the linear term $\beta(a, P)\, ds(a)$. This linearity, which makes learning from prices and market clearing tractable, holds for general preferences and payoff functions. Prior literature obtains linearity in signals only under specific preference–payoff combinations, typically CARA–Normal.[17] See Section B.2 for an intuitive discussion of why this property emerges in our framework.

[16]If the noisy supply grows proportionally to the number of agents, as in Hellwig (1980), supply and demand grow at the same rate, so their ratio remains finite and an equilibrium that is not weak-form efficient can be sustained.

[17]The class of equilibria with linear demand functions has been enlarged by extending Normal payoffs to the exponential family. See Breon-Drish (2015) for the single-asset case and Chabakauri et al. (2022) for the case of many assets.

3.2 Market clearing and price inference

Turning to market clearing, we require that the equilibrium price $\mathbf{P}_*$ be $\mathcal{F}_1$-measurable, as the price cannot reflect more information than what is available in the entire economy. Given (9), when agents observe the price realization $\mathbf{P}_* = P$, they infer a sufficient statistic:

$$s_p \equiv \frac{\int_0^1 \beta(a,P)ds(a)}{\int_0^1 \beta(a,P)da}.$$

This statistic is derived from the market-clearing condition as follows:

$$\int_0^1 dX_*(a,P) = \int_0^1 \beta(a,P)\,ds(a) + \int_0^1 \delta(a,P)\,da = \Theta(P) \Rightarrow$$
$$s_p = \frac{\Theta(P) - \int_0^1 \delta(a,P)da}{\int_0^1 \beta(a,P)da} \equiv h(P). \tag{10a}$$

Moreover, s_p can be written as

$$s_p = \int_0^1 \omega(a,P)ds(a) = v + \int_0^1 \omega(a,P)\frac{dB(a)}{\sqrt{t(a)}} \tag{10b}$$

where

$$\omega(a,P) = \frac{\beta(a,P)}{\int_0^1 \beta(a,P)da}.$$

That is, the price reveals a weighted-average signal, formed by weighting the signal of each agent a by $\omega(a,P)$, which is proportional to a's trading intensity, $\beta(a,P)$.[18]

We impose the following restriction on $h(P)$ in equilibrium.

Assumption 1. *The function $h(P)$ defined in (10a) is strictly monotone.*

We maintain Assumption 1 hereafter. Under this assumption, the equilibrium price $\mathbf{P}_*$ and the sufficient statistic s_p are informationally equivalent. The sufficient statistic s_p in (10b) has the familiar "truth plus noise" form, and both components—the truth, v, and the noise, represented by the Itô integral in (10b)—are normally distributed. Standard Bayesian updating

[18]Thus, our economy features generalized linear equilibria, as in Breon-Drish (2015), Glebkin, Gondhi, and Kuong (2021), and others.

with normal random variables then yields the following characterization.[19]

Lemma 3 (Price Inference)**.** *The conditional distribution of v given $\mathbf{P}_* = P$ is*

$$\mathcal{N}\left(\frac{\tau_p}{\tau}s_p, \frac{1}{\tau}\right).$$

The sufficient statistic s_p can be computed as $s_p = h(P)$, with $h(P)$ as in (10a). Here $\tau = \mathrm{Var}[v|P]^{-1} = \tau_p + \tau_v$, and τ_p is the precision of s_p, given as

$$\tau_p = Var[v|s_p]^{-1} - \mathrm{Var}[v]^{-1} = \left(\int_0^1 \frac{\omega(a,P)^2}{t(a)}da\right)^{-1}. \tag{11}$$

Remark 3. Our model does not require noise traders. Traditional large economies require noise traders to prevent prices from being fully revealing: the noise in private signals washes out in aggregation due to the exact Law of Large Numbers, so without noise traders the price would be fully informative. In our economy, however, the aggregate signal does not reveal the fundamental, despite aggregating a large number of private signals. As in Avdis (2018), the aggregate noise entering prices takes the form of an Itô integral—a random variable with positive and finite variance.[20]

3.3 Notion of price-taking equilibrium

We now formalize the equilibrium concept for our economy.

Definition 1. *A* ***price-taking equilibrium*** *is a cumulative demand function $X_*(a, P)$ and an $\mathcal{F}_1$-measurable price function $\mathbf{P}_*$ such that*

(i) $\mathbf{P}_*$ *clears the market, i.e.* $X_*(1, \mathbf{P}_*) = \Theta(\mathbf{P}_*)$, *and*

(ii) $X_*(a, P)$ *is optimal given* $\mathbf{P}_* = P$.

[19] See Section B.3 for a heuristic derivation and an intuitive discussion of Lemma 3.

[20] Our price noise also ensures there is trade in our large economy in a way that, unlike Avdis (2018), is consistent with the notion of perfect competition in Hellwig (1980). See Remark 5 and our literature review for details.

Definition 1 combines market clearing with demand optimality, following standard conventions. The optimality condition in (ii) requires that the limiting price belief $g(v, P, 0)$ coincide with the density of the joint distribution of v and $\mathbf{P}_*$.

Remark 4 (Aggregates vs. averages in large economies). For any quantity Y defined at the agent level, one can consider both the *aggregate*, $\sum_{i=1}^{n} Y_i$, and the *average*, $\frac{1}{n}\sum_{i=1}^{n} Y_i$. In a finite economy, both are well-defined and generically nonzero. In a large-economy limit, however, the two cannot both be finite and nonzero: if the average has a finite, nonzero limit as $n \to \infty$, the aggregate diverges; conversely, if the aggregate has a finite, nonzero limit, the average vanishes. Any large-economy model will therefore fail to match real-world behavior for at least one of these two versions of each quantity. This is not a deficiency of the modeling approach—it simply determines which version is the appropriate one to compare with data.

To see this concretely, consider the comparison between CHILE and the economies in Hellwig (1980) and Admati (1985) (hereinafter, "traditional large economies"):

	CHILE		**Traditional**	
	Average	Aggregate	Average	Aggregate
Wealth	finite	∞	finite	∞
Trade	0	finite	finite	∞
Risk premium	0	finite	finite	∞

In both frameworks, individual wealth does not scale with the number of agents, so average wealth is finite and nonzero while aggregate wealth diverges. The two approaches differ in trade and the risk premium. In traditional large economies, each agent's trade and the risk premium $\mathbb{E}[V \mid P] - P$ are finite and nonzero, so the corresponding economy-wide totals—aggregate trade $\sum_i x_i$ and aggregate risk compensation $n\,(\mathbb{E}[V \mid P] - P)$—diverge.[21] In CHILE, individual trade and the per-agent risk premium vanish in the limit, reflecting the fact that each agent's risk exposure is infinitesimal. In contrast, the aggregate trade and the aggregate risk premium are finite and are the economically meaningful objects in our framework, as we discuss below. Appendix J provides a self-contained illustration of this phenomenon in the simplest possible

[21] See Hellwig (1980), Section 5, eq. (B.2).

setting: a standard competitive economy with identical uninformed agents.

4 Price-taking equilibrium in CHILE

We present the main theorem of the paper. Section B provides a heuristic derivation of its key statements, while a rigorous proof is given in the Appendix.

Theorem 1. *There exists a unique equilibrium. The equilibrium price function has the representation* $\mathbf{P}_* = \mathcal{P}(s_p)$*, where*

$$s_p = \int_0^1 \omega(a) ds(a) = v + \int_0^1 \frac{\omega(a)}{\sqrt{t(a)}} dB(a) \tag{12}$$

is the equilibrium sufficient statistic and $\omega(a)$ *is a weighting function given by*

$$\omega(a) = \frac{t(a)}{\rho(a)} \left(\int_0^1 \frac{t(b)}{\rho(b)} db \right)^{-1}. \tag{13}$$

The function $\mathcal{P}(x)$ *is given by*

$$\mathcal{P}(x) = \int V \left(\frac{\tau_p}{\tau} x + \frac{z}{\sqrt{\tau}} \right) d\Phi(z). \tag{14}$$

Here $\Phi(z)$ *denotes the standard normal cumulative distribution function (cdf). Consequently, the price function is completely determined by* $V(\cdot)$ *and two other quantities, the precision of* s_p*, given by*

$$\tau_p = \frac{\left(\int_0^1 \frac{t(a)}{\rho(a)} da \right)^2}{\int_0^1 \frac{t(a)}{\rho(a)^2} da}, \tag{15}$$

and the posterior precision of v*, given by* $\tau = \mathbb{V}ar(v|P)^{-1} = \tau_v + \tau_p$*.*

The equilibrium cumulative demand function has the representation $dX(a) = \beta(a, P) ds(a) + \delta(a, P) da$*, where*

$$\beta(a, P) = \frac{t(a)}{\rho(a)} \frac{\tau^{-1}\, \mathbb{E}[V'(v)|s_p]}{\mathrm{Var}\,[V(v)|s_p]}, \text{ and}$$

$$\delta(a, P) = \frac{\beta(a, P)^2}{2t(a)} \pi(a) \frac{\mathrm{Sk}[V(v)|s_p]}{\mathrm{Var}[V(v)|s_p]} - \beta(a, P) \frac{\mathbb{E}[v(V(v) - P)^2|s_p]}{\mathrm{Var}[V(v)|s_p]} + \frac{\psi(P)}{\rho(a)\mathrm{Var}[V(v)|s_p]}. \tag{16}$$

Here, $\rho(a)$ denotes the absolute risk aversion and $\pi(a)$ denotes the absolute prudence coefficient, defined as

$$\rho(a) = -\frac{u''(W_0(a))}{u'(W_0(a))}, \quad \pi(a) = -\frac{u'''(W_0(a))}{u''(W_0(a))}. \tag{17}$$

The sufficient statistic s_p is related to the price P as follows:

$$s_p = \mathcal{P}^{-1}(P),$$

where $\mathcal{P}^{-1}(\cdot)$ is the inverse of the function $\mathcal{P}(\cdot)$ defined in (14). The conditional moments of $V(v)$ and the function $\psi(P)$ are given in the closed form in Appendix C.5.

The theorem above highlights the notable tractability of CHILE. All equilibrium objects are available in closed form despite rich heterogeneity and the generality of preferences. We will make use of this tractability in Section 7, where we examine the effects of changes in wealth distribution on market quality. We now discuss the main features of our equilibrium.

Trading intensity $\beta(a, P)$. The trading intensity has an intuitive structure. First, $\beta(a, P)$ is proportional to $t(a)/\rho(a)$: More informed traders (those with higher $t(a)$) and more risk-tolerant ones (those with higher $1/\rho(a)$) trade more aggressively. Second, $\beta(a, P)$ is inversely proportional to $\text{Var}(V(v)|P)$: Higher uncertainty makes all traders scale down their trading intensities. Third, $\beta(a, P)$ is proportional to $\mathbb{E}[V'(v)|P]$: Trading intensities are higher for assets with payoffs that are more sensitive to changes in fundamentals.[22]

The key results of our paper operate through *wealth effects*. With non-CARA utility, the risk tolerance $1/\rho(a)$ depends on the initial wealth $W_0(a)$. In a plausible case of decreasing absolute risk aversion, risk tolerance increases with wealth. In this scenario, Theorem 1 suggests that wealthier investors trade more aggressively, and their signals receive greater weight in the price.

Price function. The price reflects the weighted average of traders' signals $ds(a)$. Moreover, the weights are proportional to $t(a)/\rho(a)$: the signals of more informed and more risk-tolerant traders have greater weights. This is intuitive as such traders trade more aggressively.

[22] The last property is intuitive because higher $\mathbb{E}[V'(v)|P]$ implies that news about fundamentals is more payoff-relevant: signals about fundamental v tell more about payoffs $V(\cdot)$.

We also note that the closed-form expression (14) for the price function is a restatement of the efficiency condition (31). Indeed, Lemma 3 implies that conditional distribution of v given P is Normal with a mean of $\tau_p/\tau \cdot s_p$ and a variance of $1/\tau$. Thus, a random variable $z = \sqrt{\tau}(v - \tau_p/\tau \cdot s_p)$ has standard normal distribution. Therefore, one can write $v = \tau_p/\tau \cdot s_p + z/\sqrt{\tau}$. Substituting this into the efficiency condition yields $\mathbb{E}[V(v) \mid P] = \mathbb{E}[V(\tau_p/\tau \cdot s_p + z/\sqrt{\tau})] = \int V(\tau_p/\tau \cdot s_p + z/\sqrt{\tau})\, d\Phi(z)$. The same change of variable is used below to obtain closed-form expressions for other conditional moments of $V(v)$.

The coefficients $\delta(a, P)$. The first two terms in (16) indicate that our equilibrium is influenced by the higher moments of the payoff (third central moment ("skewness"), $\mathrm{Sk}(V(v)|P)$), as well as by the higher derivatives of the utility function (prudence, $\pi(a)$). This contrasts with the approach of Peress (2004), where these effects do not play a role. Peress (2004) employs a "small risk" approximation (assuming the variance of the fundamental is small), rendering higher-order risk negligible in his model. In contrast, we use a "small information" approximation, where the risk faced by each trader remains substantial even in the limit.[23]

The last term in (16) represents the demand of a trader with no private signal, referred to as "uninformed demand" hereafter. To see this, note that if we set $t(a) = 0$, the term $\psi(P)/(\rho(a)\mathrm{Var}[V(v)|P])$ becomes the only non-zero component in the coefficients $\beta(a, P)$ and $\delta(a, P)$. To better understand this term, let us first discuss the risk premium in our economy.

Risk premium. At first glance, the efficiency condition (31) appears to imply the absence of a risk premium in our economy. However, a more accurate characterization is that the risk premium in CHILE is infinitesimal rather than exactly zero:

$$\mathbb{E}[V(v) - P \mid P] = \psi(P)\, da,$$

[23] Other papers that use a local second-order approximation to the utility function, such as Samuelson (1970), Campbell and Viceira (2002), Farboodi, Singal, Veldkamp, and Venkateswaran (2022b), and Mihet (2022), also do not account for higher-order effects. However, our results demonstrate that a second-order approximation is not always sufficient. Under small information asymptotics employed here, a third-order approximation to utility is necessary. Even under small-risk asymptotics, the second-order approximation may not be valid. For example, in the setting of Peress (2004), it is valid when traders learn about the mean of the fundamental, but it fails when they learn about the payoff itself (see Peress (2011)).

where

$$\psi(P) \equiv \lim_{\mu \to 0} \frac{1}{\mu} \frac{\int_{\mathbb{R}} (V(v) - P)\, g(v, P, \mu)\, dv}{\int_{\mathbb{R}} g(v, P, \mu)\, dv}.$$

That is, $\psi(P)$ is the rate at which the conditional risk premium vanishes as the economy grows large. This mirrors how individual demands in our model are infinitesimal rather than strictly zero.

The expression above indicates that the risk premium in a discrete economy with trader mass $\mu = da$ is given by $\psi(P)\, da$. The quantity $\psi(P)$ can be interpreted as the *aggregate risk premium* in the economy—that is, the total dollar excess return earned when all traders invest one dollar each in the risky asset. This contrasts with the traditional notion of a risk premium, which refers to the excess return earned by a *single* trader investing one dollar.

In CHILE, the risk premium is infinitesimal because a finite amount of aggregate risk is distributed across an infinite number of traders. While each trader earns an infinitesimal premium, these accumulate to a finite aggregate risk premium.

With this understanding, the demand of an uninformed trader (i.e., one with $t(a) = 0$) takes the form:

$$dX^u = \frac{\psi(P)}{\rho(a)\mathrm{Var}[V(v) \mid P]}\, da = \frac{\mathbb{E}[V(v) - P \mid P]}{\rho(a)\mathrm{Var}[V(v) \mid P]}.$$

This expression coincides with the standard mean-variance demand function.

Remark 5 (Well-defined equilibrium without noise). Our equilibrium exists and features trade despite lacking external noise—a property that may seem surprising given well-known results (Milgrom and Stokey, 1982; Tirole, 1982). By conjecturing that the noise in their signals is independent of that in the price, our agents treat their signals as incrementally informative relative to the price, in contrast to Grossman (1976)'s agents who treat their signals as dominated by the price. As a result, our agents employ their signals to formulate their demands, and trade ensues. It is this belief structure—the conditional independence conjecture—rather than the continuous heterogeneous information technology per se that sustains trade and prevents full revelation. Indeed, Appendices G and H show that under the REE and BNE equilibrium concepts, where agents correctly account for the informational content of prices, trade breaks down without exogenous noise.

Nevertheless, we stress that our agents make the right price conjectures in the large limit. Although it may thus appear that trade would disappear in the limit, within the logic of our model the no-trade intuition works differently. As the economy grows, the size of individual demands shrinks to zero, but the number of agents grows, with cumulative demand counterbalancing their shrinking size by summing up more demand functions. This balance is maintained all the way to the large limit, guaranteeing that aggregate demand converges to a well-defined and non-trivial quantity. Restated colloquially, and perhaps paradoxically, our model features a no-trade theorem at the individual level—since each individual's demand converges to zero—but not at the aggregate level, as cumulative demands remain nonzero.

Remark 6 (Price-taking equilibrium vs competitive REE)*.* As highlighted in Hellwig (1980), another concern with traditional models of competition under asymmetric information lies within their version of noiseless equilibrium: the information conveyed by prices is unaffected by preferences (see also Grossman, 1976). As Hellwig points out, this is somewhat counter-intuitive: "One would expect that the weight with which agent i 's information I_i affects the equilibrium price should depend on the strength of agent i 's reaction to this information, which in turn should depend on his preferences. Presumably, it should make a difference whether the news of an increase in a firm's profits is passed to somebody who is almost risk-neutral and responds by buying a large number of shares or whether this piece of news is passed to a risk averter who hardly responds at all." As we discuss below, the informational content of prices *does* depend on preferences in our equilibrium, and in a way that aligns with the intuition in the quote.

5 Market quality

Our measure of information efficiency is based on how much prices reduce uncertainty about the fundamental and is defined as

$$\mathcal{I} = 1 - \frac{\mathbb{V}\text{ar}(v|P)}{\mathbb{V}\text{ar}(v)}.$$

This measure is common in both theoretical (e.g., Rostek and Weretka, 2012) and empirical work (e.g., Dessaint, Foucault, and Fresard, 2024; Dávila and Parlatore, 2023). In practice, $\mathcal{I}$ corresponds to the R^2 of predicting fundamentals by prices, where the fundamental v is typically proxied by earnings in empirical work.

Our measure of liquidity is based on how much supply shocks can move prices

$$\mathcal{L} = -\left(\frac{\partial \mathcal{P}(s_p, \bar{\theta})}{\partial \bar{\theta}} \bigg|_{\mathcal{P}=P} \frac{1}{\mathbb{V}\mathrm{ar}(V|P)} \right)^{-1}. \tag{18}$$

The first term in (18) captures price sensitivity to unexpected price-inelastic supply shocks, a standard measure of liquidity in the theoretical literature (e.g. Vayanos and Wang, 2013).[24] As our equilibrium is generally non-linear, this term depends on the realized price signal s_p, a stochastic quantity with few (if any) empirical counterparts. As we show next, scaling by $\mathbb{V}\mathrm{ar}(V|P)$ in (18) allows us to obtain the risk-adjusted liquidity measure that does not depend on the aggregate signal, and we thus adopt it as our primary notion of liquidity.[25]

Proposition 1. *The equilibrium expressions for information efficiency $\mathcal{I}$ and liquidity $\mathcal{L}$ are*

$$\mathcal{I} = \left(1 + \tau_v \frac{\int_0^1 \frac{t(a)}{\rho(a)^2} da}{\left(\int_0^1 \frac{t(a)}{\rho(a)} da \right)^2} \right)^{-1}, \quad \mathcal{L} = \frac{\int_0^1 \frac{t(a)}{\rho(a)^2} da}{\int_0^1 \frac{t(a)}{\rho(a)} da}.$$

As we see above, information efficiency and liquidity are in closed form. What is more, they are parsimoniously characterized by just three primitives of the economy, two profiles and one parameter: risk tolerances $1/\rho(a)$, precisions $t(a)$, and prior uncertainty $1/\tau_v$. This enables tractable comparative statics that we turn to next.

[24]By "unexpected" we mean that agents do not adjust their demand coefficients β and δ in response to a change in $\bar{\theta}$.

[25]Our liquidity measure is the inverse of price impact per unit of risk. Price impact is proportional to variance, as shown in Kyle (1985) and Kyle (1989) for the single-asset case, and in Rostek and Weretka (2015) and Malamud and Rostek (2017) for the multi-asset case. Without normality, the price impact is proportional to risk-neutral variance, as demonstrated in Glebkin, Malamud, and Teguia (2023a). Therefore, scaling by variance, as in (18), allows for better cross-sectional comparisons consistent with empirical practice.

6 A CRRA–log-normal example

To build intuition for the general comparative statics that follow, we specialize the equilibrium of Theorem 1 to a tractable parametric case. We assume (i) CRRA preferences with common relative risk aversion η, so that $\rho(a) = \eta/W_0(a)$; (ii) common signal precision $t(a) = t$; and (iii) log-normal payoffs $V(v) = e^v$.

Equilibrium in log prices. Under these assumptions, the price function (14) becomes log-linear in the sufficient statistic:

$$\ln P = \frac{\tau_p}{\tau}\, s_p + \frac{1}{2\tau}.$$

The weighting function (13) reduces to $\omega(a) = W_0(a)/\bar{W}$, where $\bar{W} = \int_0^1 W_0(a)\, da$ is average wealth: each trader's weight in the price equals her wealth share.

The dollar trading intensity, $P\,\beta(a, P)$—the trading intensity converted from shares to dollars—is price-independent:

$$P\,\beta(a, P) = \frac{t\, W_0(a)}{\eta\, \tau\, (e^{1/\tau} - 1)}, \tag{19}$$

so aggregate dollar demand is linear in the log price, with slope proportional to aggregate risk tolerance $\bar{W}/\eta$.

A convenient feature of the CRRA–log-normal case is that market quality can be expressed in terms of a single, empirically observable statistic: the Herfindahl–Hirschman Index (HHI) of the wealth distribution, which is widely used to measure AUM concentration (see, e.g., Xiong et al., 2024).

Price informativeness and liquidity via the HHI. Define the Herfindahl–Hirschman Index (HHI) of the wealth distribution as

$$\text{HHI} = \int_0^1 \left(\frac{W_0(a)}{\bar{W}}\right)^2 da = \frac{\overline{W^2}}{\bar{W}^2},$$

where $\overline{W^2} = \int_0^1 W_0(a)^2\,da$. By Jensen's inequality HHI ≥ 1, with equality if and only if all agents have the same wealth. Substituting $\rho(a) = \eta/W_0(a)$ and $t(a) = t$ into the expressions in Proposition 1:

$$\tau_p = \frac{t}{\text{HHI}}, \qquad \mathcal{L} = \frac{\bar{W}}{\eta}\,\text{HHI}. \tag{20}$$

Information efficiency becomes

$$\mathcal{I} = \frac{t}{t + \tau_v\,\text{HHI}}. \tag{21}$$

Equations (20)–(21) deliver a sharp message. Greater wealth concentration (higher HHI) reduces price informativeness and information efficiency, because it tilts the price weights toward a smaller effective number of signals. At the same time, greater concentration increases liquidity, because a few wealthy, risk-tolerant traders absorb supply shocks more aggressively. Note that for a fixed wealth distribution, a higher average wealth $\bar{W}$ raises liquidity but leaves informativeness unchanged.

In the next two sections we show that these findings—the opposing effects of inequality on informativeness and liquidity—hold under much more general conditions.

The aggregate risk premium $\psi(P)$ also depends on HHI through the posterior precision $\tau = \tau_v + t/\text{HHI}$. Because the dependence of ψ on HHI is mediated by nonlinear conditional moments of the log-normal payoff, the comparative statics of ψ are richer than those of $\mathcal{I}$ and $\mathcal{L}$; we explore this in Appendix I.

7 Comparative statics

We now explore how market quality is affected by inequality, starting with inequality in wealth, followed by inequality in precision. In more concrete terms, we consider an equilibrium object $\mathcal{O}$—our placeholder notation for either information efficiency $\mathcal{I}$ or liquidity $\mathcal{L}$—and we change the primitives $W_0(a)$ and $t(a)$ around those associated with the original equilibrium. That is, the task at hand for this section is to carry out comparative statics with respect to functions. The right tool for this job is the Gateaux derivative.

Definition 2. *The* ***Gateaux derivative*** *an equilibrium object* $\mathcal{O}$ *with respect to a parameter*

$h(a)$ in the direction $h^{\Delta}(a)$ is

$$\mathcal{O}'\left(h(a)\right)\left[h^{\Delta}(a)\right] = \lim_{\varepsilon \to 0} \frac{\mathcal{O}\left(h(a) + \varepsilon h^{\Delta}(a)\right) - \mathcal{O}\left(h(a)\right)}{\varepsilon}.$$

To facilitate exposition, we use the following conventions and types of notation in our definition above. The *parameter* (namely, a function) with respect to which we differentiate appears inside round brackets, while the *direction* (another function) along which we perturb the parameter appears inside square brackets. Our notation also distinguishes the parameter from the direction by indicating the direction with a Δ superscript.[26]

Of particular interest are directions that correspond to reduced inequality. To this end, we refer to a variation of model parameters that makes poor agents better off (either richer or more informed) and rich agents worse off (either poorer or less informed) as a *Robin Hood variation.* We then index our agents in the same order as their wealth, implying that $W_0(a)$ *increases* in a, an assumption maintained hereafter without loss of generality. The poor and rich are defined with respect to thresholds $\underline{a}$ and $\bar{a}$, with the poor lying below $\underline{a}$ and the rich above $\bar{a}$.

Definition 3. *A* ***Robin-Hood variation*** *of parameter $h(a)$ is a direction $h^{\Delta}(a)$, bounded over $a \in [0, 1)$, and two associated thresholds $\underline{a} < \bar{a}$ such that*

(i) $h^{\Delta}(a) \geq 0$ *for* $a < \underline{a}$*,*

(ii) $h^{\Delta}(a) \leq 0$ *for* $a > \bar{a}$*,*

(iii) $h^{\Delta}(a) = 0$ *for* $a \in [\underline{a}, \bar{a}]$*, and*

(iv) $h^{\Delta}(a)$ *is not zero for some set of indices* a *with positive Lebesgue measure.*

Returning to our questions on wealth inequality, we can arrive at answers by considering how Robin-Hood variations in wealth affect market quality. As, however, wealth effects may also enter through information acquisition, we must do so with care, isolating different possible effects. We thus first take Robin-Hood variations with respect to wealth keeping precisions

[26] Once we fix a direction, computing a Gateaux derivative can be done with standard calculus. One differentiates $\mathcal{O}\left(h(a) + \varepsilon h^{\Delta}(a)\right)$ with respect to the scalar ε, evaluating the result at $\varepsilon = 0$.

fixed, then with respect to precisions keeping wealth fixed, then allowing for both effects by letting precisions depend on wealth.

As we see below, this sequence of comparative statics not only uncovers useful intuition but also reveals some surprising results. Before we proceed, however, we establish an important benchmark for the discussion of later subsections.

7.1 Ideal information aggregation in CHILE

We begin by pointing out that prices reveal the weighted average of all private signals—see equation (12). Revisiting how information is aggregated in our economy, we ask: is there an "informationally ideal" version of the weighting function $\omega(a)$, in the sense that the corresponding price would reveal maximum information to the agents?

We answer this question for a hypothetical weighting function $\omega^H(a)$, using it to define the notion of an *aggregate signal* as

$$s\left[\omega^H(a)\right] = \int_0^1 \omega^H(b) ds(b), \qquad \omega^H(a) \geq 0, \qquad \int_0^1 \omega^H(a) da = 1.$$

Writing $s_p = s[\omega(a)]$ then shows that the equilibrium-price weights $\omega(a)$ are but one possibility for $\omega^H(a)$, with the more general $\omega^H(a)$ yielding an aggregate signal with precision

$$\mathbb{V}\text{ar}\big(v \big| s\left[\omega^H(a)\right]\big)^{-1} - \mathbb{V}\text{ar}(v)^{-1}.$$

The informationally-ideal weighting function maximizes this precision, and is characterized in the following result.

Lemma 4 (Ideal information aggregation)**.** *No aggregate signal can exceed the cumulative precision of the entire economy, that is,*

$$\mathbb{V}ar\big(v \big| s\left[\omega^H(a)\right]\big)^{-1} - \mathbb{V}ar(v)^{-1} \leq \int_0^1 t(a) da \tag{22}$$

for any weighting function $\omega^H(a)$. The weighting function that maximizes signal precision *satisfies* (22) *as an equality, and is given by*

$$\omega^I(a) = \frac{t(a)}{\int_0^1 t(a)da}. \tag{23}$$

As we can see in (23), if we were to design an aggregate signal with maximizing informativeness as our only goal, we should be weighing each signal in proportion to its precision. Comparing (23) with (13) now highlights the key to understanding information inefficiency: the informationally-ideal weights are proportional to precisions, $\omega^I \propto t(a)$, whereas the price weights ω are distorted by risk tolerances, $\omega \propto t(a)/\rho(a)$. With DARA, it is the wealthier agents that can tolerate more risk. Consequently, DARA preferences yield equilibria where prices overweigh the signals of the rich, underweighing those of the poor.

7.2 Wealth inequality

Using the result above as an illuminating benchmark, we examine the effects of transferring wealth from the rich to the poor on market quality holding precisions fixed. To proceed, we need the following technical conditions.

Assumption 2. *The following hold for the profiles of wealth,* $W_0(a)$*, absolute risk tolerances,* $1/\rho(a)$*, and relative risk aversions,* $\rho(a)/W_0(a)$*.*

1. *The cross-sectional cdf of wealth is continuous, strictly increasing, and has support* $[0, \infty)$*;*
2. *The cross-sectional cdf of absolute risk tolerances* $1/\rho(a)$ *is continuous and strictly increasing;*
3. *There exists constants* $\underline{\eta}$ *and* $\overline{\eta}$ *such that* $0 < \underline{\eta} \leq \rho(a)\, W_0(a) \leq \bar{\eta} < \infty$.[27]

We also assume $t(a) > 0$, which means that in our comparative statics exercises, we only consider wealth transfers among informed traders. Changing wealth of uninformed (i.e., those with $t(a) = 0$) has no effect on market quality as is clear from Proposition 1. As in earlier parts

[27]The cross-sectional cdf of wealth (resp., absolute risk tolerances) is defined as $F_{W_0}(x) = \Lambda(a : W_0(a) \leq x)$ (resp., $F_{1/\rho}(x) = \Lambda(a : 1/\rho(a) \leq x)$. Here $\Lambda(\cdot)$ denotes the Lebesgue measure.

of the paper, we maintain the above assumption, from its statement onwards. We next state the main result of this section.

Proposition 2. *Suppose that agent preferences are DARA. Then, there exist thresholds* $0 < a_1^W \leq a_2^W < 1$*, such that for any Robin Hood variation* $W_0^\Delta(a)$ *with* $\underline{a} \leq a_1^W \leq a_2^W \leq \bar{a}$ *we have*

$$\mathcal{I}'(W_0(a))[W_0^\Delta(a)] > 0 \text{ and } \mathcal{I}'(W_0(a))[-W_0^\Delta(a)] < 0; \tag{24}$$

$$\mathcal{L}'(W_0(a))[W_0^\Delta(a)] < 0 \text{ and } \mathcal{L}'(W_0(a))[-W_0^\Delta(a)] > 0. \tag{25}$$

Reducing inequality by transferring wealth from the sufficiently rich to the sufficiently poor leads to higher informational efficiency. The intuition behind this is rooted in the distortion highlighted in the previous section: the price overweights the signals of the rich and underweights the signals of the poor. By redistributing wealth from the wealthy to the poor, this distortion is corrected, thereby improving informational efficiency.

We now turn to the liquidity result (25). There are two channels at play. First, a reduction in inequality is associated with increased information efficiency. When prices are more informative, traders become less willing to provide liquidity—for instance, by selling when prices increase—since higher prices are more likely to reflect stronger fundamentals. Second, there is the uncertainty reduction channel: higher information efficiency leads to less uncertainty about fundamentals, potentially decreasing $\text{Var}[V(v)|P]$ and making traders more willing to provide liquidity. By examining the risk-adjusted measure $\mathcal{L}$, we isolate the first effect. As a result, a reduction in inequality negatively impacts $\mathcal{L}$.

Beyond providing comparative statics, our results connect with several sets of empirical observations. First, regarding secular trends, it is known that while stocks have become more liquid since the beginning of the 20th century (e.g., Chordia, Roll, and Subrahmanyam, 2001), the informativeness of the average US stock has deteriorated (Farboodi, Matray, Veldkamp, and Venkateswaran, 2022a). This is puzzling given the improved data availability in modern markets.[28] Our model can explain both trends by appealing to the growing wealth inequality

[28]There is also evidence that the trend in price informativeness is not uniform: Bai, Philippon, and Savov (2016) find that the informativeness of S&P 500 firms has increased over time, while Farboodi et al. (2022a) show that the negative average trend is driven by small firms.

among individual investors (Saez and Zucman, 2016), or to the unequal distribution of assets under management for institutional investors (Ben-David et al., 2021).

Second, our results are consistent with recent empirical findings by Xiong et al. (2024), who document a negative relationship between wealth inequality—measured by the Herfindahl-Hirschman Index (HHI) of fund assets under management (AUM) or the ownership share of the top five investors—and information efficiency, as defined in Bai et al. (2016).[29] This relationship holds both at the market level and for individual stocks. Notably, what matters in their analysis is the concentration among *active* (i.e., informed) investors—consistent with our model, where uninformed demand can be absorbed into the supply function $\Theta(P)$ and plays no role in determining price informativeness. Importantly, our mechanism does not rely on market power, which plays a central role in the theoretical framework proposed by Xiong et al. (2024).[30]

Third, our model speaks to the recent rise in retail investor participation. If the smallest market participants are interpreted as retail investors, their private signals are systematically underweighted in equilibrium—a consequence of wealth effects under DARA preferences. Increasing retail participation generates two predictions: price informativeness should increase, while liquidity should decrease.[31] The first prediction is consistent with Boehmer, Jones, Zhang, and Zhang (2021), who find that retail order imbalances positively predict future stock returns, with predictive power that persists beyond what order-flow persistence, contrarian trading, or public news sentiment can explain—indicating that retail orders carry information not yet incorporated into prices. The second prediction is consistent with Eaton, Green, Roseman, and Wu (2022), who exploit brokerage outages as quasi-natural experiments and find that removing small, inexperienced investors from the market is associated with improved liquidity and lower volatility for stocks with high retail interest.

[29] They also examine the effects of an exogenous increase in inequality driven by fund mergers.

[30] Their framework builds on Kacperczyk, Nosal, and Sundaresan (2024).

[31] Formally, we model the rise of retail participation as a Robin Hood variation that increases the wealth of the smallest market participants while leaving the rest of the distribution unchanged. Because these agents lie below the threshold a_2^W in Proposition 2, this variation improves informational efficiency and reduces risk-adjusted liquidity.

7.2.1 The role and economic interpretation of thresholds a_1^W and a_2^W

To build intuition for the thresholds in Proposition 2, consider the special case of homogeneous preferences. Denoting risk tolerance by $y(a) = 1/\rho(a)$, precision τ_p (monotonically related to $\mathcal{I}$) and liquidity $\mathcal{L}$ become

$$\tau_p = \frac{\left(\int_0^1 t(a)y(a)\,da\right)^2}{\int_0^1 t(a)y(a)^2\,da}, \quad \mathcal{L} = \frac{\int_0^1 t(a)y(a)^2\,da}{\int_0^1 t(a)y(a)\,da}. \tag{26}$$

Index traders so that $y(a)$ is increasing in a; under DARA with homogeneous preferences, more risk-tolerant traders are also wealthier. A first-order expansion shows that increasing risk tolerance by ϵ for traders in $[a, a+da)$ changes precision and liquidity by

$$\begin{aligned}
\delta\tau_p &= t(a)\frac{2\epsilon\,da}{\mathcal{L}}\left(1 - \frac{y(a)}{\mathcal{L}}\right), \\
\delta\mathcal{L} &= t(a)\frac{2\epsilon\,da}{\int_0^1 t(x)y(x)\,dx}\left(y(a) - \frac{\mathcal{L}}{2}\right).
\end{aligned}$$

Here $\mathcal{L}$ can be interpreted as a weighted-average level of risk tolerance: it has the same units as y and lies between its minimum and maximum values.

These expressions pin down the thresholds. Precision τ_p increases with risk tolerance for agents below the average level $\mathcal{L}$ and decreases above it, yielding $y(a_2^W) = \mathcal{L}$. Liquidity $\mathcal{L}$ increases with risk tolerance for agents above $\mathcal{L}/2$ and decreases below it, yielding $y(a_1^W) = \mathcal{L}/2$. Since $a_1^W \leq a_2^W$, a Robin Hood transfer that spans both thresholds improves informational efficiency but reduces liquidity.

Thus, both thresholds are determined by the weighted-average risk tolerance, which in equilibrium coincides with the liquidity measure $\mathcal{L}$. This connection is natural: markets populated by more risk-tolerant agents are, ceteris paribus, more liquid, so $\mathcal{L}$ simultaneously captures the representative level of risk-bearing capacity and the ease of trading.

7.3 Information inequality

Here, we examine the effects of changing the distribution of information across agents on market quality holding the wealth profile fixed.

Proposition 3. *Suppose that traders have DARA utilities. Then, there exist thresholds* $0 < a_1^t \leq a_2^t < 1$*, such that for any Robin Hood variation* $t^{\Delta}(a)$ *with* $\underline{a} \leq a_1^t \leq a_2^t \leq \bar{a}$

$$\mathcal{I}'(t(a))[t^{\Delta}(a)] > 0 \text{ and } \mathcal{I}'(t(a))[-t^{\Delta}(a)] < 0; \quad (27)$$
$$\mathcal{L}'(t(a))[t^{\Delta}(a)] < 0 \text{ and } \mathcal{L}'(t(a))[-t^{\Delta}(a)] > 0.$$

Making the rich less informed and the poor more informed improves information efficiency but reduces liquidity. We discuss the intuition for the information result, from which the liquidity result follows. Recall the key distortion highlighted in Section 7.1: the trading intensities of the rich are too large, while those of the poor are too small. To correct this inefficiency, one needs to increase the trading intensities of the poor relative to the rich. This can be achieved by making the poor more informed and the rich less informed.

Note that the definition of a Robin Hood variation allows for weak inequalities. Therefore, the proposition above applies to a variation that decreases the precisions of the rich while leaving other precisions unchanged.

Corollary 1 (An information-aggregation paradox)**.** *Suppose that traders have DARA utilities. Then, there exists a threshold* a_2^t *such that for any* $t^{\Delta}(a) \neq 0$ *such that* $t^{\Delta}(a) \leq 0$ *for* $a > a_2^t$*, and* $t^{\Delta}(a) = 0$ *otherwise,* $\mathcal{I}'(t(a))[t^{\Delta}(a)] > 0$ *and* $\mathcal{I}'(t(a))[-t^{\Delta}(a)] < 0$.

Weakly increasing the information of all traders can hurt information efficiency, while weakly decreasing it can have the opposite effect. Banerjee, Davis, and Gondhi (2018), Dugast and Foucault (2017), and Glebkin and Kuong (2023) also showed that improving the quality of private information can reduce information efficiency. In their studies, the mechanism is that giving more information to some traders invites more noise from another source. In our paper, we focus on a pure information aggregation channel: the only noise comes from the traders' signals themselves. Yet, reducing the noise in some traders' signals can harm information efficiency. This occurs because better information is aggregated less effectively: increasing the precisions of the rich makes them trade more aggressively, causing their signals to be even more overweighted in price, exacerbating the existing distortion.

The aforementioned papers also do not feature wealth effects, so the effect there is not specific to large traders (i.e., traders with large wealth). An implication of our result is that making large investors differentially more informed could harm information efficiency. One of the changes introduced in the MiFID II regulation was to unbundle investment research from trading costs. Simplifying, before MiFID, everyone who traded obtained information. After that, only whoever is willing to pay gets it. Since large traders have a higher value of information (see Section D), such regulation makes large traders differentially more informed, potentially negatively affecting price informativeness.

7.3.1 The role and economic interpretation of thresholds a_1^t and a_2^t

To build intuition for the thresholds in Proposition 3, consider again the case of homogeneous preferences. Using the expressions for τ_p and $\mathcal{L}$ in (26), a first-order expansion shows that increasing precision by ϵ for traders in $[a, a + da)$ changes precision and liquidity by

$$\delta\tau_p = \frac{\epsilon\, da\, y(a)}{\mathcal{L}} \left(2 - \frac{y(a)}{\mathcal{L}}\right),$$
$$\delta\mathcal{L} = \frac{\epsilon\, da\, y(a)}{\int_0^1 t(x)y(x)\, dx} \left(y(a) - \mathcal{L}\right).$$

These expressions pin down the thresholds. Increasing precision improves τ_p (and hence $\mathcal{I}$) for agents with risk tolerance below $2\mathcal{L}$ and worsens it above, yielding $y(a_2^t) = 2\mathcal{L}$. Liquidity $\mathcal{L}$ increases with precision for agents above $\mathcal{L}$ and decreases below it, yielding $y(a_1^t) = \mathcal{L}$. Since $a_1^t \le a_2^t$, a Robin Hood transfer in information that spans both thresholds improves informational efficiency but reduces liquidity.

The sign of $\delta\tau_p$ above the threshold $y(a_2^t) = 2\mathcal{L}$ reflects the information-aggregation paradox. The expression (26) for τ_p can be interpreted as the signal-to-noise ratio of the price signal $\int_0^1 \beta(a)\, ds(a) \propto v \int_0^1 t(a)y(a)\, da + \int_0^1 (t(a)y(a))/\sqrt{t(a)}\, dB(a)$: the numerator $N = \left(\int_0^1 t(a)y(a)\, da\right)^2$

is proportional to the variance of the signal component, while the denominator $D = \int_0^1 t(a)y(a)^2\,da$ captures the variance of the noise. Agent a's marginal contribution to the signal is proportional to $y(a)$, whereas the contribution to the noise is proportional to $y(a)^2$. For sufficiently risk-tolerant agents, the noise contribution dominates, so increasing their precision worsens price informativeness despite adding information to the economy.

7.4 Wealth inequality redux: the role of information acquisition

Do the results in Proposition 2 hold when the precisions are endogenous (i.e., can depend on $W_0(a)$)? Consider the information efficiency result. The mechanism there is that the signals of the rich are overweighted, while the signals of the poor are underweighted. With endogenous information acquisition, one can expect the rich to acquire higher quality signals: they trade more aggressively and thus have more use for their information, making them value it more.[32] Is overweighting the signals of better quality a bad idea?

The transfer of wealth from rich to poor has two effects, one direct and one indirect. Proposition 2 captures the direct one. Proposition 3 captures the indirect one, by decreasing the precisions of the rich and increasing that of the poor, via the dependence of precisions on wealth. Since both effects work in the same direction, the results (24)–(25) (see p.27)are reinforced in the presence of information acquisition. We summarize this in the proposition below.

Proposition 4. *Suppose that Assumption 2 holds. Suppose that traders have DARA utilities. Suppose that precisions are a function of wealth $t(W_0(a), a)$ and are increasing in $W_0(a)$. Then, for any Robin Hood variation $W_0^{\Delta}(a)$ with $\underline{a} \leq \min\{a_1^t, a_1^W\} \leq \max\{a_2^t, a_2^W\} \leq \bar{a}$, results (24)–(25) hold.*

The result of Proposition 4 is perhaps surprising. Since the rich acquire more information than the poor, a Robin Hood variation could lead to a smaller overall amount of information being produced in the economy. Why does the information efficiency improve? Similar to our information aggregation paradox (Corollary 1), the effect of better aggregation of information dominates: Less information, but aggregated better results in more information efficiency.

[32] In Section D, we consider the information acquisition problem and show that agent a's precision $t(a)$ is indeed an increasing function of his wealth $W_0(a)$.

8 A summary of extensions and robustness exercises

This section summarizes additional robustness checks and model extensions, with full analyses presented in the Appendix.

- Section E shows that our main results are unique to the CHILE framework and do not extend to traditional large-economy models based on law-of-large-numbers (LLN) aggregation, such as Hellwig (1980), Admati (1985), and Peress (2004). The key insight is an aggregation result: even when investors differ in wealth, precision, and preferences, the economy remains observationally equivalent to one with homogeneous agents. As a result, Robin Hood variations that preserve aggregate trading intensity have no effect on market quality—unlike in CHILE.

- Section F incorporates noise traders into our main model. The resulting equilibrium captures both the classical mechanism—where inequality improves information efficiency via greater aggregate trading intensity—and the novel mechanism introduced in this paper, where inequality disrupts signal aggregation. The key result is that the latter effect dominates, making our core conclusion—that transferring wealth from rich to poor improves information efficiency but reduces liquidity—robust to the presence of noise.

- Section G compares three equilibrium concepts in the CARA-normal setting: (i) Bayes-Nash Equilibrium, where traders internalize both their price impact and their impact on the informational content of prices; (ii) Rational-Expectations Equilibrium, where they internalize only the latter; and (iii) the price-taking equilibrium, where both effects are ignored. We show that our main results—wealth redistribution from rich to poor improves information efficiency but lowers liquidity—hold under both competitive Rational Expectations and Bayes-Nash, and are not artifacts of the price-taking equilibrium concept. Section H extends the REE and BNE analysis to general preferences and payoff structures.

9 Literature review

There are several branches of literature related to our paper. First, there is literature on REE models that go beyond the CARA-Normal framework.[33] Breon-Drish (2015) extends the CARA-Normal framework beyond normality in a single asset setup. Chabakauri et al. (2022) further extends Breon-Drish (2015) by allowing for multiple assets. Albagli, Hellwig, and Tsyvinski (2021) consider a setup with general distribution and risk-neutral traders subject to position limits. All of these papers assume CARA utility and so abstract away from wealth effects that are central to our paper.[34]

Malamud (2015) considers an REE model with a continuum of assets. Central to the tractability of his framework is the assumption of market completeness.[35] In contrast, we have one asset and a continuum of states of the world, hence our market is incomplete. One of the central results in Malamud (2015) is that with non-CARA utility, the equilibrium is fully revealing.[36] In contrast, in our incomplete market setup, there is no full revelation for any utility function, thanks to the aggregate price noise.

Peress (2004) was the first (to our knowledge) to study wealth effects in noisy REE.[37] His model features log-normally distributed payoffs and non-CARA utilities. The key difference from our paper is that Peress (2004) relies on a "small risk" approximation, where the riskiness of the asset is small. In his limit, the variance of risky asset return is zero, making such an approach not suitable for quantitative work (as it would be hard to match variance). Our approximation is essentially "small information." In contrast to Peress (2004), in our model the asset stays risky even in the limit. Beyond different limit behaviors, our paper differs from Peress (2004) in several ways. First, in our model, the equilibrium quantities are affected

[33] CARA-Normal framework is also used to model markets where traders have market power (see Rostek and Yoon (2020) for a review). Glebkin et al. (2023a) and Glebkin, Malamud, and Teguia (2023b) allow for, respectively, non-normal payoffs in a setup with CARA traders and non-normal payoffs in a setup that also allows for non-CARA utilities. These papers abstract from information frictions that are central to this paper.

[34] Here, we consider risk-neutral preferences as a special case of CARA with risk aversion equal to zero.

[35] Relatedly, DeMarzo and Skiadas (1998) and DeMarzo and Skiadas (1999) analyze REE models, where the market is *quasi-complete*.

[36] See Theorem 2.1 in Malamud (2015). See also Chabakauri (2024) who shows that the results about full revelation with non-CARA utility are more nuanced.

[37] The wealth effect can arise even in CARA model, because the wealth may affect the tightness of financial constraints, as in Glebkin et al. (2021).

by absolute risk aversion and absolute prudence, whereas in Peress (2004) only risk aversion matters. What is more; the conditional third central moment ("skewness") plays a role in our model, but not in Peress (2004). Finally, our model allows for generally-distributed asset payoffs.

Our paper is also related to asset pricing literature studying the implications of heterogeneity in preferences and wealth for asset prices. Examples include Dumas (1989), Gârleanu and Panageas (2015) and Gomez et al. (2016)–see Panageas (2020) for a review. This literature focuses on the implications of wealth heterogeneity on risk premia and risk-free rates but abstracts away from informational frictions and does not derive implications for market quality that are central to our paper.

Next, our paper is also related to the literature on mean-field games. See Lasry and Lions (2007) and Achdou, Han, Lasry, Lions, and Moll (2022) and, for a review, Guéant, Lasry, and Lions (2011). As Achdou et al. (2022) note: "The name (Mean Field Games) comes from an analogy to the continuum limit taken in 'Mean Field theory' which approximates large systems of interacting particles by assuming that these interact only with the statistical mean of other particles." This analogy holds in our model. The effect of other traders on a trader of interest in our economy is summarized by several statistics of the cross-sectional distribution of traders' characteristics. These statistics can be viewed as a "mean field" that influences each trader's equilibrium behavior. As in Mean Field theory, other traders do not affect a trader of interest directly, but only through their (infinitesimal) contribution to the mean field.

On the technical side, our paper is also related to literature that uses stochastic calculus tools outside the domain of continuous time finance and economics. Examples include Malamud (2015) who models the noise in a continuum of assets as a cross-sectional stochastic process; Gârleanu, Panageas, and Yu (2015) who use a Brownian bridge to represent the dividends for firms located on a circle; and Glebkin et al. (2021) who use stochastic calculus techniques to derive a marginal value of information in a static model.[38] Finally, the most closely related paper is Avdis (2018), which introduces a model with continuous heterogeneous information, albeit with CARA preferences and, as a result, without wealth effects.

[38] There is also a related econometric literature on the unit roots. A good example is Phillips (1987).

10 Conclusion

We introduce a new asymmetric-information asset-pricing framework called "Continuous-and-Heterogeneous Information in a Large Economy" (CHILE). In this economy, we study perfect competition with rich agent heterogeneity, arbitrary preferences, and general payoff distributions. A unique equilibrium features all quantities in closed form. Leveraging the tractability of our model and its ability to work with wealth effects, we show how changes in the distribution of wealth affect different aspects of market quality: information efficiency and liquidity.

There are many potentially fruitful extensions. While we focus on a competitive CHILE equilibrium in this paper, in ongoing and preliminary work we show that the strategic equilibrium has a different limit. Moreover, even though we model markets as uniform-price auctions, our techniques can also find applications in discriminatory-price auctions. Finally, our framework can help study several interesting environments, such as those with dynamic trading, feedback effects, and endogenous growth.

References

Yves Achdou, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll. Income and wealth distribution in macroeconomics: A continuous-time approach. The review of economic studies, 89(1):45–86, 2022. 35

Anat R Admati. A noisy rational expectations equilibrium for multi-asset securities markets. Econometrica: Journal of the Econometric Society, pages 629–657, 1985. 6, 15, 33, 76

Elias Albagli, Christian Hellwig, and Aleh Tsyvinski. Dispersed information and asset prices. 2021. 34

Efstathios Avdis. Risk seekers: trade, noise, and the rationalizing effect of market impact on convex preferences. FTG Working Paper Series, (No. 00051-00), 2018. URL `https://financetheory.org/public/storage/working_paper/00051-00.pdf`. 14, 35

Efstathios Avdis, Sergei Glebkin, and Joel Peress. On competitive equilibria in large economies. Working paper, 2025. 2, 3, 88

Jennie Bai, Thomas Philippon, and Alexi Savov. Have financial markets become more informative? Journal of Financial Economics, 122(3):625 – 654, 2016. ISSN 0304-405X. 27, 28

Snehal Banerjee, Jesse Davis, and Naveen Gondhi. When transparency improves, must prices reflect fundamentals better? The Review of Financial Studies, 31(6):2377–2414, 2018. 30

Itzhak Ben-David, Francesco Franzoni, Rabih Moussawi, and John Sedunov. The granular nature of large institutional investors. Management Science, 67(11):6629–6659, 2021. 3, 28

Fischer Black. Studies of stock price volatility changes. Proceedings of the 1976 Meetings of the American Statistical Association, Business and Economic Statistics Section, pages 177–181, 1976. 81

Ekkehart Boehmer, Charles M. Jones, Xiaoyan Zhang, and Xinran Zhang. Tracking retail investor activity. The Journal of Finance, 76(5):2249–2305, 2021. 28

Bradyn Breon-Drish. On existence and uniqueness of equilibrium in a class of noisy rational expectations models. The Review of Economic Studies, 82(3):868–921, 2015. doi: 10.1093/restud/rdv012. URL http://restud.oxfordjournals.org/content/82/3/868.abstract. 6, 12, 13, 34

John Y Campbell and Luis M Viceira. Strategic asset allocation: portfolio choice for long-term investors. Clarendon Lectures in Economic, 2002. 18

Georgy Chabakauri. Informational efficiency and asset prices in large markets. Available at SSRN, 2024. 34

Georgy Chabakauri, Kathy Yuan, and Konstantinos E Zachariadis. Multi-asset noisy rational expectations equilibrium with contingent claims. The Review of Economic Studies, 89(5): 2445–2490, 2022. 6, 12, 34

Tarun Chordia, Richard Roll, and Avanidhar Subrahmanyam. Market liquidity and trading activity. The journal of finance, 56(2):501–530, 2001. 27

John Cochrane. Asset pricing: Revised edition. Princeton university press, 2009. 43

Eduardo Dávila and Cecilia Parlatore. Identifying price informativeness. working paper, 2023. 21

P. DeMarzo and C. Skiadas. Aggregation, determinacy, and informational efficiency for a class of economies with asymmetric information. Journal of Economic Theory, 80(1):123–152, 1998. 34

P. DeMarzo and C. Skiadas. On the uniqueness of fully informative rational expectations equilibria. Economic Theory, 13(1):1–24, 1999. 34

Oolivier Dessaint, Thierry Foucault, and Laurent Fresard. Does alternative data improve financial forecasting? the horizon effect. The Journal of Finance, 79(3):2237–2287, 2024. 21

Jérôme Dugast and Thierry Foucault. Data abundance and asset price informativeness. Working paper, 2017. 30

Bernard Dumas. Two-person dynamic equilibrium in the capital market. The Review of Financial Studies, 2(2):157–188, 1989. 35

Gregory W. Eaton, T. Clifton Green, Brian S. Roseman, and Yanbin Wu. Retail trader sophistication and stock market quality: Evidence from brokerage outages. Journal of Financial Economics, 146(2):502–528, 2022. 28

Maryam Farboodi, Adrien Matray, Laura Veldkamp, and Venky Venkateswaran. Where has all the data gone? The Review of Financial Studies, 35(7):3101–3138, 2022a. 27

Maryam Farboodi, Dhruv Singal, Laura Veldkamp, and Venky Venkateswaran. Valuing financial data. Technical report, National Bureau of Economic Research, 2022b. 18, 76

Xavier Gabaix. The granular origins of aggregate fluctuations. Econometrica, 79(3):733–772, 2011. 1

Xavier Gabaix and Ralph SJ Koijen. In search of the origins of financial fluctuations: The inelastic markets hypothesis. Technical report, National Bureau of Economic Research, 2021. 1

Xavier Gabaix and Ralph SJ Koijen. Granular instrumental variables. Journal of Political Economy, 132(7):2274–2303, 2024. 1

Nicolae Gârleanu and Stavros Panageas. Young, old, conservative, and bold: The implications of heterogeneity and finite lives for asset pricing. Journal of Political Economy, 123(3): 670–685, 2015. 35

Nicolae Gârleanu, Stavros Panageas, and Jianfeng Yu. Financial entanglement: A theory of incomplete integration, leverage, crashes, and contagion. American Economic Review, 105(7):1979–2010, July 2015. doi: 10.1257/aer.20131076. URL http://www.aeaweb.org/articles?id=10.1257/aer.20131076. 35

Sergei Glebkin and John Chi-Fong Kuong. When large traders create noise. Journal of Financial Economics, 150(2):103709, 2023. 30

Sergei Glebkin, Naveen Gondhi, and John Chi-Fong Kuong. Funding constraints and informational efficiency. The Review of Financial Studies, 34(9):4269–4322, 2021. 13, 34, 35

Sergei Glebkin, Semyon Malamud, and Alberto Teguia. Illiquidity and higher cumulants. The Review of Financial Studies, 36(5):2131–2173, 2023a. 21, 34

Sergei Glebkin, Semyon Malamud, and Alberto Teguia. Strategic trading with wealth effects. 2023b. 34

Arthur Stanley Goldberger. A course in econometrics. Harvard University Press, 1991. 48

Matthieu Gomez et al. Asset prices and wealth inequality. Unpublished paper: Princeton, 2016. 35

Sanford Grossman. On the efficiency of competitive stock markets where trades have diverse information. The Journal of finance, 31(2):573–585, 1976. 19, 20

Sanford J. Grossman and Joseph E. Stiglitz. On the impossibility of informationally efficient markets. The American Economic Review, 70(3):393–408, June 1980. 4

Olivier Guéant, Jean-Michel Lasry, and Pierre-Louis Lions. Mean field games and applications. In Paris-Princeton lectures on mathematical finance 2010, pages 205–266. Springer, 2011. 35

Martin F. Hellwig. On the aggregation of information in competitive markets. Journal of Economic Theory, 22(3):477–498, 1980. URL http://linkinghub.elsevier.com/retrieve/pii/0022053180900563. 1, 2, 5, 6, 9, 12, 14, 15, 20, 33, 45, 46, 76, 78

Marcin Kacperczyk, Jaromir Nosal, and Savitar Sundaresan. Market power and price informativeness. Review of Economic Studies, page rdae077, 2024. 28

Alexander Kovalenkov and Xavier Vives. Competitive rational expectations equilibria without apology. Journal of Economic Theory, 149:211 – 235, 2014. ISSN 0022-0531. doi: http://dx.doi.org/10.1016/j.jet.2013.05.002. URL http://www.sciencedirect.com/science/article/pii/S0022053113001026. Financial Economics. 2, 96

Pablo Kurlat and Laura Veldkamp. Should we regulate financial information? Journal of Economic Theory, 158:697–720, 2015. 5, 88

Albert S. Kyle. Continuous auctions and insider trading. Econometrica, 53(6):1315–1335, 1985. URL http://www.jstor.org/stable/1913210. 21

Albert S. Kyle. Informed speculation with imperfect competition. Review of Economic Studies, 56:317–356, 1989. 6, 21, 96, 97

Jean-Michel Lasry and Pierre-Louis Lions. Mean field games. Japanese journal of mathematics, 2(1):229–260, 2007. 35

Dmitry Makarov and Astrid V Schornick. A note on wealth effect under cara utility. Finance Research Letters, 7(3):170–177, 2010. 5, 88

Semyon Malamud. Noisy arrow-debreu equilibria. Available at SSRN 2572881, 2015. 6, 34, 35

Semyon Malamud and Marzena Rostek. Decentralized exchange. American Economic Review, 107(11):3320–3362, 2017. 21

Roxana Mihet. Financial technology and the inequality gap. Swiss Finance Institute Research Paper, (21-04), 2018. 88

Roxana Mihet. Financial information technology and the inequality gap. 2022. 5, 18, 76

P. Milgrom and N. Stokey. Information, trade and common knowledge. Journal of Economic Theory, 26(1):17–27, 1982. 19

Stavros Panageas. The implications of heterogeneity and inequality for asset pricing. Foundations and Trends in Finance, 12(3):199–275, 2020. ISSN 1567-2395. doi: 10.1561/0500000057. URL http://dx.doi.org/10.1561/0500000057. 1, 35

Joel Peress. Wealth, information acquisition, and portfolio choice. The Review of Financial Studies, 17(3):879–914, 2004. 1, 5, 6, 18, 33, 34, 35, 76

Joel Peress. Erratum: Wealth, information acquisition and portfolio choice, a correction. The Review of Financial Studies, 24(9):3187–3195, 2011. 18, 76, 77

Joel Peress. Learning from stock prices and economic growth. The Review of Financial Studies, 27(10):2998–3059, 2014. 6

Peter CB Phillips. Time series regression with a unit root. Econometrica: Journal of the Econometric Society, pages 277–301, 1987. 35

Marzena Rostek and Marek Weretka. Price inference in small markets. Econometrica, 80(2): 687–711, 2012. 21

Marzena Rostek and Marek Weretka. Dynamic thin markets. The Review of Financial Studies, 28(10):2946–2992, 2015. 21

Marzena J Rostek and Ji Hee Yoon. Equilibrium theory of financial markets: Recent developments. Available at SSRN 3710206, 2020. 34

Emmanuel Saez and Gabriel Zucman. Wealth inequality in the united states since 1913: Evidence from capitalized income tax data. The Quarterly Journal of Economics, 131(2):519–578, 2016. 4, 28

Paul A Samuelson. The fundamental approximation theorem of portfolio analysis in terms of means, variances and higher moments. The Review of Economic Studies, 37(4):537–542, 1970. 18

J Michael Steele. Stochastic calculus and financial applications, volume 1. Springer, 2001. 43

Jean Tirole. On the possibility of speculation under rational expectations. Econometrica, 50(5): 1163–1181, 1982. ISSN 00129682, 14680262. URL http://www.jstor.org/stable/1911868. 19

Dimitri Vayanos and Jiang Wang. Market liquidity—theory and empirical evidence. In Handbook of the Economics of Finance, volume 2, pages 1289–1361. Elsevier, 2013. 21

Robert E Verrecchia. Information acquisition in a noisy rational expectations economy. Econometrica: Journal of the Econometric Society, pages 1415–1430, 1982. 4

Robert Wilson. Auctions of shares. The Quarterly Journal of Economics, 93(4):675–689, 1979. 116, 131

Yan Xiong, Liyan Yang, and Zexin Zheng. Institutional ownership concentration and informational efficiency. Available at SSRN, 2024. 4, 5, 22, 28

Appendices

A Summary of notation

Notation	Explanation
	General mathematical notation
$X = \text{plim}_{n\to\infty} X_n$ or $X_n \xrightarrow[n\to\infty]{\mathbb{P}} X$	X_n converges to X in probability
$dB(a)$	An increment of the Brownian motion
$u^{(l)}(\cdot)$	l-th derivative of $u(\cdot)$
a, b	Index of agents in the continuous economy
i, j	Index of agents in the discrete economy
n	Number of agents in the discrete economy
m_i	Mass of an agent in the discrete economy
$\mu = 1/n$	Average mass of an agent in the discrete economy
$\text{Sk}[Y] = \mathbb{E}[(Y - E[Y])^3]$	Third central moment of r.v. Y (unstandardized skewness)
$\Phi(\cdot)$	Standard normal cdf
$\mathcal{O}'\left(h(a)\right)\left[h^{\Delta}(a)\right]$	Gateaux derivative of an equilibrium object $\mathcal{O}$ with respect to a parameter $h(a)$ in the direction $h^{\Delta}(a)$
	Model variables
$v \sim N(0, \tau_v^{-1})$	Fundamental, with ex-ante precision τ_v
$V(v)$	Payoff function
$\Theta(P) = \bar{\theta} + \theta(P)$	Supply of risky asset
$ds(a) = v\,da + \frac{1}{\sqrt{t(a)}}dB(a)$	Trader a's signal, with precision $t(a)$
$\mathcal{F}_{b,c} = \sigma\left(\{s(z) - s(b)\}_{b\le z<c}\right)$	Information available to agents living in $[b, c)$
$\mathcal{F}_1 = \mathcal{F}_{0,1}$	Information in the entire economy
$dX(a) = \beta(a,P)ds(a) + \delta(a,P)da$	Demand of trader a; $\beta(a,P)$ is a's trading intensity

Notation	Explanation
$\rho(a) = -u''(a)/u'(a)$	Absolute risk aversion of trader a
$\pi(a) = -u'''(a)/u''(a)$	Absolute prudence of trader a
$s\left[\omega^H(a)\right] = \int_0^1 \omega^H(b)ds(b)$	Aggregate signal; $\omega^H(a)$ is a weighting function
$s_p = s[\omega(a)] = \int_0^1 \omega(a)ds(a)$	Equilibrium sufficient statistic; $\omega(a)$ is equilibrium weighting function
$\mathcal{P}(s_p)$	Equilibrium price function
$h(\cdot)$	The inverse of $\mathcal{P}(\cdot)$

B Heuristic derivation of equilibrium

In this section, we sketch out the derivation of key equilibrium objects, applying familiar heuristics that have become available in CHILE due to its setup.[39]

B.1 Key stochastic calculus heuristics

We begin by recalling some properties of Brownian Motion and its increments. For $a \neq b \in [0, 1)$, we have

$$dB(a) \perp\!\!\!\perp dB(b) \qquad dB(a), dB(b) \sim \mathcal{N}(0, da),$$

allowing us to interpret the signals in (5) as a continuum of pointwise mutually-independent Normal random variables.

In what follows, we employ second-order Taylor expansions ignoring terms of orders higher than da, following a method commonly referred to as the "box calculus" (Steele, 2001, Chapter

[39]See, e.g., Cochrane (2009), Appendix A.3, for an application of these heuristics over time, instead of over agents.

8.4):

$$da \cdot da = 0, \quad dB(a) \cdot da = 0, \quad dB(a) \cdot dB(a) = da; \quad dB(a) \cdot dB(b) = 0. \tag{28}$$

B.2 Information continuity linearizes demand

As our Representation Lemma demonstrates, the demands *must* be linear in signals (see (9)). Suppose, by contradiction, that demands are non-linear in signals—for example, assume that (9) contained a squared-signal term. Denoting the coefficient of the quadratic signal as $\zeta(\cdot)$ and denoting the new version of the remaining quantities by adding tildes on top, we have

$$\begin{aligned} dX(a) &= \tilde{\beta}(P,a)ds(a) + \tilde{\delta}(P,a)da + \zeta(P,a)ds^2(a) \\ &= \tilde{\beta}(P,a)ds(a) + \left[\tilde{\delta}(a,P) + \frac{\zeta(a,P)}{t(a)}\right]da, \end{aligned} \tag{29}$$

where the second equality follows by the box-calculus heuristic. What is more, by comparing (29) to (9) we can see that the demand function in (9) can subsume the extra term through its da term without affecting the signal term, giving us a contradiction. To summarize, the functional form in (9) already accounts for squared signals.[40]

B.3 Price inference

Equation (10) implies that information in the price is summarized by

$$s_p = v + \int_0^1 \omega(a,P)\frac{dB(a)}{\sqrt{t(a)}}.$$

This signal has a familiar "truth plus normally distributed noise" structure. Once the precision τ_p is known, the inference from prices, summarized in Lemma 3, follows from Bayes's rule for

[40] Restricting attention to quadratic adjustments is without loss of generality, as any sufficiently differentiable non-linear function can be expanded into a polynomial of $ds(a)$, with terms of order three and above becoming zero due to (28).

normal random variables. Here, we derive expression (11) for τ_p heuristically; see Appendix C.4 for a formal proof.

The "noise" term $\int_0^1 \omega(a,P)/\sqrt{t(a)}\,dB(a)$ is normally distributed with mean 0 and variance $\int_0^1 \omega(a,P)^2/t(a)\,da$. This follows from the basic properties of stochastic integrals but can also be derived heuristically using the heuristics outlined at the beginning of this section. Indeed, the noise $\int_0^1 \omega(a,P)/\sqrt{t(a)}\,dB(a)$ accumulates terms $\omega(a,P)/\sqrt{t(a)}\,dB(a)$, each normally distributed with mean zero and variance $\omega(a,P)^2/t(a)\,da$, and independent from each other. As an accumulation of normal mean-zero independent random variables, the integral is normally distributed with a mean of zero and a variance that sums the variances of the individual terms, i.e.,

$$\mathrm{Var}\left[\int_0^1 \omega(a,P)\frac{dB(a)}{\sqrt{t(a)}}\right] = \int_0^1 \omega(a,P)^2/t(a)\,da.$$

This variance is the reciprocal of τ_p.

B.3.1 Price inference for a price-taking agent

Note that equation (10b) can be expressed alternatively as:

$$s_p = v + \frac{\omega(a,P)}{\sqrt{t(a)}}dB(a) + \int_{b\in[0,1)\setminus[a,a+da)} \frac{\omega(b,P)}{\sqrt{t(b)}}dB(b).$$

However, a trader viewing the price in this manner would not be acting as a price taker. Consistent with Hellwig (1980)'s critique, such a trader would "account for the covariance between the 'noise' in their own information and the 'noise' embedded in the price." Indeed, using the heuristics presented above, one can obtain:

$$\mathrm{cov}(dB(a), s_p) = \frac{\omega(a,P)}{\sqrt{t(a)}}da.$$

A price-taking trader, who believes their signal does not influence the price, must perceive this covariance as zero.

We take a page right out of Hellwig (1980)'s critique: our agents assume that the noise in their private information is independent of the noise in the price. Specifically, trader a perceives the price as equivalent to a sufficient statistic $\hat{s}_p$, defined by

$$\hat{s}_p = v + \frac{\omega(a,P)}{\sqrt{t(a)}} d\hat{B}(a) + \int_{b\in[0,1)\setminus[a,a+da)} \frac{\omega(b,P)}{\sqrt{t(b)}} dB(b),$$

where the Brownian motions $\hat{B}$ and B are independent. Under this assumption, it is straightforward to see that

$$\text{cov}(dB(a), \hat{s}_p) = 0.$$

This represents the minimal deviation in traders' beliefs consistent with price-taking behavior. Indeed, our traders correctly perceive their private signals $ds(a)$, the marginal distribution of s_p, and the joint distribution of s_p and v. Their only mistake lies in misunderstanding the correlation between the noise in their own signals and the noise embedded in the price. Consequently, the conditional distribution of v given $\hat{s}_p$ coincides with that of v given s_p. In particular, this implies that $\hat{s}_p$ and s_p have identical precision. We also emphasize that the mistakes agents make are infinitesimal. In the limit as $da \to 0$, their perceived price statistic $\hat{s}_p$ coincides exactly with the actual statistic s_p.

B.4 A market-efficiency result

Consider trader a, who has size da and receives signal ds. Trader a's optimal demand x satisfies the first-order condition:

$$\mathbb{E}[u'\left(W_0(a) + (V-P)x\right)(V-P)|ds(a), \hat{s}_p] = 0, \tag{30}$$

where we have dropped the dependence of the payoff function on v to lighten notation.

We note that demands must go to zero as the number of agents goes to infinity—if that was not the case, aggregate demand would sum up infinitely many terms of finite size, and it would explode. Consequently, $x = 0$ must solve (30) in the large economy limit. Further noting that the information set of $(ds(a), \hat{s}_p)$ collapses to that of just $\hat{s}_p$ in the limit, setting $x = 0$ in (30) yields

$$\mathbb{E}[u'(W_0(a))(V-P)|\hat{s}_p] = 0 \iff P = \mathbb{E}[V|\hat{s}_p].$$

Since the conditional distribution of v given $\hat{s}_p$ coincides with that of v given s_p, we can replace $\hat{s}_p$ with s_p, yielding:

$$P = \mathbb{E}[V|s_p]. \tag{31}$$

This result implies that equilibrium prices in our model are weak-form efficient.

B.5 Deriving trading intensity

As we will show below, market quality is completely determined by the trading-intensity $\beta(a, P)$, on which we now focus. We Taylor-expand (30) over x up to the second order, and then we replace x by $dX(a)$, obtaining

$$\begin{aligned} 0 = u'(W_0(a))\,\mathbb{E}[(V-P)|ds(a), \hat{s}_p] \\ + u''(W_0(a))\,\mathbb{E}\left[(V-P)^2\middle|ds(a), \hat{s}_p\right] dX(a) \\ + \frac{1}{2}u'''(W_0(a))\,\mathbb{E}\left[(V-P)^3\middle|ds(a), \hat{s}_p\right] dX(a)^2. \end{aligned} \tag{32}$$

To solve for trading intensity, we plug (9) into (32) and compare the result with (9). We then solve for $\beta(a, P)$ by matching $ds(a)$ coefficients (for $\delta(a, P)$, we can match da terms).

After some simplifications explained in detail in Appendix C.11, we obtain

$$0 \;=\; u'(W_0(a))\,\mathbb{E}[(V-P)|ds(a), \hat{s}_p] + u''(W_0(a))\,\mathbb{E}\left[(V-P)^2\middle|s_p\right]\beta(a, P)ds(a) + \cdots, \tag{33}$$

where we have omitted terms that do not contain the signal, as they do not influence our subsequent calculations. Such "non-ds" terms are denoted by "$\cdots$" in the remainder of the subsection.

Next, note that by the market efficiency condition (31) we have $P = E[V|s_p]$ and so $\mathbb{E}[(V-P)^2|s_p] = \mathrm{Var}[V|s_p]$. Note also that we can replace $\rho(a) = -u''(W_0(a)/u'(W_0(a))$, where $\rho(a)$ is the risk-aversion coefficient of agent a. This simplifies the above even further, eventually yielding

$$\beta(a,P)ds(a) = \frac{\mathbb{E}[(V-P)|ds(a),\hat{s}_p]}{\rho(a)\,\mathbb{V}\mathrm{ar}(V|P)} + \cdots \tag{34}$$

Juxtaposing (34) with (9) brings forth a striking property: even though we use general preferences, the demand function depends on the signal as if preferences were mean-variance. We can think of this property as the converse of linearizing demand—by applying box calculus on the expanded first-order condition, all higher-order signal terms drop out, and thus the only way for signals to influence demand is through the first two terms in (32).[41]

To pin down $\beta(a,P)$ we need to separate out the $ds(a)$ term on the right-hand side of (34). To this end, we compute the $ds(a)$ term in the conditional expectation $\mathbb{E}[(V-P)|ds(a),\hat{s}_p]$. Note that for small $ds(a)$, this conditional expectation is linear in $ds(a)$ and so can be computed using a familiar linear regression formula[42]

$$\mathbb{E}[(V-P)|ds(a),\hat{s}_p] = \frac{\mathbb{C}\mathrm{ov}(V(v),ds(a)|\hat{s}_p)}{\mathbb{V}\mathrm{ar}(ds(a)|\hat{s}_p)}ds(a)+\cdots = \frac{\mathbb{C}\mathrm{ov}(v,ds(a)|\hat{s}_p)}{\mathbb{V}\mathrm{ar}(ds(a)|\hat{s}_p)}\,\mathbb{E}[V'(v)|\hat{s}_p]ds(a)+\cdots \tag{35}$$

where the second equality follows from Stein's Lemma.[43]

[41]In equilibrium, preferences cannot be substituted by a mean-variance tradeoff locally approximating the utility. While higher-order moments of returns and higher-order marginal preferences (i.e. prudence) may not affect $\beta(a,P)$, they still affect $\delta(a,P)$.

[42]More precisely, the linearity of expectation allows to substitute it with the Best Linear Predictor formula (see, e.g., Goldberger (1991), Chapter 5.4.) that is written following the first equality in (35).

[43]Stein's Lemma states that if X and Y are jointly normally distributed, then for any differentiable function g such that $\mathbb{E}|g'(X)|<\infty$, $\mathbb{C}\mathrm{ov}(g(X),Y) = \mathbb{C}\mathrm{ov}(X,Y)\cdot\mathbb{E}\left[g'(X)\right]$.

Moreover, up to terms of order higher than da, we have

$$\mathbb{C}\text{ov}(v, ds(a)|\hat{s}_p) = \mathbb{C}\text{ov}(v, v\, da|\hat{s}_p) = da \cdot \mathbb{V}\text{ar}(v|\hat{s}_p) = \frac{da}{\tau}.$$

The first equality above uses the independence of $dB(a)$ and $\hat{s}_p$, and the third equality applies the identity $\tau \equiv 1/\mathbb{V}\text{ar}(v|\hat{s}_p)$.

Similarly, we compute

$$\mathbb{V}\text{ar}(ds(a)|\hat{s}_p) = \mathbb{V}\text{ar}(dB(a)/\sqrt{t(a)}|\hat{s}_p) = da/t(a).$$

Combining everything, (34) becomes

$$\beta(a, P)ds(a) = \frac{t(a)}{\tau} \frac{\mathbb{E}[V'(v)|\hat{s}_p]}{\rho(a)\, \mathbb{V}\text{ar}(V|\hat{s}_p)} ds(a) + \cdots \iff \beta(a, P) = \frac{t(a)}{\rho(a)} \frac{\mathbb{E}[V'(v)|s_p]}{\tau\, \mathbb{V}\text{ar}(V|s_p)}.$$

In the last step above, we have replaced $\hat{s}_p$ with s_p inside the terms $\mathbb{E}[V'(v)|s_p]$ and $\mathbb{V}\text{ar}(V|s_p)$, relying on the fact that the conditional distributions of v given $\hat{s}_p$ and v given s_p coincide.

C Derivations and proofs

C.1 Technical conditions

The equilibrium construction requires differentiating the first-order conditions for optimality and evaluating the resulting expressions in the large-economy limit ($\mu \to 0$, $s \to 0$). We impose the following conditions on the primitives.

Assumption 3 (Technical restrictions on the primitives)**.** *We impose the following restrictions:*

1. *There exist $q \geq 0$ and $C_u < \infty$ such that for all $w \in \mathbb{R}$ and $a \in [0, 1)$, $|u'(w, a)| + |u''(w, a)|$ $+ |u'''(w, a)| \leq C_u(1 + |w|^q)$.*

2. *$V(v)$ satisfies $\lim_{v\to\pm\infty} \frac{\ln|V(v)|}{v^2} \leq 0$.*

3. *There exist $\varepsilon > 0$ and constants $A, k > 0$ such that for all $v \in \mathbb{R}$,*

$$\sup_{|\mu|\leq\varepsilon} |g(v, P, \mu)| \leq Ae^{-kv^2},$$

and the same bounds hold for the partial derivatives:

$$\sup_{|\mu|\leq\varepsilon} |g_\mu(v, P, \mu)| \leq Ae^{-kv^2}, \qquad \sup_{|\mu|\leq\varepsilon} |g_P(v, P, \mu)| \leq Ae^{-kv^2}.$$

4. *The price belief $g(v, P, \mu)$ is continuously differentiable in (μ, P) in a neighborhood of $\mu = 0$.*

5. *The agent primitives satisfy the integrability conditions*

$$\int_0^1 \frac{1}{\rho(a)}\, da < \infty, \quad \int_0^1 \frac{t(a)}{\rho(a)}\, da < \infty, \quad \int_0^1 \frac{t(a)}{\rho^2(a)}\, da < \infty, \quad \int_0^1 \frac{t(a)\,\pi(a)}{\rho^2(a)}\, da < \infty.$$

Parts 1–3 provide the growth and integrability bounds needed to invoke the Dominated Convergence Theorem, allowing us to pass limits as $\mu \to 0$ and derivatives under the integral sign. Part 4 ensures that the price belief is smooth enough for the relevant derivatives to be well defined. Part 5 ensures that the equilibrium objects—price informativeness τ_p, the cumulative demand integrals, and the market-clearing integrals—are finite. These conditions are easily satisfied when $\rho(a)$, $\pi(a)$, and $t(a)$ are continuous on $[0,1]$.

C.2 The Aggregation Lemma

Lemma 5. ***(Aggregation lemma)*** *Consider a sequence of functions* $\hat{x}^n(P,s,m,a):\mathbb{R}\times\mathbb{R}\times[0,1]\times[0,1]\to\mathbb{R}$ *with partial derivatives* $\hat{x}^n_s(P,s,m,a)$, $\hat{x}^n_{ss}(P,s,m,a)$, *and* $\hat{x}^n_m(P,s,m,a)$ *that are continuous functions of* s *and* m *in the neighbourhood of* $s=m=0$ *for every* $P\in\mathbb{R}$ *and* $a\in(0,1)$. *Suppose that for every* $P\in\mathbb{R}$ *and every* $y\in(0,1)$, *the functions* $a\mapsto 1/\hat{t}^n(a)$, $a\mapsto\hat{x}^n_s(P,0,0,a)$, $a\mapsto\hat{x}^n_{ss}(P,0,0,a)$, *and* $a\mapsto\hat{x}^n_m(P,0,0,a)$ *converge uniformly on* $(0,y)$. *Denote the respective limits by* $1/t(a)$, $x_s(P,a)$, $x_{ss}(P,a)$, *and* $x_m(P,a)$. *Define*

$$\beta(P,a)\equiv x_s(P,a) \quad \textit{and} \quad \delta(P,a)\equiv\frac{1}{2t(a)}x_{ss}(P,a)+x_m(P,a).$$

Assume that for all n *and all* $a\in(0,1)$, $\hat{t}^n(a)\geq\varepsilon>0$ *for some constant* ε, *and that for every* $P\in\mathbb{R}$ *and every* $y\in(0,1)$ *the following conditions hold:*

(i) ***Lipschitz:*** *There exists* $K>0$ *such that for all* n, $a\in[0,y)$, *and sufficiently small* s,m,

$$|\hat{x}^n_\alpha(P,s,m,a)-\hat{x}^n_\alpha(P,0,0,a)|\leq K(|s|+|m|), \quad \textit{for } \alpha\in\{s,ss,m\}.$$

(ii) ***Integrability:*** *For every* $P\in\mathbb{R}$ *and every* $y\in(0,1)$,

$$\int_0^y \frac{x_s(P,a)^2}{t(a)}\,da<\infty,\ \int_0^y x_s(P,a)^2\,da<\infty,\ \int_0^y x_{ss}(P,a)^2\,da<\infty,\ \textit{and}\ \int_0^y x_m(P,a)^2\,da<\infty.$$

(iii) ***Continuity:*** *The functions* $a\mapsto t(a)$, $a\mapsto x_s(P,a)$, $a\mapsto x_{ss}(P,a)$, *and* $a\mapsto x_m(P,a)$ *are continuous on* $[0,y)$.

For a fixed $y \in (0,1)$, let $[0,y) = \bigcup_i [a_i, a_i + m_i)$ be a sequence of partitions with mesh size $\bar{m} = \max_i m_i \to 0$. Let the increments be defined as $\Delta s_i = v m_i + [\hat{t}^n(a_i)]^{-1/2}(B(a_i+m_i) - B(a_i))$. Then, letting s_a denote the process $ds_a = v\, da + [t(a)]^{-1/2} dB_a$, we have:

$$\sum_{i:a_i<y} (\hat{x}^n(P, \Delta s_i, m_i, a_i) - \hat{x}^n(P, 0, 0, a_i)) \xrightarrow{p} \int_0^y \beta(P,a)\, ds_a + \int_0^y \delta(P,a)\, da,$$

along any sequence with $\bar{m} \to 0$ and $n \to \infty$.

Proof of Lemma 5.

Fix $P \in \mathbb{R}$ and write $\Delta B_i \equiv B(a_i + m_i) - B(a_i)$, so that

$$\Delta s_i = v m_i + [\hat{t}^n(a_i)]^{-1/2} \Delta B_i.$$

For each i (suppressing (P, a_i) in the notation), a Taylor expansion of $\hat{x}^n(s,m)$ around $(s,m) = (0,0)$ (second order in s, first order in m) gives

$$\hat{x}^n(\Delta s_i, m_i) - \hat{x}^n(0,0) = \hat{x}^n_s(0,0)\Delta s_i + \frac{1}{2}\hat{x}^n_{ss}(0,0)(\Delta s_i)^2 + \hat{x}^n_m(0,0) m_i + r^n_i,$$

where the remainder can be written in mean-value form as

$$r^n_i = (\hat{x}^n_s(0,m_i) - \hat{x}^n_s(0,0))\Delta s_i + \frac{1}{2}(\hat{x}^n_{ss}(\tilde{s}_i, m_i) - \hat{x}^n_{ss}(0,0))(\Delta s_i)^2 + (\hat{x}^n_m(0,\tilde{m}_i) - \hat{x}^n_m(0,0)) m_i$$

for some $\tilde{s}_i$ between 0 and Δs_i and some $\tilde{m}_i$ between 0 and m_i. By the Lipschitz assumption (i),

$$|r^n_i| \le C\Big(m_i|\Delta s_i| + (|\Delta s_i| + m_i)(\Delta s_i)^2 + m_i^2\Big)$$

for a constant C independent of n, i.[44]

[44] Brownian paths are almost surely uniformly continuous on $[0,y]$, hence $\max_i |\Delta B_i| \to 0$ almost surely as $\bar{m} \to 0$, and therefore $\max_i |\Delta s^n_i| \to 0$ almost surely (uniformly in n). Thus, for $\bar{m}$ small enough, the neighborhood required for condition (i) applies almost surely to all increments.

By assumption, $\hat{t}^n(a) \geq \varepsilon > 0$ for all n and all $a \in (0, y)$, so there is a constant $C_t < \infty$ (e.g. $C_t \equiv 1/\varepsilon$) such that

$$\sup_n \sup_{a\in(0,y)} \frac{1}{\hat{t}^n(a)} \leq C_t.$$

Moreover, since $1/\hat{t}^n \to 1/t$ uniformly on $(0, y)$, we have $1/t(a) \leq C_t$ on $[0, y)$ and hence $t(a) \geq 1/C_t > 0$ there. Since $\Delta B_i \sim N(0, m_i)$ and $(\hat{t}^n(a_i))^{-1} \leq C_t$, there exists $C' > 0$ (independent of n, i) such that

$$\mathbb{E}(\Delta s_i)^2 \leq C' m_i \qquad \text{and} \qquad \mathbb{E}|\Delta s_i|^3 \leq C' m_i^{3/2}.$$

Therefore,

$$\mathbb{E}\Big[\sum_{i:a_i<y} |r_i^n|\Big] \leq C'' \sum_{i:a_i<y} \Big(m_i^{3/2} + m_i^2\Big) \leq C''(y\sqrt{\bar{m}} + y\bar{m}) \xrightarrow[\bar{m}\to 0]{} 0,$$

and hence by Markov's inequality, for any $\eta > 0$,

$$\mathbb{P}\left(\sum_{i:a_i<y} |r_i^n| > \eta\right) \leq \frac{1}{\eta}\,\mathbb{E}\Big[\sum_{i:a_i<y} |r_i^n|\Big] \xrightarrow[\bar{m}\to 0]{} 0,$$

In particular,

$$\sum_{i:a_i<y} r_i^n \xrightarrow{p} 0, \qquad \text{equivalently} \qquad \sum_{i:a_i<y} r_i^n = o_p(1)$$

Define the shorthand coefficients

$$\beta_i^n \equiv \hat{x}_s^n(P, 0, 0, a_i), \qquad \gamma_i^n \equiv \hat{x}_{ss}^n(P, 0, 0, a_i), \qquad \mu_i^n \equiv \hat{x}_m^n(P, 0, 0, a_i), \qquad \tau_i^n \equiv \hat{t}^n(a_i).$$

Summing the expansion gives

$$\sum_{i:a_i<y} (\hat{x}^n(P, \Delta s_i, m_i, a_i) - \hat{x}^n(P, 0, 0, a_i)) = \sum_{i:a_i<y} \beta_i^n \Delta s_i + \frac{1}{2} \sum_{i:a_i<y} \gamma_i^n (\Delta s_i)^2 + \sum_{i:a_i<y} \mu_i^n m_i + o_p(1).$$

Step 1 (linear term). Decompose

$$\sum_{i:a_i<y} \beta_i^n \Delta s_i = v \sum_{i:a_i<y} \beta_i^n m_i + \sum_{i:a_i<y} \beta_i^n (\tau_i^n)^{-1/2} \Delta B_i.$$

Because $\int_0^y \beta(P,a)^2/t(a)\, da < \infty$, the limiting integrand $a \mapsto \beta(P,a)t(a)^{-1/2}$ lies in $L^2([0,y])$. By the uniform convergence of $\hat{x}_s^n(P,0,0,\cdot)$ to $\beta(P,\cdot)$ and the uniform convergence $(\tau^n)^{-1/2} \to t^{-1/2}$, the step functions $a \mapsto \beta_i^n(\tau_i^n)^{-1/2}$ (constant on $[a_i, a_i+m_i)$) converge in $L^2([0,y])$ to $a \mapsto \beta(P,a)t(a)^{-1/2}$ along any refining sequence of partitions. By Itô isometry this implies

$$\sum_{i:a_i<y} \beta_i^n (\tau_i^n)^{-1/2} \Delta B_i \xrightarrow{L^2} \int_0^y \frac{\beta(P,a)}{\sqrt{t(a)}}\, dB_a.$$

Similarly, using $\int_0^y |\beta(P,a)|\, da < \infty$ and the uniform convergence of $\hat{x}_s^n(P,0,0,\cdot)$ to $\beta(P,\cdot)$,

$$v \sum_{i:a_i<y} \beta_i^n m_i \to v \int_0^y \beta(P,a)\, da.$$

Combining yields

$$\sum_{i:a_i<y} \beta_i^n \Delta s_i \xrightarrow{p} \int_0^y \beta(P,a)\, ds_a, \qquad ds_a = v\, da + t(a)^{-1/2} dB_a.$$

Step 2 (quadratic term). Expand

$$(\Delta s_i)^2 = v^2 m_i^2 + 2v m_i (\tau_i^n)^{-1/2} \Delta B_i + (\tau_i^n)^{-1} (\Delta B_i)^2.$$

The $v^2 m_i^2$ contribution is $o(1)$. Indeed,

$$\frac{1}{2} \sum_{i:a_i<y} |\gamma_i^n| v^2 m_i^2 = \frac{v^2}{2} \sum_{i:a_i<y} (|\gamma_i^n| \sqrt{m_i})\, m_i^{3/2} \le \frac{v^2}{2} \Big(\sum_{i:a_i<y} (\gamma_i^n)^2 m_i \Big)^{1/2} \Big(\sum_{i:a_i<y} m_i^3 \Big)^{1/2}.$$

Since $m_i^3 \le \bar{m}^2 m_i$ we have $\sum_{i:a_i<y} m_i^3 \le \bar{m}^2 \sum_{i:a_i<y} m_i = \bar{m}^2 y \to 0$. Moreover, using

$\int_0^y x_{ss}(P,a)^2\,da < \infty$ and the uniform convergence of $\hat{x}^n_{ss}(P,0,0,\cdot)$ to $x_{ss}(P,\cdot)$,

$$\sum_{i:a_i<y} (\gamma_i^n)^2 m_i \to \int_0^y x_{ss}(P,a)^2\,da,$$

and hence the product above vanishes.

The cross term is a martingale. Using independence of the Brownian increments and $\mathbb{E}[(\Delta B_i)^2] = m_i$,

$$\mathbb{E}\left[\left(\sum_{i:a_i<y} \gamma_i^n m_i (\tau_i^n)^{-1/2}\Delta B_i\right)^2\right] = \sum_{i:a_i<y} (\gamma_i^n)^2 m_i^2 (\tau_i^n)^{-1}\mathbb{E}[(\Delta B_i)^2] \le C_t \sum_{i:a_i<y} (\gamma_i^n)^2 m_i^3.$$

Since $m_i^3 \le \bar{m}^2 m_i$ and $\sum_i (\gamma_i^n)^2 m_i \to \int_0^y x_{ss}(P,a)^2\,da < \infty$, this second moment is $O(\bar{m}^2) \to 0$, hence, by Chebyshev's inequality,

$$\sum_{i:a_i<y} \gamma_i^n m_i (\tau_i^n)^{-1/2}\Delta B_i \xrightarrow{p} 0.$$

For the remaining part, write $(\Delta B_i)^2 = m_i + ((\Delta B_i)^2 - m_i)$ to obtain

$$\frac{1}{2}\sum_{i:a_i<y} \gamma_i^n (\tau_i^n)^{-1}(\Delta B_i)^2 = \frac{1}{2}\sum_{i:a_i<y} \gamma_i^n (\tau_i^n)^{-1} m_i + \frac{1}{2}\sum_{i:a_i<y} \gamma_i^n (\tau_i^n)^{-1}((\Delta B_i)^2 - m_i).$$

The last term is again a martingale. Since $\mathrm{Var}((\Delta B_i)^2 - m_i) = 2m_i^2$,

$$\mathbb{E}\left[\left(\frac{1}{2}\sum_{i:a_i<y} \gamma_i^n (\tau_i^n)^{-1}((\Delta B_i)^2 - m_i)\right)^2\right] = \frac{1}{4}\sum_{i:a_i<y} (\gamma_i^n)^2 (\tau_i^n)^{-2}\mathrm{Var}((\Delta B_i)^2 - m_i) \le \frac{1}{2}C_t^2 \sum_{i:a_i<y} (\gamma_i^n)^2 m_i^2.$$

Using $m_i^2 \le \bar{m}\, m_i$ and the boundedness of $\sum_i (\gamma_i^n)^2 m_i$, this second moment is $O(\bar{m}) \to 0$, hence the martingale term converges to 0 in probability by Chebyshev's inequality. The first term converges to $\int_0^y \frac{1}{2t(a)} x_{ss}(P,a)\,da$ by the uniform convergence of $\hat{x}^n_{ss}(P,0,0,\cdot)$ to $x_{ss}(P,\cdot)$ and uniform convergence of $(\tau^n)^{-1}$. Hence

$$\frac{1}{2}\sum_{i:a_i<y}\gamma_i^n(\Delta s_i)^2 \xrightarrow{p} \int_0^y \frac{1}{2t(a)}x_{ss}(P,a)\,da.$$

Step 3 (the m term). Since $x_m(P,\cdot) \in L^2(0,y) \subset L^1(0,y)$ and $\hat{x}_m^n(P,0,0,\cdot) \to x_m(P,\cdot)$ uniformly on $(0,y)$,

$$\sum_{i:a_i<y}\mu_i^n m_i \to \int_0^y x_m(P,a)\,da.$$

Putting Steps 1–3 together and recalling $\delta(P,a) = \frac{1}{2t(a)}x_{ss}(P,a)+x_m(P,a)$ yields the desired convergence in probability. ■

C.3 Proof of Lemma 1 and Lemma 2

Recall that in the nth economy (with $\mu = 1/n$), the cumulative demand is

$$X(y) = \sum_{i:\,a_i<y} x^*(\Delta s_i, P, a_i, \mu).$$

Define $x^*(P,a) \equiv x^*(0,P,a,0)$—the optimal demand when $s=0$ in the limiting economy. We decompose:

$$X(y) = \sum_{i:\,a_i<y}(x^*(\Delta s_i,P,a_i,\mu) - x^*(0,P,a_i,0)) + \sum_{i:\,a_i<y} x^*(P,a_i).$$

To apply the Aggregation Lemma, we identify $\hat{x}(P,s,m,a) = x^*(s,P,a,m)$, so that s plays the role of the signal increment and $m=\mu$ is the mesh size (which equals the economy parameter, since the partition is uniform). By Aggregation Lemma,

$$\sum_{i:\,a_i<y}(x^*(\Delta s_i,P,a_i,\mu) - x^*(0,P,a_i,0)) \xrightarrow[\mu\to 0]{\mathbb{P}} \int_0^y \beta(P,a)\,ds_a + \int_0^y \delta(P,a)\,da, \tag{36}$$

where $\beta(P, a) = x_s^*(0, P, a, 0)$ and $\delta(P, a) = \frac{1}{2t(a)} x_{ss}^*(0, P, a, 0) + x_\mu^*(0, P, a, 0)$.

The hypotheses of the Aggregation Lemma hold. Continuity of x_s^*, x_{ss}^*, and x_μ^* in (s, μ) near $(0, 0)$ follows from the Implicit Function Theorem applied to the first-order condition (39). Uniform convergence of these derivatives over $a \in [0, y]$ as $\mu \to 0$ follows from their joint continuity in (a, μ)—guaranteed by the IFT and the continuity of primitives in a (Assumption 3)—together with the compactness of $[0, y]$. The integrability conditions in Assumption 3, part 5 ensure that the limiting integrals are finite.

For $X(y)$ to have a finite limit, the residual sum $\sum_{i:\, a_i < y} x^*(P, a_i)$ must also be finite. This leads to the efficiency condition.

C.3.1 Efficiency condition

Setting $s = 0$ and $\mu = 0$ in trader a's first-order condition (39) gives

$$\int u'(W_0(a) + x^*(P, a)\,(V(v) - P),\, a)\,(V(v) - P)\, g(v, P, 0)\, dv = 0. \tag{37}$$

The passage to the limit is justified by the Dominated Convergence Theorem: Assumption 3, parts 1–3 imply that the integrand admits an integrable majorant of the form $A \exp(-kv^2)$, and part 4 ensures the smooth convergence $g(\cdot, \mu) \to g(\cdot, 0)$.

Lemma 6. *For a fixed $y \in (0, 1)$, $\lim_{\mu \to 0} \sum_{i:\, a_i < y} x^*(P, a_i) < \infty$ if, and only if, $x^*(P, a) = 0$ for all $a \in [0, y]$ and*

$$\int (V(v) - P)\, g(v, P, 0)\, dv = 0. \tag{38}$$

Proof. Suppose $\int (V(v) - P)\, g(v, P, 0)\, dv > 0$ (the opposite case is analogous). Then (37) implies $x^*(P, a) > 0$ for every $a \in [0, y]$. Since $x^*(P, \cdot)$ is continuous in a (by the IFT and continuity of primitives), there exist $\epsilon > 0$ and an interval $(a_0 - \delta,\, a_0 + \delta) \subset [0, y]$ on which $x^*(P, a) > \epsilon$. As $\mu \to 0$, the number of partition points falling in this interval grows without bound, so $\sum_{i:\, a_i < y} x^*(P, a_i) \geq \sum_{|a_i - a_0| < \delta} x^*(P, a_i) \to \infty$—a contradiction. Conversely, if (38) holds, then $x^*(P, a) = 0$ for all a (the unique solution to (37) when $\mathbb{E}[V - P \mid P] = 0$, by strict concavity), and the sum is identically zero. ■

Lemma 6 proves Lemma 1. Substituting the efficiency condition into (36) proves Lemma 2, with β and δ characterized explicitly in Lemmas 7 and 8.

Note that (38) and the efficiency condition (8) are equivalent.

C.4 Proof of Lemma 3

Proof of Lemma 3. By the properties of the Ito integral, $\int_0^1 \omega(a)/\sqrt{(}t(a))dB(a)$ is distributed normally with a mean of zero. By Ito isometry, the variance of the integral is given by $\int_0^1 \frac{w(a)^2}{t(a)} da$. The statements of the lemma then follow from the standard results on Bayes rule with normal random variables. ■

C.5 Proof of Theorem 1

Consider a trader at location a, living on the interval $[a, a + \mu)$ in the economy with parameter $\mu = 1/n$. The price belief in that economy is $g(v, P, \mu)$, and we denote the trader's signal realization by s. Since the partition is uniform, the mesh size equals μ, so the optimal demand $x^*(s, P, a, \mu)$ is determined by the first-order condition

$$\int u'(W_0(a) + x^*(s, P, a, \mu)\,(V(v) - P),\, a)\,(V(v) - P) \cdot \exp\big((sv - \mu v^2/2)\,t(a)\big)\,g(v, P, \mu)\,dv = 0, \quad (39)$$

where we have dropped the normalizing constant from the conditional density (3), as it does not affect the optimization. The cumulative demand in the nth economy is

$$X(y) = \sum_{i:\, a_i < y} x^*(\Delta s_i, P, a_i, \mu).$$

C.5.1 Deriving $\beta(P, a)$

By the Aggregation Lemma, the trading intensity equals the signal sensitivity of optimal demand evaluated in the large-economy limit: $\beta(P, a) = x_s^*(0, P, a, 0)$. Computing this derivative requires differentiating the first-order condition (39) with respect to s and evaluating at $s = 0$, $\mu = 0$.

Lemma 7. *Suppose that Assumption 3 is satisfied. Then*

$$\beta(P, a) = \frac{t(a)\ \mathrm{Cov}(V(v), v \mid P)}{\rho(a)\,\mathrm{Var}(V(v) \mid P)}. \tag{40}$$

In particular, $\beta(P, a)$ is multiplicatively separable: $\beta(P, a) = \beta_a(a)\,\beta_P(P)$ where

$$\beta_a(a) = \frac{t(a)}{\rho(a)} \qquad \text{and} \qquad \beta_P(P) = \frac{\mathrm{Cov}(V(v), v \mid P)}{\mathrm{Var}(V(v) \mid P)}.$$

If $v \mid \mathbf{P}_*$ is normally distributed with precision τ, then $\beta_P(P) = \frac{\tau^{-1}\,\mathbb{E}[V'(v)|P]}{\mathrm{Var}(V(v)|P)}$.[45]

Proof. The optimal demand x^* satisfies the first-order condition (FOC):

$$G(x^*, s) \equiv \int_{\mathbb{R}} (V(v) - P)\, u'(W_0(a) + x^*(V(v) - P)) \exp(t(a)\, s\, v)\, g(v, P, 0)\, dv = 0.$$

By the Implicit Function Theorem, the sensitivity $\beta(P, a)$ at the limit ($s = 0$, $x^* = 0$) is

$$\beta(P, a) = \frac{\partial x^*}{\partial s} = -\frac{\partial G/\partial s}{\partial G/\partial x^*}.$$

Assumption 3, parts 1–3 imply that the derivatives can be passed under the integral sign and

[45] This follows via integration by parts:

$$\mathbb{E}[v(V(v) - P) \mid P] = \int_{\mathbb{R}} (v - \mathbb{E}[v \mid P])\,(V(v) - P) \frac{1}{\sqrt{2\pi/\tau}} \exp\left(-\frac{(v - \mathbb{E}[v \mid P])^2 \tau}{2}\right) dv$$
$$= \tau^{-1} \int_{\mathbb{R}} \frac{1}{\sqrt{2\pi/\tau}} \exp\left(-\frac{(v - \mathbb{E}[v \mid P])^2 \tau}{2}\right) V'(v)\, dv.$$

that G is continuously differentiable at $x^* = 0$, $s = 0$. Part 3 further ensures that $\partial G/\partial x^* \neq 0$, validating the Implicit Function Theorem.

Step 1: The Numerator ($\partial G/\partial s$). Differentiating G with respect to s and evaluating at $s = 0$, $x^* = 0$ (where $x^* = 0$ follows from Lemma 6):

$$\frac{\partial G}{\partial s} = u'(W_0)\, t(a) \int_{\mathbb{R}} (V(v) - P)\, v\, g(v, P, 0)\, dv.$$

Using $\mathbb{E}[V(v) - P \mid P] = 0$ (Lemma 6), the integral equals $\text{Cov}(V(v), v \mid P)$:

$$\frac{\partial G}{\partial s} = u'(W_0)\, t(a)\ \text{Cov}(V(v), v \mid P).$$

Step 2: The Denominator ($\partial G/\partial x^*$). Differentiating G with respect to x^* and evaluating at $x^* = 0$. Since the price belief $g(v, P, 0)$ does not depend on x^* (price-taking assumption), only the utility derivative contributes:

$$\frac{\partial G}{\partial x^*} = u''(W_0) \int_{\mathbb{R}} (V(v) - P)^2\, g(v, P, 0)\, dv = u''(W_0)\, \text{Var}(V(v) \mid P).$$

Step 3: Combining Terms. Substituting into the IFT formula and using $\rho(a) = -u''(W_0)/u'(W_0)$:

$$\beta(P, a) = -\frac{u'(W_0)\, t(a)\ \text{Cov}(V(v), v \mid P)}{u''(W_0)\, \text{Var}(V(v) \mid P)} = \frac{t(a)\ \text{Cov}(V(v), v \mid P)}{\rho(a)\, \text{Var}(V(v) \mid P)}.$$

■

C.5.2 Deriving $\delta(P, a)$

By the Aggregation Lemma, the coefficient $\delta(P, a)$ combines the curvature of optimal demand in the signal and the sensitivity to the economy parameter μ:

$$\delta(P, a) = \frac{1}{2\,t(a)}\, x^*_{ss}(0, P, a, 0) + x^*_\mu(0, P, a, 0).$$

Lemma 8. *Suppose that Assumption 3 is satisfied. Let* $\Sigma_{VV}(P) = \mathrm{Var}(V(v) \mid P)$*. Then*

$$\delta(P, a) = \frac{1}{\rho(a)\, \Sigma_{VV}(P)} \left(\psi(P) + \frac{\beta(P, a)^2}{2\,t(a)}\, \pi(a)\, \rho(a)\, \mathrm{Sk}[V(v) \mid P] - \beta(P, a)\, \rho(a)\, \mathbb{E}\big[(V(v) - P)^2 v \mid P\big] \right), \tag{41}$$

where $\pi(a) = -u'''(W_0)/u''(W_0)$ *is the coefficient of absolute prudence and* $\psi(P) = \frac{\partial}{\partial \mu} \mathbb{E}[V(v) - P \mid P]$.

Proof. We follow the same strategy as the computation of $\beta(P, a)$. The optimal demand satisfies $G(x^*, s, \mu) = 0$, where μ appears in both the exponential $\exp((sv - \mu v^2/2)\, t(a))$ and the price belief $g(v, P, \mu)$. Since the first-order condition is homogeneous in the price belief, we can normalize $g(v, P, \mu)$ into the corresponding conditional density of v given P and express all moments below as conditional expectations. From the proof of Lemma 7, $G_{x^*} = -u'(W_0)\, \rho(a)\, \Sigma_{VV}(P)$.

Step 1: Computing x^*_μ. By the Implicit Function Theorem, $x^*_\mu = -G_\mu / G_{x^*}$. Differentiating G with respect to μ at $s = 0$, $\mu = 0$, $x^* = 0$, both the exponential and the price belief contribute:

$$G_\mu = u'(W_0) \left[-\frac{t(a)}{2}\, \mathbb{E}\big[(V(v) - P)\, v^2 \mid P\big] + \frac{\partial}{\partial \mu} \mathbb{E}[V(v) - P \mid P] \Big|_{\mu=0} \right].$$

By the efficiency condition, $\mathbb{E}[V - P \mid P] = 0$, so the first term equals $-\frac{t(a)}{2}\, \mathrm{Cov}(V - P,\, v^2 \mid P)$.

Using the earlier definition $\psi(P) = \frac{\partial}{\partial \mu}\mathbb{E}[V(v) - P \mid P]$, we obtain

$$x^*_\mu = \frac{\psi(P) - \frac{t(a)}{2}\,\mathrm{Cov}(V{-}P,\, v^2 \mid P)}{\rho(a)\,\Sigma_{VV}(P)}. \tag{42}$$

The derivatives under the integral are justified by Assumption 3, parts 3–4.

Step 2: Computing x^*_{ss}. By implicit differentiation, $x^*_{ss} = -\frac{1}{G_{x^*}}[G_{ss} + 2\beta\, G_{x^*s} + \beta^2\, G_{x^*x^*}]$. Since g is independent of x^* (price-taking), the cross-derivatives involve only utility terms:

$$\begin{aligned} G_{ss} &= u'(W_0)\,t(a)^2\,\mathbb{E}\big[(V(v) - P)\,v^2 \mid P\big] = u'(W_0)\,t(a)^2\;\mathrm{Cov}(V{-}P,\, v^2 \mid P), \\ G_{x^*s} &= u''(W_0)\,t(a)\,\mathbb{E}\big[(V(v) - P)^2 v \mid P\big]\,, \\ G_{x^*x^*} &= u'''(W_0)\,\mathbb{E}\big[(V(v) - P)^3 \mid P\big]\,. \end{aligned}$$

Step 3: Combining. In the combination $\delta = \frac{1}{2t(a)}x^*_{ss} + x^*_\mu$, the $\mathrm{Cov}(V{-}P,\, v^2 \mid P)$ contribution from $G_{ss}/(2t\,G_{x^*})$ exactly cancels the corresponding term in (42). Using $-u'''(W_0)/u''(W_0) = \pi(a)$, $x^*_s = \beta(P, a)$, $\mathbb{E}[(V(v) - P)^3 \mid P] = \mathrm{Sk}[V \mid P]$, and $\mathbb{E}[(V(v) - P)^2 \mid P] = \Sigma_{VV}(P)$, the surviving terms yield the expression in the lemma. ∎

C.5.3 Proof of Theorem 1

Proof. The proof proceeds in three steps.

Step 1: Demand coefficients. By Lemma 7, the trading intensity is

$$\beta(P, a) = \frac{t(a)\;\mathrm{Cov}(V(v), v \mid P)}{\rho(a)\,\mathrm{Var}(V(v) \mid P)}.$$

Since $\lambda(P) = 0$ in the price-taking case, this is the REE expression (Lemma 13) with $\lambda = 0$. By Lemma 8, the drift coefficient $\delta(P, a)$ is given by (41), which is the REE expression (Lemma 15) with all λ-terms set to zero.

Step 2: Price function. The multiplicative separability $\beta(P, a) = \beta_a(a)\,\beta_P(P)$ implies that the cumulative demand depends on the price only through the sufficient statistic

$$s_p = \frac{\int_0^1 \beta_a(a)\,ds(a)}{\int_0^1 \beta_a(a)\,da} = v + \int_0^1 \frac{\omega(a)}{\sqrt{t(a)}}\,dB(a),$$

where $\omega(a) = \beta_a(a)/\int_0^1 \beta_a(b)\,db$. By Lemma 3, $v \mid s_p$ is normally distributed with precision $\tau = \tau_v + \tau_p$ and mean $(\tau_p/\tau)\,s_p$. The efficiency condition $\mathbb{E}[V(v) - P \mid P] = 0$ (Lemma 6) then yields the equilibrium price function

$$\mathcal{P}(s_p) = \int V\left(\frac{\tau_p}{\tau}\,s_p + \frac{z}{\sqrt{\tau}}\right) d\Phi(z) = P, \tag{43}$$

where Φ is the standard normal CDF. The conditional moments of $V(v)$ can be computed from this representation:

$$\mathbb{E}[(V(v) - P)^k | P] = \int \left(V\left(\frac{\tau_p}{\tau}\,h(P) + \frac{z}{\sqrt{\tau}}\right) - P\right)^k d\Phi(z), \tag{44}$$

where $h(P) = \mathcal{P}^{-1}(P)$.

Step 3: Price informativeness and uniqueness. The price informativeness is

$$\tau_p = \frac{\left(\int_0^1 \beta_a(a)\,da\right)^2}{\int_0^1 \frac{\beta_a(a)^2}{t(a)}\,da} = \frac{\left(\int_0^1 \frac{t(a)}{\rho(a)}\,da\right)^2}{\int_0^1 \frac{t(a)}{\rho(a)^2}\,da}.$$

Unlike in the REE case (Theorem 7), τ_p does not involve a fixed-point equation: $\beta_a(a) = t(a)/\rho(a)$ depends only on primitives, so τ_p is determined in closed form. Given τ_p, the price function $\mathcal{P}(\cdot)$ is uniquely determined by (43), and the drift coefficient $\delta(P, a)$ is determined by Lemma 8, with $\psi(P)$ pinned down by the market-clearing condition $\int_0^1 \delta(a, P)\,da + h(P)\int_0^1 \beta(a, P)\,da = \Theta(P)$. The equilibrium is therefore unique.

■

C.6 Proof of Proposition 1

Proof of Proposition 1. From definition of $\mathcal{I}$ we have $\mathcal{I} = \tau_p/\tau$. Substituting (15) into the last equation and rearranging, we obtain the stated expression for $\mathcal{I}$.

We turn to deriving expression for $\mathcal{L}$. The market clearing price $\mathcal{P}(s_p, \bar{\theta})$ is P that solves

$$s_p = \frac{\bar{\theta} + \theta(P) - \int \delta(a, P)da}{\int \beta(a, P)da}.$$

Then define

$$\begin{aligned}
\text{Liquidity} &\equiv -\left(\frac{\partial}{\partial \bar{\theta}} \mathcal{P}(s_p, \bar{\theta})\right)^{-1} \\
&= -s_p \frac{\partial}{\partial P}\left(\int \beta(a, P)da\right) - \frac{\partial}{\partial P}\left(\int \delta(a, P)da - \theta(P)\right)
\end{aligned}$$

On the other hand,

$$\begin{aligned}
\frac{\partial}{\partial s_p} \mathcal{P}(s_p, \bar{\theta}) &= \frac{-\int \beta(a, P)da}{s_p \frac{\partial}{\partial P}\left(\int \beta(a, P)da\right) + \frac{\partial}{\partial P}\left(\int \delta(a, P)da - \theta(P)\right)} \\
&= \text{Liquidity}^{-1} \cdot \int \beta(a, P)da.
\end{aligned}$$

From (14) we get that

$$\frac{\partial}{\partial s_p} \mathcal{P}(s_p, \bar{\theta}) = \frac{\tau_p}{\tau} \int V'\left(\frac{\tau_p}{\tau} s_p + \frac{z}{\sqrt{\tau}}\right) d\Phi(z) = \frac{\tau_p}{\tau} E[V'(v)|P].$$

Combining the two preceding equations and substituting the expression for $\beta(a, P)$ we get

$$\frac{\tau_p}{\tau} E[V'(v)|P] = \text{Liquidity}^{-1} \cdot \int \frac{t(a)}{\rho(a)} \frac{\tau^{-1}\,\mathbb{E}[V'(v)|P]}{\text{Var}\,[V(v)|P]} da \implies$$

$$\text{Liquidity} = \frac{1}{\tau_p \text{Var}[V(v)|P]} \int \frac{t(a)}{\rho(a)} da.$$

Multiplying by $\text{Var}[V(v)|P]$ and substituting (15) we get the stated equation for $\mathcal{L}$. ■

C.7 Proof of Lemma 4

Proof of Lemma 4. First, from Lemma 3 we have $\tau_{agg} \equiv \mathbb{V}\text{ar}\big(v \big| s\left[\omega^H(a)\right]\big)^{-1} - \mathbb{V}\text{ar}(v)^{-1} = 1/\Big(\int_0^1 \omega^H(a)^2/t(a)da\Big)$. Second, we apply the Cauchy-Bunyakovsky-Schwartz inequality

$$\left(\int_0^1 f(a)g(a)da\right)^2 \leq \int_0^1 f(a)^2 da \int_0^1 g(a)^2 da \tag{45}$$

with $f(a) = \omega^H(a)/\sqrt{t(a)}$ and $g(a) = \sqrt{t(a)}$ to obtain $\tau_{agg}(b(a)) \leq \int_0^1 t(a)da$. The equality in (45) is attained if, and only if, $f(a)$ and $g(a)$ are linearly dependent, i.e., when $\omega^H(a)/\sqrt{t(a)} = c\sqrt{t(a)}$. The constant c is pinned down by the condition $\int_0^1 w(a)da = 1$. ■

C.8 Proof of Proposition 2

Proof of Proposition 2. Denote absolute risk tolerance $y(a) \equiv 1/\rho(a)$. Without loss of generality, index traders such that $y(b)$ increases in b. (This is in contrast to index a, which is such that $W_0(a)$ is increasing in a.) We first compute the Gateaux derivatives of $\mathcal{I}$ and $\mathcal{L}$ with respect to $y(b)$. We then show that under DARA utilities and the technical conditions imposed, the signs of the derivatives with respect to $y(b)$ and $W_0(a)$ are the same. We start by proving the following statement.

Step 1. There exist thresholds $0 < b_1^y \leq b_2^y < 1$, such that for for any Robin Hood variation $y^\Delta(b)$ with $\underline{b} \leq b_1^y \leq b_2^y \leq \bar{b}$

$$\begin{aligned} &\mathcal{I}'(y(b))[y^\Delta(b)] > 0 \text{ and } \mathcal{I}'(y(b))[-y^\Delta(b)] < 0; \\ &(\mathcal{L})'(y(b))[y^\Delta(b)] < 0 \text{ and } (\mathcal{L})'(y(b))[-y^\Delta(b)] > 0. \end{aligned} \tag{46}$$

From (15) we obtain

$$\tau_p = \left(\int_0^1 y(b)t(b)db\right)^2 \Big/ \left(\int_0^1 y(b)^2 t(b)db\right). \tag{47}$$

Substituting this expression into $\mathcal{I} = \tau_p/(\tau_p + \tau_v)$ and computing the Gateaux derivative (this entails substituting $y(b)+\epsilon y^{\Delta}(b)$ instead of $y(b)$, differentiating with respect to ϵ, and evaluating the resulting expression at $\epsilon = 0$) yields:

$$\mathcal{I}'(y(b))[y^{\Delta}(b)] = C_{\mathcal{I}} \int_0^1 t(b) y^{\Delta}(b)(I_2 - I_1 y(b))\, db.$$

Here, $C_{\mathcal{I}} > 0$ is positive (we have the closed-form expressions for $C_{\mathcal{I}}$ via parameters of the model, but it is not important here), $I_1 = \int_0^1 t(b)y(b)\, db$ and $I_2 = \int_0^1 t(b)y(b)^2\, db$. Lemma 9 (to follow) implies that there exists a unique b_y^* such that $I_2 - I_1 y(b) \gtreqless 0$ iff $b \lesseqgtr b_*^y$. Then, for a $y^{\Delta}(b)$ that is Robin Hood with $\underline{b} < b_*^y < \bar{b}$, $y^{\Delta}(b)(I_2 - I_1 y(b)) > 0$ and the first statement of this step follows by letting $b_1^y = b_2^y = b_*^y$.

One can obtain $\mathcal{L} = \int_0^1 t(b)y(b)db/\tau_p$. Substituting (47) in this equation and computing the Gateaux derivative yields

$$(\mathcal{L})'(y(b))[y^{\Delta}(b)] = C_{\mathcal{L}} \int_0^1 t(b) y^{\Delta}(b)(2I_1 y(b) - I_2)\, db.$$

Here, $C_{\mathcal{L}} > 0$ is positive, and Lemma 9 (to follow) implies that there exists a unique $b_{**}^y < b_*^y$ such that $2I_1 y(b) - I_2 \gtreqless 0$ iff $b \lesseqgtr b_{**}^y$. Then, for a $y^{\Delta}(b)$ that is Robin Hood with $\underline{b} < b_{**}^y < \bar{b}$, $y^{\Delta}(b)(2I_1 y(b) - I_2) > 0$ and the first three statements of this step follow by letting $b_2^y = b_*^y$ and $b_1^y = b_{**}^y$.

Step 2. There exist thresholds $0 < a_1^W \le a_2^W < 1$*, such that for for any Robin Hood variation* $W_0^{\Delta}(a)$ *with* $\underline{a} \le a_1^W \le a_2^W \le \bar{a}$ (46) *follows.*

Let a_2^W be the largest solution to $W_0(a) = \overline{\eta} y(b_2^y)$. Note that for any $a > a_2^W$, we have $y(a) = W_0(a)/\eta(a) > W_0(a)/\overline{\eta} > y(b_2^y)$. Thus, by the previous step of the proposition, decreasing risk tolerances for traders $a > a_2^W$ leads to improvement in information efficiency and reduction in liquidity. Note also that decreasing wealth for traders $a > a_2^W$ induces their risk tolerances to decrease as well (DARA utilities). Similarly, letting a_1^W be the smallest solution to $W_0(a) = \underline{\eta} b_2^y$, we get that for any $a < a_1^W$, $y(a) < y(b_1^y)$ and that increasing wealth for traders $a < a_a^W$ induces their risk tolerances to increase as well (DARA utilities). Then, by the previous step, the statement of the proposition holds. ■

Lemma 9. *Assume $0 < \underline{\eta} \leq \eta(a) \leq \bar{\eta} < \infty$. Assume that absolute risk tolerance $y(a)$ is a continuous and strictly increasing function of a. For any $c > 0$, there exists a unique solution $\hat{a}(c)$ to $y(a) = c$, moreover, $\hat{a}(c)$ increases in c and $0 < \hat{a}(c) < 1$ for any $0 < c < \infty$.*

Proof of Lemma 9. Since $y(a)$ increases in a, at most one solution to $y(a) = c$ exists. Monotonicity also implies $\hat{a}(c)$ increases in c. For risk tolerance $y(a)$ we can write $y(a) = W_0(a)/\eta(a)$. We have $y(a) < W_0(a)/\underline{\eta}$ and so $0 \leq \lim_{a\to 0} y(a) = \inf y(a) \leq \inf\{W_0(a)\}/\underline{\eta} = 0$. Thus, $\lim_{a\to 0} y(a) = 0$. One can show analogously that $\lim_{a\to\infty} y(a) = \infty$. Then, by Intermediate Value Theorem, $\hat{a}(c)$ exists and $0 < \hat{a}(c) < 1$. ■

C.9 Proof of Proposition 3

Proof of Proposition 3. This proof follows the same steps as the proof of Proposition 2. Without loss of generality, index traders such that $y(b)$ increases in b.

Step 1. There exist thresholds $0 < b_1^t \leq b_2^t < 1$, such that for for any Robin Hood variation $t^\Delta(b)$ with $\underline{b} \leq b_1^t \leq b_2^t \leq \bar{b}$

$$\mathcal{I}'(t(b))[t^\Delta(b)] > 0 \text{ and } \mathcal{I}'(t(b))[-t^\Delta(b)] < 0;$$
$$(\mathcal{L})'(t(b))[t^\Delta(b)] < 0 \text{ and } (\mathcal{L})'(t(b))[-t^\Delta(b)] > 0.$$

Here, the proof is identical to step 1 of Proposition 2, with the difference of expressions for the Gateaux derivatives, which we reproduce below:

$$\mathcal{I}'(t(b))[t^\Delta(b)] = C_\mathcal{I} \int_0^1 y(b)t^\Delta(b)(2I_2 - I_1 y(b))\, db;$$

$$(\mathcal{L})'(t(b))[t^{\Delta}(b)] = C_{\mathcal{L}} \int_0^1 y(b) t^{\Delta}(b)(I_1 y(b) - I_2)\, db;$$

Step 2. There exist thresholds $0 < a_1^t \leq a_2^t < 1$, such that for for any Robin Hood variation $W_0^{\Delta}(a)$ with $\underline{a} \leq a_1^t \leq a_2^t \leq \bar{a}$ (27) holds.

Let a_2^t be the largest solution to $W_0(a) = \overline{\eta} y(b_2^t)$. Note that for any $a > a_2^W$, we have $y(a) > W_0(a)/\overline{\eta} > y(b_2^t)$. Similarly, letting a_1^t be the smallest solution to $W_0(a) = \underline{\eta} b_2^t$, we get that for any $a < a_1^t$, $y(a) < y(b_1^t)$. Then, by the previous step, the statement of the proposition holds. ■

C.10 Proof of Proposition 4

Proof of Proposition 4. With endogenous information acquisition and DARA utilities, the precision of information is increasing in wealth. The transfer of wealth from rich to poor has two effects. The indirect one of decreasing the precisions of the rich and increasing that of the poor (via endogenous information acquisition) is captured by the Proposition 3. Proposition 2 captures the direct one. By taking the thresholds $\underline{a} \leq \min\{a_1^t, a_1^W\}$ and $\bar{a} \geq \max\{a_2^t, a_2^W\}$ we make sure that both propositions apply ■

C.11 Completing heuristic derivation in Section B

Here, we justify the transition from (32) to (33). Note that by the stochastic calculus heuristics (28), $dX(a)^2 = \beta(a,P)^2/t(a)da$ and so $\mathbb{E}[(V-P)^3|ds(a), \hat{s}_p]\, dX(a)^2$ does not contain any $ds(a)$ terms. (Indeed, the terms in $\mathbb{E}[(V-P)^3|ds(a), \hat{s}_p]$ that contain $ds(a)$ will be zeroed out after multiplying by $dX(a)^2 = \beta(a,P)^2/t(a)da$ and applying (28).)

Similarly, the term $\mathbb{E}[(V-P)^2|ds(a), \hat{s}_p]$ in (32) can be replaced by $\mathbb{E}[(V-P)^2|s_p]$ in (33). The difference $\mathbb{E}[(V-P)^2|ds(a), \hat{s}_p] - \mathbb{E}[(V-P)^2|\hat{s}_p]$ contains a leading $ds(a)$ term, which will become a da term after being multiplied by $\beta(a,P)ds(a)$ in (33). Finally, $\mathbb{E}[(V-P)^2|\hat{s}_p]$ can be replaced by $\mathbb{E}[(V-P)^2|s_p]$ since conditional distributions $v|\hat{s}_p$ and $v|s_p$ are the same.

C.12 Proof of Proposition 5

Proof of Proposition 5.

Fix a trader i. Given the price P, his realized utility at time $t=2$ is

$$\mathcal{U}_{i,t=2}\left(\Delta s_i, m_i\right) = u\left(W_0(a_i) + x(\Delta s_i, m_i; a_i)(V(v) - P)\right).$$

The Δs_i is a finite increment of a diffusion process

$$ds(b) = vdb + \frac{1}{\sqrt{t(a_i)}}dB(b)$$

between $b = a_i$ and $b = a_i + m_i$. Similarly, $\mathcal{U}_{i,t=2}\left(\Delta s_i, m_i\right) - \mathcal{U}_{i,t=2}\left(0,0\right)$ can be viewed as a finite increment of a diffusion process driven by $ds(b)$, between $b = a_i$ and $b = a_i + m_i$. By Ito's lemma, we can write

$$\mathcal{U}_{i,t=2}\left(\Delta s_i, m_i\right) - \hat{\mathcal{U}}_{i,t=2}\left(0,0\right) = \int_0^m \mu_u(b)db + \int_0^m \sigma_u(b)dB,$$

where μ_u and σ_u denote the drift (the "db" coefficient) and the diffusion coefficients (the "$dB(b)$" coefficient) of $\mathcal{U}_{i,t=2}$ process.

By Lemma 10 (to follow), the optimal precision solves

$$t(a_i) \in \arg\max_t \left\{ \left. \frac{\partial\, \mathbb{E}[\mathcal{U}_{i,t=2}]}{\partial m_i} \right|_{m_i=0} \right\}. \tag{48}$$

Thus, $t(a_i)$ maximized the expected drift of $\hat{\mathcal{U}}_{i,t=2}$ at $m_i = 0$, $\mathbb{E}[\mu_u(0)]$. The expected drift is then computed as in Section D.1, with the first order (necessary and sufficient) condition in (48) reducing to (55). ■

Lemma 10. *Consider a continuously differentiable function $f(t,m)$ such that $f(t,0)$ does not depend on t. Consider $t(m) \in \arg\max_t f(t,m)$. Suppose that $t(m)$ is bounded for small enough m. Then, $t(0) \in \arg\max_t f_m(t,0)$.*

Proof of Lemma 10. Suppose, on the contrary, that there exists some $\check{t}$ such that $f_m(\check{t},0) > f_m(t(0),0)$. Then, by continuity, there exists $\bar{m}$ such that

$$f_m(\check{t},m) > f_m(t(m),m) \text{ for } m < \bar{m}. \tag{49}$$

Integrate (49) with respect to m:[46]

$$f(\check{t},m) - f(\check{t},0) > f(t(m),m) - f(t(0),0).$$

Since $f(t,0)$ does not depend on t we have $f(t(0),0) = f(\check{t},0)$ and so

$$f(\check{t},m) > f(t(m),m).$$

We obtained a contradiction with $t(m) \in \arg\max_t f(t,m)$. ■

[46]To integrate the right-hand side we use the Envelope Theorem $\frac{df(t(m),m)}{dm} = f_m(t(m),m)$

D Information acquisition

Information acquisition happens at $t = 0$. The precisions $t(a)$ are endogenized by requiring them to be optimal, given an information acquisition cost. We define the notion of optimality of the profile of precisions $t(a)$ similarly to how we defined the optimality of demands: precision profile $t(a)$ is optimal in CHILE if it is a limit of optimal precisions in the discrete economies. We make this definition precise after we describe the information acquisition in the discrete economy. We assume that agents are uncertain about the distribution of wealth $W_0(a)$ (but they know their wealth).[47] This distribution becomes known before the start of the trade at $t = 1$.

The new primitive in CHILE is the information acquisition cost $c(t, a)$. We assume that the cost of acquiring an infinitesimal signal $ds(a) = vda + 1/\sqrt{t}dB(a)$ for a trader a is $c(t; a)da$, where $c(\cdot)$ is continuous, strictly increasing and convex function of t. Thus, the cost of acquiring a finite signal

$$\Delta s = \int_{a_i}^{a_i+m_i} vda + \int_{a_i}^{a_i+m_i} \frac{1}{\sqrt{t(a)}} dB(a) \text{ is } \int_{a_i}^{a_i+m_i} c(t(a), a_i)da.$$

A finite signal can be split into a collection of infinitesimal ones, with associated costs. Using Jensen's inequality, one can show that it is not optimal to split a finite signal into infinitesimal ones of varying precisions. If a trader wants to get information of precision tm_i, he should acquire a signal

$$\Delta s = \int_{a_i}^{a_i+m_i} vda + \frac{1}{\sqrt{t}} dB(a) = vm_i + \frac{B(a_i + m_i) - B(a_i)}{\sqrt{t}} \text{ at a cost } c(t; a_i)m_i. \tag{50}$$

Thus, without loss of generality, we restrict finite signals in the discrete economy to be of the form (50).

[47]Done this way, the information choice of agent a will only depend on his wealth, but not the wealth of others, which helps to simplify some of the analysis.

Denote $\mathcal{U}_i(W_0^i, t_i, m_i)$ the maximum utility in (**??**) for a given precision t_i and initial wealth W_0^i.[48] Define $c(t_i, a_i)m_i$ as a (monetary) cost of acquiring signal Δs_i. We require that precisions t_i are *optimal*, i.e.

$$t_i = \arg\max_t \left\{ \int \mathcal{U}_i(W_0^i - c(t, a_i)m_i, t, m_i) g(v, P, \mu) dv dP \right\}. \tag{51}$$

Here $g(\cdot)$ denotes the PDF of the joint distribution of $\mathbf{P}_i^n$ and v. We denote $t^n(a)$ the profile of optimal precisions in the discrete economy, $t^n(a) = t_i$, for all $a \in [a_i, a_i + m_i)$, where t_i solves (51).

We introduce some technical restrictions on $c(t; a)$.

Assumption 4. *The information acquisition cost $c(t; a)$ is such that there exists M_t such that for any i, $\hat{t}_i > M_t$ is not optimal.*

The additional technical restriction allows us to make a choice set in (51) compact. Thus, without loss of generality, we assume $t \in [0, M_t]$ everywhere in the sequel. We define the notion of optimal profile of precisions $t(a)$ by continuity.

Definition 4. *A profile $t(a)$ is* ***optimal*** *if, for every a, the optimal precision in discrete economy $t^n(a)$ converges to $t(a)$ as $n \to \infty$.*

Finally, we define the information acquisition equilibrium at $t = 0$.

Definition 5. *A profile $t(a)$ is an information acquisition* ***equilibrium*** *if for every a, $t(a)$ is optimal.*

With these definitions at hand, we are ready to state this section's main result in the subsection below.

[48]Here we follow the setup of the main paper, where the profile of preferences in the discrete and continuous economy are the same. This approach can be generalized to allow them to differ for finite n but coincide in the $n \to \infty$ limit. Such an alternative approach could allow to lift the technical restrictions from the primitives of CHILE and "shift" those restrictions to the primitives of discrete economies instead.

D.1 Information acquisition: heuristic derivation

It is illuminating to start with the heuristic derivation. Consider a change in trader a's time-2 realized utility due to trade

$$\begin{aligned} d\mathcal{U}_{t=2}(dX(a);a) &= u(W_0(a) - c(t(a))da + dX(a)(V(v) - P);a) - u(W_0(a);a) \\ &= u'(W_0(a))\left(dX(a)(V(v) - P) - c(t(a))da - \frac{\rho(a)}{2}dX(a)^2(V(v) - P)^2\right) \end{aligned}$$

In the second line, we Taylor expanded the $u(\cdot)$, up to terms of order da. Due to the heuristics of Section B, we have $dX(a)^2 = \beta(a,P)^2/t(a)da$. Now substitute $P = \mathbf{P}(a)$ (trader a's conjecture about the market-clearing price) and take the expectation:

$$\frac{\mathbb{E}[d\mathcal{U}_{t=2}(dX(a);a)]}{u'(W_0(a);a)} = \mathbb{E}[dX(a)(V(v) - \mathbf{P}(a)] - c(t(a))da - \frac{\rho(a)}{2t(a)}\mathbb{E}[\beta(a,\mathbf{P}(a))^2(V(v) - \mathbf{P}(a))^2]da. \tag{52}$$

Now simplify

$$\begin{aligned} \mathbb{E}[dX(a)(V(v) - \mathbf{P}(a))] &= da\,\mathbb{E}[\beta(a,\mathbf{P}(a))v(V(v) - \mathbf{P}(a))] \\ &\quad + 1/\sqrt{t(a)}\,\mathbb{E}[dB(a)\beta(a,\mathbf{P}(a))(V(v) - \mathbf{P}(a))] \\ &\quad + da\,\mathbb{E}[\delta(a,\mathbf{P}(a))(V(v) - \mathbf{P}(a))] \\ &= da\,\mathbb{E}[\beta(a,\mathbf{P}(a))v(V(v) - \mathbf{P}(a))]. \end{aligned} \tag{53}$$

In the first transition, we substituted $dX(a) = \beta(a,P)ds(a) + \delta(a,P)da$. To get (53), we substituted $\mathbb{E}[dB(a)\beta(a,\mathbf{P}(a))(V(v) - \mathbf{P}(a))] = 0$ ($dB(a)$ is independent of $V(v)$ and $\mathbf{P}(a)$ as the traders are price takers and assume the noise in their signals in independent from that in the price) and $\mathbb{E}[\delta(a,\mathbf{P}(a))(V(v) - \mathbf{P}(a))] = 0$ (market efficiency condition).

To proceed further, we divide (52) by da and pass to $da \to 0$ limit. In that limit, $\mathbf{P}(a)$ becomes the market clearing price $\mathbf{P}_*$, which for ease of notation we simply denote P going forward. We can further simplify (53) by noting that $\mathbb{E}[v(V(v) - P)|P] = \beta(a,P)Var[V(v)|P]\rho(a)/t(a)$

(see (40)). Then, after substituting $\mathbb{E}[(V(v)-P)^2|P] = \text{Var}[V(v)|P]$, and $\beta(a,P) = t(a)/\rho(a)\beta_P(P)$ we can finally obtain

$$\frac{\mathbb{E}[d\mathcal{U}_{t=2}(dX(a);a)]}{u'(W_0(a);a)da} = \frac{t(a)}{2\rho(a)}\,\mathbb{E}[\beta_P(P)^2\text{Var}[V(v)|P]] - c(t(a)) \tag{54}$$

The optimal precision maximizes (54) with respect to $t(a)$. We summarize in the Proposition below. The rigorous proof is in the Appendix C.12.

Proposition 5. *The optimal precision choice for a trader a solves*

$$c'(t,a) = \frac{1}{2\rho(a)}\,\mathbb{E}[\beta_P(P)^2\text{Var}[V(v)|P]]. \tag{55}$$

Here $\beta_P(P) = \frac{E[v(V(v)-P)|P]}{\text{Var}[V(v)|P]}$. When $V(v) = \exp(v)$, the optimal precision solves

$$c'(t,a) = \frac{1}{2\rho(a)}\,\mathbb{E}\left[\frac{1}{\tau^2\left(\exp\left(\tau^{-1}\right)-1\right)}\right]. \tag{56}$$

Provided that the equilibrium exists, the equilibrium precision for trader a is an increasing function of his wealth $W_0(a)$ under DARA preferences.

Note that the expectations in (55) and (56) are over prices P, distribution of aggregate signal paths $\{s(a)\}_a$, and potential wealth distributions.

Online appendix

E A benchmark model with LLN aggregation

This section demonstrates that our main results do not obtain if one adopts the traditional approach to modeling a large economy based on law of large numbers (LLN) aggregation, as in Hellwig (1980), Admati (1985), Peress (2004), among others. The key finding here is an aggregation result: an economy with investors differing in wealth, precision, and preferences appears observationally equivalent to one with homogeneous investors. Consequently, in such economies, Robin Hood variations in wealth that leave aggregate trading intensity unchanged have no effect on market quality, unlike in CHILE.

This section derives our main results using the model of Peress (2011), which is, to the best of our knowledge, the only model in the literature with micro-founded wealth effects and heterogeneous information.[49] We present the Peress (2011) model using notation similar to ours and provide brief derivations of the main results. For more details, we refer the reader to the original paper.

The economy unfolds over two time periods, $t = 1$ and $t = 2$. It consists of a continuum of agents of total mass one. There are two assets: one risk-free and one risky, with the returns of both assets, realized at $t = 2$. The risk-free asset has a perfectly elastic supply and gross return normalized to 1, while the risky asset has a liquidation value of V that is log-normally distributed.[50]

Agents trade at $t = 1$, determining the price P of the risky asset. There are rational traders indexed by $a \in [0, 1)$ and noise traders. Noise traders submit price-inelastic demand $\theta \sim N(0, \tau_\theta^{-1})$ (measured in units of stocks). Agent a has initial wealth $W_0(a)$, and he trades a

[49]It is straightforward to derive the results of this section within a mean-variance preference framework that includes ad-hoc wealth effects, where the "variance" coefficient depends on wealth, as in Farboodi et al. (2022b) and Mihet (2022).

[50]It is also straightforward to derive our results for the case of normal distribution of V. However, existing methods are limited in their ability to accommodate the broad range of payoff functions that CHILE supports.

total amount of x *dollars*—i.e. price per share times number of shares. His realized utility is

$$u(W, a),$$

where his wealth at time $t = 2$ is

$$W = W_0(a) + x(R - 1),$$

and where

$$R = V/P.$$

Eaach trader has a private signal $s(a) = v + Z(a)/\sqrt{t(a)}$, where $Z(a)$ are i.i.d. standard Normal random variables.

The analytical tractability is achieved by employing a "small risk" approximation. It is assumed that

$$\ln V = \zeta \cdot v + v_n, \tag{57}$$

where $v \sim N(0, \tau_v^{-1})$ is a learnable component of asset payoff, and $v_n \sim N(0, \zeta\tau_{v,n}^{-1})$ is a non-learnable component.[51] The approach consists of characterizing the equilibrium asymptotics as $\zeta \to 0$.

Remark 7. Relative to the model in the paper, the model here features two new ingredients. The first is noise θ. The second one is the unlearnable component v_u of asset payoff. Both are essential for the equilibrium here to be well-defined. Without θ, there will be no noise in the price, and the equilibrium will not exist. Without v_u, the asymptotics of equilibrium as $\zeta \to 0$ will not be well-behaved (see Peress (2011)). Our model in the paper is well-defined without these two elements, so we did not introduce them there. However, we verified that introducing them does not change our main conclusions. Thus, the comparison to the benchmark here is

[51] This is equivalent to Peress (2011) formulation, saying that agents learn about the mean of log-payoff as opposed to log-payoff itself. Indeed our assumption can equivalently be formulated as $\ln V \sim N(\zeta v, \zeta\tau_{v,u}^{-1})$, the mean $v \sim N(0, \tau_v^{-1})$ is not known, and agents learn about mean v as opposed to about log-payoff $\ln V$.

fair.

Remark 8. The aggregation here will be performed by computing $\int x(a)da$. It should be noted that $\int x(a)da$ represents the average, not aggregate demand. Indeed, the integral is approximately $\sum_a x(a)\frac{1}{n}$, which is average demand. (Here, n is the number of traders in the economy.) In the large economy here, each demand is not infinitesimal $1/x(a) < \infty$, so their sum is infinite. Starting from Hellwig (1980), the literature solves this problem by equalizing the average, not aggregate demand $\int x(a)da$ to the noisy supply θ. Consequently, the actual aggregate noisy supply, $\theta \cdot n$, has infinite variance.

E.1 Equilibrium

Solving for equilibrium involves taking the standard steps. We postulate that for small ζ, traders have the following conjecture about the price function

$$\ln P \equiv p = \hat{p}\zeta, \text{ where } \hat{p} = c_0 + c_1 v - c_2\theta. \tag{58}$$

Given such conjecture, agents form their demands, which then results in an equilibrium price function $p^{eq}(\zeta)$. Equilibrium requires traders' conjecture to be consistent in the $\zeta \to 0$ limit: $p^{eq}(\zeta) = \hat{p}\zeta + o(\zeta)$.

Substituting (57) and conjecture (58) into agent a's first order condition $E_a[u'(W_0(a) + x(\exp(\ln V - p) - 1)(\exp(\ln V - p) - 1)] = 0$, expanding it up to the terms of order ζ, we obtain $x(p) = \hat{x}(\hat{p}) + o(1)$, where

$$\hat{x}(\hat{p}) = \frac{E_a[v] - \hat{p} + 0.5\tau_{v,n}^{-1}}{\rho(a)\tau_{v,n}^{-1}},$$

where $\rho(a) = -u''(W_0(a))/U'(W_0(a))$ is trader a's absolute risk aversion and

$$E_a[v] = \frac{t(a)}{\tau_v + \tau_p + t(a)} s(a) + \frac{\tau_p}{\tau_v + \tau_p + t(a)} \frac{\hat{p} - c_0}{c_1}, \text{ and } \tau_p \equiv \mathrm{Var}[v|p]^{-1} - \tau_v = \frac{c_1^2}{c_2^2}\tau_\theta.$$

Applying market clearing, we get

$$v \cdot \tau_{v,n} \int_{\nu}^{1} \frac{t(a)/\rho(a)}{\tau_v + \tau_p + t(a)} da + (\text{affine function of } p) = \theta,$$

and hence

$$\frac{c_1}{c_2} = \sqrt{\frac{\tau_p}{\tau_\theta}} = \tau_{v,n} \int_0^1 \frac{t(a)/\rho(a)}{\tau_v + \tau_p + t(a)} da.$$

The main result of this section follows.

Theorem 2. *There exists a unique equilibrium. The equilibrium demands can be written as* $\hat{x}(\hat{p}) = \alpha(a) + \beta(a)s(a) - \gamma(a)\hat{p}$, *where*

$$\beta(a) = \frac{t(a)\tau_{v,n}}{\rho(a)(\tau_v + \tau_p + t(a))}.$$

The price informativeness τ_p *can be written as*

$$\tau_p = \left(\int_0^1 \beta(a) da \right)^2 \tau_\theta$$

and is the unique solution to

$$\sqrt{\frac{\tau_p}{\tau_\theta}} = \tau_{v,n} \int_0^1 \frac{t(a)/\rho(a)}{\tau_v + \tau_p + t(a)} da.$$

The price function and the aggregate demand is the same as in the economy with homogeneous investors with absolute risk aversion ρ *and precision* t *satisfying* $\int_0^1 \frac{t(a)/\rho(a)}{\tau_v+\tau_p+t(a)} da = \frac{t/\rho}{\tau_v+\tau_p+t}$.

The economy with LLN aggregation allows for a "representative agent" formulation. Aggregate quantities, such as the price function and informativeness, remain consistent between the original heterogeneous economy and the equivalent homogeneous one. In this case, heterogeneity is irrelevant, unlike in CHILE. Consequently, we also get the following result, further illustrating the distinctions of CHILE and traditional economies in terms of comparative statics of information efficiency.

Corollary 2. *Aggregate trading intensity* $\int_0^1 \beta(a)da$ *is a sufficient statistic of price informativeness. Changes in wealth distribution that leave the aggregate trading intensity unchanged leave price informativeness unchanged.*

In CHILE, under the assumptions of Proposition 2, *any* Robin Hood variation of wealth with thresholds $\underline{a} \leq a_1^W \leq a_2^W \leq \bar{a}$ results in increased information efficiency. This implies that the result holds even for redistributions that leave aggregate trading intensity unchanged. This distinction underscores the difference between CHILE and traditional large economies: changes in parameters that do not affect the parameters of an equivalent homogeneous economy have no impact on traditional economies, but they do in CHILE.

F A model with noise traders

In this section, we introduce noise traders. The resulting equilibrium captures both the mechanism emphasized in previous literature—where inequality enhances information efficiency by increasing the aggregate trading intensity of informed traders—and the new mechanism highlighted previously in this paper, which shows that inequality hinders efficiency by disrupting the optimal aggregation of informed traders' signals. The main result is that the new mechanism tends to dominate, ensuring that our findings from the main part of the paper remain robust to the introduction of noise.

F.1 Setup

The model follows the setup in Section 3, with the addition of noise traders. There is a measure ν of these traders, and we index all traders by $a \in [0, 1+\nu)$. Traders in the range $a \in [0, 1)$ are rational, while those in $a \in [1, 1+\nu)$ are noise traders.

To maintain analytical tractability, we model noise traders following Black (1976), who describes them as traders "trading on noise as if it were information." Specifically, we assume that noise traders observe

$$ds(a) = u\,da + \frac{1}{\sqrt{t(a)}}dB(a), \quad a \in [1, 1+\nu). \tag{59}$$

Here, $u \sim N(u, \tau_v^{-1})$ is distributed identically to v but is independent of it. Although their signals follow (59), noise traders act as if they were receiving signals given by (5). Noise traders are identical to rational traders in all aspects except for the information they receive and how they respond to it.

F.2 Derivation of equilibrium

Derivation identical to that of Theorem 1 yields that the demand of noise traders is given by

$$dX(a) = \beta(a, P)ds(a) + \delta(a, P)da,$$

Where

$$\beta(a, P) = \frac{t(a)}{\rho(a)} \frac{\tau^{-1}\,\mathbb{E}[V'(v)|s_p]}{\mathrm{Var}\,[V(v)|s_p]}.$$

From market clearing, agents can extract the sufficient statistic

$$s_p = v + \frac{\int_1^{1+\nu} \frac{t(a)}{\rho(a)}da}{\int_0^1 \frac{t(a)}{\rho(a)}da} u + \int_0^{1+\nu} \frac{t(a)/\rho(a)}{\int_0^1 t(a)/\rho(a)da} \frac{dB(a)}{\sqrt{t(a)}}, \tag{60}$$

which has precision

$$\tau_p = \left(\left(\frac{\int_1^{1+\nu} \frac{t(a)}{\rho(a)}da}{\int_0^1 \frac{t(a)}{\rho(a)}da} \right)^2 \tau_v^{-1} + \left(\frac{1}{\int_0^1 t(a)/\rho(a)da} \right)^2 \int_0^{1+\nu} \frac{t(a)}{\rho(a)^2}da \right)^{-1}. \tag{61}$$

As in the main model, prices must be weak-form efficient, which implies

$$P = \mathbb{E}[V(v) \mid s_p].$$

The conditional distribution of v given P is normal with mean $(\tau_p/\tau) \cdot s_p$ and variance $1/\tau$. Defining the standardized variable $z = \sqrt{\tau}(v - (\tau_p/\tau) \cdot s_p)$, we note that z follows a standard normal distribution.

Thus, we can express v as

$$v = \frac{\tau_p}{\tau} s_p + \frac{z}{\sqrt{\tau}}.$$

Substituting this into the efficiency condition gives

$$\mathbb{E}[V(v) \mid P] = \mathbb{E}\left[V\left(\frac{\tau_p}{\tau}s_p + \frac{z}{\sqrt{\tau}}\right)\right] = \int V\left(\frac{\tau_p}{\tau}s_p + \frac{z}{\sqrt{\tau}}\right) d\Phi(z),$$

where $\Phi(z)$ is the standard normal cumulative distribution function.

The rest of the derivation follows the steps of Theorem 1, leading to the final result.

Theorem 3. *There exists a unique equilibrium. The equilibrium price function has the representation $\mathbf{P}_* = \mathcal{P}(s_p)$, where the equilibrium sufficient statistic s_p is given by (60). The function $\mathcal{P}(x)$ is given by*

$$\mathcal{P}(x) = \int V\left(\frac{\tau_p}{\tau}x + \frac{z}{\sqrt{\tau}}\right) d\Phi(z). \tag{62}$$

Here $\Phi(z)$ denotes the standard normal cumulative distribution function (cdf). Consequently, the price function is completely determined by $V(\cdot)$ and two other quantities, the precision of s_p, given by (61) and the posterior precision of v, given by $\tau = \mathbb{V}ar(v|P)^{-1} = \tau_v + \tau_p$.

The equilibrium cumulative demand function has the representation $dX(a) = \beta(a,P)ds(a) + \delta(a,P)da$, for all $a \in [0, 1+\nu)$, where

$$\beta(a,P) = \frac{t(a)}{\rho(a)} \frac{\tau^{-1}\,\mathbb{E}[V'(v)|s_p]}{\mathrm{Var}\left[V(v)|s_p\right]}, \text{ and}$$

$$\delta(a,P) = \frac{\beta(a,P)^2}{2t(a)}\pi(a)\frac{\mathrm{Sk}[V(v)|s_p]}{\mathrm{Var}[V(v)|s_p]} - \beta(a,P)\frac{\mathbb{E}[v(V(v)-P)^2|s_p]}{\mathrm{Var}[V(v)|s_p]} + \frac{\psi(P)}{\rho(a)\mathrm{Var}[V(v)|s_p]}.$$

Here $\rho(a)$ and $\pi(a)$ denote the absolute risk aversion and prudence coefficients, defined in (17). The sufficient statistic s_p is related to the price P as follows:

$$s_p = \mathcal{P}^{-1}(P).$$

Here $\mathcal{P}^{-1}(\cdot)$ is the inverse of the function $\mathcal{P}(\cdot)$ defined in (62). The conditional moments of $V(v)$ and the function $\psi(P)$ are given in the closed form in (44).

F.3 Market quality

Our definitions of liquidity $\mathcal{L}$ and information efficiency $\mathcal{I}$ follow those in the main paper. Using the derivation steps from Proposition 1, we obtain

$$\mathcal{I} = \frac{\tau_p}{\tau_v + \tau_p},$$

and

$$\mathcal{L} = \frac{1}{\tau_p} \int_0^1 \frac{t(a)}{\rho(a)} da.$$

Since τ_p is given by (61), this leads to the following result.

Proposition 6. *The equilibrium information efficiency and liquidity are given by*

$$\mathcal{I} = \frac{\tau_p}{\tau_v + \tau_p}, \quad \mathcal{L} = \frac{1}{\tau_p} \int_0^1 \frac{t(a)}{\rho(a)} da,$$

where τ_p is given by (61).

F.4 Comparative statics

To ensure consistency with previous literature, which assumes exogenous noise remains unaffected by the parameters of interest, we impose that comparative statics do not influence noise traders. Formally, our definition of the Robin Hood variation assumes $h^{\Delta}(a) = 0$ for all $a \in [1, 1+\nu)$ and coincides with Definition 3 in all other respects.

F.4.1 Wealth inequality

First, we examine changes in the wealth distribution $W_0(a)$ while keeping the precisions $t(a)$ fixed. This leads to the following proposition, which exactly matches Proposition 2.

Proposition 7. *Suppose that agent preferences are DARA. Then, there exist thresholds* $0 <$

$a_1^W \leq a_2^W < 1$, *such that for any Robin Hood variation* $W_0^\Delta(a)$ *with* $\underline{a} \leq a_1^W \leq a_2^W \leq \bar{a}$ *we have*

$$\mathcal{I}'(W_0(a))[W_0^\Delta(a)] > 0 \text{ and } \mathcal{I}'(W_0(a))[-W_0^\Delta(a)] < 0; \tag{63}$$

$$\mathcal{L}'(W_0(a))[W_0^\Delta(a)] < 0 \text{ and } \mathcal{L}'(W_0(a))[-W_0^\Delta(a)] > 0. \tag{64}$$

The mechanism highlighted in previous literature—where inequality enhances information efficiency by increasing the aggregate trading intensity of informed traders—does not alter Proposition 2 in the presence of noise. However, this does not imply that exogenous noise is irrelevant. Its presence may influence the thresholds a_1^W and a_2^W.

F.4.2 Information inequality

Here, we examine the effects of changing the distribution of information across agents on market quality holding the wealth profile fixed.

Proposition 8. *Suppose that traders have DARA utilities. Then, there exist thresholds* $0 < a_1^t \leq a_2^t < 1$, *such that for any Robin Hood variation* $t^\Delta(a)$ *with* $\underline{a} \leq a_1^t \leq a_2^t \leq \bar{a}$

$$\mathcal{I}'(t(a))[t^\Delta(a)] > 0 \text{ and } \mathcal{I}'(t(a))[-t^\Delta(a)] < 0;$$

$$\mathcal{L}'(t(a))[t^\Delta(a)] < 0 \text{ and } \mathcal{L}'(t(a))[-t^\Delta(a)] > 0.$$

The formulation of Proposition 3 remains unchanged. Corollary 1 also holds with noise.

Corollary 3 (An information-aggregation paradox)**.** *Suppose that traders have DARA utilities. Then, there exists a threshold* a_2^t *such that for any* $h^\Delta(a) \neq 0$ *such that* $h^\Delta(a) \geq 0$ *for* $a > a_2^t$, *and* $h^\Delta(a) = 0$ *otherwise,* $\mathcal{I}'(t(a))[t^\Delta(a)] > 0$ *and* $\mathcal{I}'(t(a))[-t^\Delta(a)] < 0$.

F.4.3 The role of information acquisition

Combining the results of Propositions 7 and 8, we conclude that the statement of Proposition 4 remains unchanged.

Proposition 9. *Suppose that Assumption 2 holds. Suppose that traders have DARA utilities. Suppose that precisions are a function of wealth $t(W_0(a), a)$ and are increasing in $W_0(a)$. Then, for any Robin Hood variation $W_0^{\Delta}(a)$ with $\underline{a} \leq \min\{a_1^t, a_1^W\} \leq \max\{a_2^t, a_2^W\} \leq \bar{a}$, results (63)–(64) hold.*

F.5 Proofs for Section F

F.6 Proof of Proposition 7

Proof of Proposition 7. Following the proof of Proposition 2, we denote absolute risk tolerance $y(a) \equiv 1/\rho(a)$. Without loss of generality, index traders such that $y(b)$ increases in b. (This is in contrast to index a, which is such that $W_0(a)$ is increasing in a.) We first compute the Gateaux derivatives of $\mathcal{I}$ and $\mathcal{L}$ with respect to $y(b)$.

Substituting (61) into $\mathcal{I} = \tau_p/(\tau_p + \tau_v)$ and computing the Gateaux derivative (this entails substituting $y(b) + \epsilon y^{\Delta}(b)$ instead of $y(b)$, differentiating with respect to ϵ, and evaluating the resulting expression at $\epsilon = 0$) yields:

$$\mathcal{I}'(y(b))[y^{\Delta}(b)] = C_{\mathcal{I}} \int_0^1 t(b) y^{\Delta}(b)(K_1 - K_2 y(b))\, db.$$

Here, $C_{\mathcal{I}} > 0$ is positive (we have the closed-form expressions for $C_{\mathcal{I}}$ via parameters of the model, but it is not important here), $I_1 = \int_0^1 t(b)y(b)\, db$, $I_2 = \int_0^{1+\nu} t(b)y(b)^2\, db$, and $I_3 = \int_1^{1+\nu} t(b)y(b)\, db$, $K_1 = I_3^2 + I_2\tau_v$, and $K_2 = I_1\tau_v$.

Lemma 9 implies the existence of a unique b_y^* such that $K_2 - K_1 y(b) \gtrless 0$ if and only if $b \lessgtr b_y^*$. The remainder of the proof follows the same steps as Proposition 2 and is omitted for brevity.

We now turn to the liquidity results. Substituting (61) in $\mathcal{L} = \int_0^1 t(b)y(b)db/\tau_p$ and computing the Gateaux derivative yields

$$(\mathcal{L})'(y(b))[y^{\Delta}(b)] = C_{\mathcal{L}} \int_0^1 t(b)y^{\Delta}(b)(k_1 y(b) - k_2)\, db.$$

Here, $C_{\mathcal{L}} > 0$ is positive, $k_1 = 2\tau_v I_1$ and $k_2 = I_2\tau_v + I_3^2$. Lemma 9 implies that there exists a unique $b_{**}^y > b_*^y$ such that $k_1 y(b) - k_2 \gtreqless 0$ iff $b \gtreqless b_{**}^y$. The remainder of the proof follows the same steps as Proposition 2 and is omitted for brevity. ■

F.7 Proof of Proposition 8

Proof of Proposition 8.

This proof follows the same steps as the proof of Proposition 2. Without loss of generality, we index traders such that $y(b)$ is increasing in b. The proof remains identical to that of Proposition 2, except for the expressions for the Gâteaux derivatives, which we reproduce below:

$$\mathcal{I}'(t(b))[t^{\Delta}(b)] = C_{\mathcal{I}} \int_0^1 y(b)t^{\Delta}(b)(K_2 - K_1 y(b))\, db;$$

$$\mathcal{L}'(t(b))[t^{\Delta}(b)] = C_{\mathcal{L}} \int_0^1 y(b)t^{\Delta}(b)(k_1 y(b) - k_2)\, db.$$

Here, $K_2 = 2I_3^2 + 2I_2\tau_v$, $K_1 = I_1\tau_v$, $k_1 = I_1\tau_v$, and $k_2 = K_2/2$. The notation I_1, I_2, and I_3 follows the definitions in the proof of Proposition 7. ■

G Price taking equilibrium, competitive REE and BNE: CARA-Normal case

In this section, we contrast three equilibrium concepts: (i) the Bayesian Nash Equilibrium (BNE), in which traders account for both (a) their individual impact on the price level and (b) the informational content of the price; (ii) the competitive Rational Expectations Equilibrium (REE), where traders internalize (b) but ignore (a); and (iii) the price-taking equilibrium, where traders disregard both. A large portion of the material presented here is drawn from Avdis et al. (2025) and is included for completeness.

A full treatment of BNE and competitive REE under general preferences and payoff structures is the subject of our ongoing work. In this section, we focus on the CARA-normal framework, modeling wealth effects in an ad hoc manner by assuming that the absolute risk aversion coefficient $\rho(a)$ decreases with initial wealth $W_0(a)$.[52]

G.1 Setup

We consider a sequence of economies with a finite number n of traders and examine the behavior of equilibrium quantities in the large economy limit as $n \to \infty$. The economy features a risky asset with payoff $v \sim N(0, \tau_v^{-1})$ and a riskless asset that serves as the numéraire.

Each trader i lives in an interval $[a_i, a_i + m)$, where $a_i = \frac{i-1}{n}$ and $m = \frac{1}{n}$. Traders exhibit Constant Absolute Risk Aversion (CARA), with individual absolute risk aversion coefficients denoted by $\rho(a_i)$. Trader i receives a private signal of the form:

$$\Delta s_i = v\, m + \frac{1}{\sqrt{t(a_i)}} \Delta B(a_i),$$

where Δs_i has precision tm.

[52]See Makarov and Schornick (2010), Kurlat and Veldkamp (2015), and Mihet (2018), who adopt a similar ad hoc approach to modeling wealth effects.

In the large economy limit, we can write the above as

$$ds(a) = v\,da + \frac{1}{\sqrt{t(a)}} dB(a).$$

Our objective is to study the limiting behavior of equilibria as $n \to \infty$. To support the existence of BNE and competitive REE, we introduce noisy supply:

$$u \sim N(0, \tau_u^{-1}).$$

We begin by analyzing standard equilibrium concepts: (i) the competitive Rational Expectations Equilibrium (REE), and (ii) the Bayesian Nash Equilibrium (BNE) in demand schedules. We then turn to the price-taking equilibrium.

G.2 Market quality

Our definitions of liquidity and information efficiency are unchanged from the main text. We define information efficiency as:

$$\mathcal{I} = 1 - \frac{\mathrm{Var}[v \mid P]}{\mathrm{Var}[v]},$$

which measures the fraction of uncertainty about the fundamental value v that is eliminated by observing the price P.

Our measure of liquidity is based on the sensitivity of prices to price-inelastic shocks. Formally, it is defined as:

$$\mathcal{L} = -\left(\frac{\partial P}{\partial u} \cdot \frac{1}{\mathrm{Var}[v \mid P]}\right)^{-1}.$$

In our discrete economies, equilibrium is linear, with individual demands taking the form:

$$x_i^n = \beta^n(a_i)\,\Delta s_i - \gamma^n(a_i)\,p\,m.$$

Under this structure, the liquidity expression simplifies to:

$$\mathcal{L} = \mathrm{Var}[v \mid P] \cdot \sum_{i=1}^{n} \gamma(a_i)\, m. \tag{65}$$

G.3 Competitive REE

We begin by defining the notion of equilibrium in the discrete economy:

Definition 6. *In the discrete economy n, a competitive REE consists of a price function P^n and a collection of demand schedules $\{x_i^n(p, \Delta s_i)\}$, such that the following two conditions are satisfied:*

1. *Optimality: For each trader i,*

$$x_i^n(p, \Delta s_i) \in \arg\max_x \mathbb{E}\left[-\exp\left(-\gamma_i(v-p)x\right) \mid \Delta s_i, P^n = p\right];$$

2. *Market clearing:*

$$\sum_i x_i^n(P^n, \Delta s_i) - u = 0.$$

We employ a guess-and-verify approach to solve for the equilibrium. We conjecture that the equilibrium strategies in the discrete economy n take the linear form:

$$x_i^n = \beta^n(a_i)\, \Delta s_i - \gamma^n(a_i)\, p\, m, \tag{66}$$

where $\beta^n(a_i)$ and $\gamma^n(a_i)$ are constants specific to each trader.

In the large economy limit, the corresponding strategy becomes:

$$dX(a) = \beta(a)\, ds(a) - \gamma(a)\, p\, da.$$

From market clearing, traders can compute the sufficient statistic:

$$s_p^n = v + \sum_{i=1}^{n} \frac{w^n(a_i)}{\sqrt{t(a_i)}} \Delta B(a_i) - \frac{u}{\sum_{i=1}^n \beta(a_i) m} = P \cdot \frac{\sum_{i=1}^n \gamma(a_i) m}{\sum_{i=1}^n \beta(a_i) m},$$

where we introduce the weights

$$w^n(a_i) = \frac{\beta^n(a_i)}{\sum_{j=1}^n \beta(a_j) m}.$$

In the large economy limit, these expressions become:

$$s_p = v + \int_0^1 \frac{w(a)}{\sqrt{t(a)}} \, dB(a) - \frac{u}{\int_0^1 \beta(a)\, da} = P \cdot \frac{\int_0^1 \gamma(a)\, da}{\int_0^1 \beta(a)\, da}, \tag{67}$$

and the weight function is defined as

$$w(a) = \frac{\beta(a)}{\int_0^1 \beta(a)\, da}.$$

G.3.1 Heuristic derivation of equilibrium

Following the approach in Section B, it is straightforward to derive that optimal demands take the form:

$$dX(a) = \frac{\mathbb{E}[v - P \mid ds(a), s_p]}{\rho(a)\ \mathbb{V}\text{ar}(v|s_p)}. \tag{68}$$

To identify $\beta(a)$, we isolate the contribution of $ds(a)$ on the right-hand side of (68), which requires computing the conditional expectation $\mathbb{E}[v \mid ds(a), s_p]$.

To facilitate this calculation, we rewrite the conditional expectation as:

$$\mathbb{E}[v \mid ds(a), s_p] = \mathbb{E}[v \mid ds(a), s_p - w(a)\, ds(a)],$$

where $w(a) = \beta(a)/\int_0^1 \beta(a)\, da$. Since $ds(a)$ and $s_p - w(a)\, ds(a)$ are conditionally independent given v, we can apply the standard linear projection formula to obtain:

$$\begin{aligned}\mathbb{E}[v \mid ds(a), s_p - w(a)\, ds(a)] &= \frac{t(a)}{\tau}\, ds(a) + \frac{\tau_p}{\tau}\,(s_p - w(a)\, ds(a)) \\ &= \left(\frac{t(a)}{\tau} - \frac{\tau_p}{\tau} w(a)\right) ds(a) + \cdots,\end{aligned}$$

where $\tau \equiv \mathbb{V}\text{ar}(v|s_p)^{-1}$, τ_p is the precision of s_p, and "$\cdots$" denotes terms that do not depend on $ds(a)$.

Matching the coefficients of $ds(a)$ in (68), and substituting for $w(a)$, we obtain:

$$\beta(a) = \frac{1}{\rho(a)}\left(t(a) - \tau_p \cdot \frac{\beta(a)}{\int_0^1 \beta(a)\, da}\right). \tag{69}$$

To solve for $\beta(a)$, we introduce the shorthand:

$$\kappa \equiv \frac{\tau_p}{\int_0^1 \beta(a)\, da}. \tag{70}$$

Then equation (78) becomes:

$$\beta(a) = \frac{1}{\rho(a)}\,(t(a) - \kappa\, \beta(a)) \qquad \Longleftrightarrow \qquad \beta(a) = \frac{t(a)}{\rho(a) + \kappa}.$$

Thus, once κ is determined, $\beta(a)$ is fully characterized. To solve for κ, we compute precision τ_p, by deriving the variance of the noise term in the sufficient statistic s_p:

$$\begin{aligned}\tau_p^{-1} &= \text{Var}\left(\int_0^1 \frac{w(a)}{\sqrt{t(a)}}\, dB(a) - \frac{u}{\int_0^1 \beta(a)\, da}\right) \\ &= \int_0^1 \frac{w(a)^2}{t(a)}\, da + \frac{1}{\left(\int_0^1 \beta(a)\, da\right)^2},\end{aligned}$$

where the second equality follows from Itô isometry and can be derived heuristically as in Section B.

Substituting $w(a) = \beta(a)/\int_0^1 \beta(a)\,da$, we obtain:

$$\tau_p = \frac{\left(\int_0^1 \beta(a)\,da\right)^2}{\int_0^1 \frac{\beta(a)^2}{t(a)}\,da + \tau_u^{-1}}. \tag{71}$$

Hence,

$$\kappa = \frac{\int_0^1 \beta(a)\,da}{\int_0^1 \frac{\beta(a)^2}{t(a)}\,da + \tau_u^{-1}}. \tag{72}$$

Substituting $\beta(a) = t(a)/(\rho(a) + \kappa)$ into (72), we obtain an equation for κ alone:

$$\kappa = \frac{\int_0^1 \frac{t(a)}{\rho(a)+\kappa}\,da}{\int_0^1 \frac{t(a)}{(\rho(a)+\kappa)^2}\,da + \tau_u^{-1}}.$$

We summarize this heuristic derivation below and provide a rigorous proof in Appendix G.7.

Theorem 4. *Let κ be the unique positive solution to*

$$\kappa = \frac{\int_0^1 \frac{t(a)}{\rho(a)+\kappa}\,da}{\int_0^1 \frac{t(a)}{(\rho(a)+\kappa)^2}\,da + \tau_u^{-1}}. \tag{73}$$

There exists a unique competitive REE with

$$\beta(a) = \frac{t(a)}{\rho(a) + \kappa}.$$

G.3.2 Market quality

The following proposition characterizes liquidity and information efficiency in equilibrium.

Proposition 10. *The equilibrium information efficiency and liquidity are given by*

$$\mathcal{I} = \frac{\tau_p}{\tau_v + \tau_p}, \quad \mathcal{L} = \frac{1}{\kappa},$$

where

$$\tau_p = \kappa \int_0^1 \frac{t(a)}{\rho(a) + \kappa} \, da, \tag{74}$$

and κ *solves* (73).

With the above proposition in hand, we are now ready to examine how changes in the wealth distribution affect market quality.

G.3.3 Comparative statics

We assume that absolute risk aversion is a decreasing function of initial wealth. With a slight abuse of notation, we write it as $\rho(W_0(a))$, where $\rho(\cdot)$ is a decreasing function. In addition, we continue to maintain Assumption 2 from the main text and assume that traders are indexed so that $W_0(a)$ is increasing in a.

Since endogenous information acquisition is not modeled here, we conduct comparative statics with respect to $W_0(a)$ only, and take $t(a)$ as exogenous.

Proposition 11. *Suppose that agent preferences are DARA. Then there exist thresholds* $0 < a_1^W \leq a_2^W < 1$ *such that for any Robin Hood variation* $W_0^\Delta(a)$ *with* $\underline{a} \leq a_1^W \leq a_2^W \leq \bar{a}$, *the following holds:*

$$\mathcal{L}'(W_0(a))[W_0^\Delta(a)] < 0 \quad \textit{and} \quad \mathcal{L}'(W_0(a))[-W_0^\Delta(a)] > 0.$$

Now suppose in addition that $\int_0^1 W_0^\Delta(a)\, da = 0$, *and that the function* $a \mapsto t(a)\, |\rho'(W_0(a))|/(c+\rho(W_0(a)))^2$ *is decreasing in* a *for every* $c > 0$, *at least for* a *sufficiently close to 0 and 1. Then, there exist thresholds* $0 < a_1^W \leq a_2^W < 1$ *such that for any Robin Hood variation* $W_0^\Delta(a)$ *with* $\underline{a} \leq a_1^W \leq a_2^W \leq \bar{a}$, *we have:*

$$\mathcal{I}'(W_0(a))[W_0^\Delta(a)] > 0 \quad \textit{and} \quad \mathcal{I}'(W_0(a))[-W_0^\Delta(a)] < 0.$$

Our results on liquidity extend directly from the main paper without any modification. In the case of information efficiency, we impose additional technical assumptions in order to complete the proof. However, the main economic message remains unchanged: transferring wealth from the rich to the poor improves information efficiency but reduces liquidity.

The proof is more intricate here due to the interaction between liquidity and information efficiency—an effect that was absent in the baseline model. As seen in equation (74), the precision τ_p depends on $\kappa = 1/\mathcal{L}$, introducing a feedback loop between the two measures.

To ensure the result goes through analytically, we impose two auxiliary conditions:

1. *Budget balance*: The requirement $\int_0^1 W_0^{\Delta}(a)\, da = 0$ guarantees that the Robin Hood variation is not wasteful—all wealth taken from the rich is transferred to the poor.

2. *Monotonicity*: We assume that the function

$$y(a) = \frac{t(a)\,|\rho'(W_0(a))|}{(c + \rho(W_0(a)))^2}$$

is decreasing in a. This condition is satisfied, for instance, when $\rho(a) = \eta/W_0(a)$ (emulating CRRA preferences with RRA equal to η) and $t(a) = 1$.[53] In that case,

$$y(a) = \frac{\eta}{(cW_0(a) + \eta)^2}$$

is decreasing in a.

While these assumptions are used for technical tractability, we expect that the underlying comparative statics continue to hold more broadly.

[53] This example can be generalized to allow for non-constant signal precisions $t(a)$.

G.4 BNE

In this section, we consider an equilibrium with strategic traders, as in Kyle (1989). We follow the approach advocated by Kovalenkov and Vives (2014): we start from the finite-agent Bayesian Nash equilibrium—where each trader fully internalizes both her impact on the price level and on the informational content of the price—and characterize the limiting equilibrium as the number of traders grows large.

Definition 7. *A Bayesian Nash Equilibrium (BNE) is a collection of demand schedules* $\{x_i^n(p, \Delta s_i)\}_{i=1}^n$ *such that for each trader* i*, the schedule* x_i^n *solves:*

$$\mathbb{E}\left[-\exp\left(\rho(a_i)(v - p(x_i^n, x_{-i}^n)) \cdot x_i^n\right)\right] \geq \mathbb{E}\left[-\exp\left(\rho(a_i)(v - p(y, x_{-i}^n)) \cdot y\right)\right]$$

for any alternative demand schedule y*.*

Here, x_{-i}^n *denotes the vector of equilibrium demand schedules for traders* $j \neq i$*, and* $p(y, x_{-i}^n)$ *is the market-clearing price when trader* i*'s schedule is* y*, and other traders follow* x_{-i}^n*.*

We employ a guess-and-verify approach to solve for the equilibrium. We conjecture that the equilibrium strategies in the discrete economy with n traders take the linear form:

$$x_i^n = \beta^n(a_i)\,\Delta s_i - \gamma^n(a_i)\,p\,m, \tag{75}$$

where $\beta^n(a_i)$ and $\gamma^n(a_i)$ are constants specific to each trader.

In the large economy limit, the corresponding strategy becomes:

$$dX(a) = \beta(a)\,ds(a) - \gamma(a)\,p\,da.$$

Following the derivation in Kyle (1989), the optimal demand for trader i is given by:

$$x_i^n = \frac{\mathbb{E}[v \mid \Delta s_i, s_p^n] - P}{\rho(a_i)\,\mathrm{Var}[v \mid \Delta s_i, s_p^n] + \lambda_i^n}.$$

Here, λ_i denotes the individual price impact, i.e., the slope of the inverse residual supply curve faced by trader i, given by:

$$\lambda_i^n = \left(\sum_{j\neq i} m\,\gamma^n(a_j)\right)^{-1}.$$

In the large economy limit, we write, heuristically:

$$dX(a) = \frac{\mathbb{E}[v - P \mid ds(a), s_p]}{\rho(a)\ \mathbb{V}\mathrm{ar}(v|s_p) + \lambda}, \tag{76}$$

where

$$\lambda = \left(\int_0^1 \gamma(a)\,da\right)^{-1} = \mathbb{V}\mathrm{ar}(v|s_p)\cdot\mathcal{L}^{-1}.$$

In the last equality, we used the limiting form of the liquidity expression from equation (65), replacing the discrete sum $\sum_{i=1}^n \gamma(a_i)\,m$ with its continuous counterpart $\int_0^1 \gamma(a)\,da$.

G.4.1 Heuristic derivation of equilibrium

We focus our derivation on the coefficient $\beta(a)$. Define the constant κ as in Section G.3, via equation (70). We will show below that the relationship $\kappa = 1/\mathcal{L}$ continues to hold in the Bayesian Nash Equilibrium (BNE). This allows us to rewrite the first-order condition (77) as:

$$dX(a) = \frac{\mathbb{E}[v - P \mid ds(a), s_p]}{\mathrm{Var}[v \mid s_p]\cdot(\rho(a)+\kappa)}. \tag{77}$$

Proceeding as in Section G.3.1 we obtain:

$$\mathbb{E}[v \mid ds(a), s_p - w(a)\, ds(a)] = \left(\frac{t(a)}{\tau} - \frac{\tau_p}{\tau} w(a)\right) ds(a) + \cdots,$$

where $\tau = \mathrm{Var}[v \mid s_p]^{-1}$, and $\cdots$ denotes terms independent of $ds(a)$.

Matching the coefficients of $ds(a)$ in equation (77), and substituting for $w(a) = \beta(a)/\int_0^1 \beta(a)\, da$, we obtain:

$$\beta(a) = \frac{1}{\rho(a) + \kappa}\left(t(a) - \tau_p \cdot \frac{\beta(a)}{\int_0^1 \beta(a)\, da}\right). \tag{78}$$

Substituting the definition of κ from (70) into the equation above gives:

$$\beta(a) = \frac{t(a) - \kappa \cdot \beta(a)}{\rho(a) + \kappa} \quad \Longleftrightarrow \quad \beta(a) = \frac{t(a)}{\rho(a) + 2\kappa}.$$

Moreover, equation (72) continues to hold in BNE. Substituting the above expression for $\beta(a)$ into it, we obtain the equation that determines κ:

$$\kappa = \frac{\int_0^1 \frac{t(a)}{\rho(a)+2\kappa}\, da}{\int_0^1 \frac{t(a)}{(\rho(a)+2\kappa)^2}\, da + \tau_u^{-1}}.$$

We summarize this heuristic derivation below and provide a rigorous proof in Appendix G.7.

Theorem 5. *Let k be the unique positive solution to*

$$\kappa = \frac{\int_0^1 \frac{t(a)}{\rho(a)+2\kappa}\, da}{\int_0^1 \frac{t(a)}{(\rho(a)+2\kappa)^2}\, da + \tau_u^{-1}}. \tag{79}$$

There exists a unique equilibrium with

$$\beta(a) = \frac{t(a)}{\rho(a) + 2\kappa}.$$

G.4.2 Market quality

The characterization of liquidity and information efficiency in equilibrium remains unchanged from the competitive REE case.[54] For completeness, we restate the result below.

Proposition 12. *The equilibrium information efficiency and liquidity are given by*

$$\mathcal{I} = \frac{\tau_p}{\tau_v + \tau_p}, \quad \mathcal{L} = \frac{1}{\kappa},$$

where

$$\tau_p = \kappa \int_0^1 \frac{t(a)}{\rho(a) + 2\kappa} \, da,$$

and κ is the unique solution to equation (79).

We now turn to the comparative statics analysis.

G.4.3 Comparative statics

We assume that absolute risk aversion is a decreasing function of initial wealth. With a slight abuse of notation, we write it as $\rho(W_0(a))$, where $\rho(\cdot)$ is a decreasing function. In addition, we continue to maintain Assumption 2 from the main text and assume that traders are indexed so that $W_0(a)$ is increasing in a.

The comparative statics results in the Bayesian Nash Equilibrium closely mirror those in the competitive REE case. However, the technical conditions that were previously required only for proving the results on information efficiency $\mathcal{I}$ are now also necessary for establishing the results on liquidity $\mathcal{L}$. As discussed earlier, the core economic message remains unchanged: transferring wealth from the rich to the poor increases information efficiency but decreases liquidity.

[54] The only difference lies in the expression for $\beta(a)$, which is given by $\beta(a) = t(a)/(\rho(a) + 2\kappa)$ in BNE, as opposed to $\beta(a) = t(a)/(\rho(a) + \kappa)$ in REE.

Proposition 13. *Suppose that agent preferences exhibit decreasing absolute risk aversion (DARA). Assume, in addition, that the function*

$$a \mapsto \frac{t(a)\,|\rho'(W_0(a))|}{(c+\rho(W_0(a)))^2}$$

is decreasing in a for every $c > 0$, at least for a sufficiently close to 0 and 1. Then, there exist thresholds $0 < a_1^W \leq a_2^W < 1$ such that for any Robin Hood variation $W_0^\Delta(a)$, satisfying $\int_0^1 W_0^\Delta(a)\,da = 0$ and $\underline{a} \leq a_1^W \leq a_2^W \leq \bar{a}$, the following comparative statics hold:

$$\mathcal{I}'(W_0(a))[W_0^\Delta(a)] > 0 \text{ and } \mathcal{I}'(W_0(a))[-W_0^\Delta(a)] < 0;$$

$$\mathcal{L}'(W_0(a))[W_0^\Delta(a)] < 0 \text{ and } \mathcal{L}'(W_0(a))[-W_0^\Delta(a)] > 0.$$

G.5 Price-taking equilibrium

The definition of equilibrium and the derivation are as in the main part of the paper. For completeness, we state the characterization of the equilibrium in the theorem below.

Theorem 6. *There exists a unique price-taking equilibrium with*

$$\beta(a) = \frac{t(a)}{\rho(a)}.$$

G.5.1 Market quality

Substituting $\beta(a) = t(a)/\rho(a)$ into equation (71), we obtain the expression for the precision of the sufficient statistic of the price:

$$\tau_p = \frac{\left(\int_0^1 \frac{t(a)}{\rho(a)}\,da\right)^2}{\int_0^1 \frac{t(a)}{\rho(a)^2}\,da + \tau_u^{-1}}.$$

The expression for information efficiency $\mathcal{I}$ then follows from:

$$\mathcal{I} = \frac{\tau_p}{\tau_v + \tau_p}.$$

Similarly, substituting $\beta(a) = t(a)/\rho(a)$ into equation (72), and using the identity $\kappa = 1/\mathcal{L}$, we obtain:

$$\mathcal{L} = \frac{\int_0^1 \frac{t(a)}{\rho(a)^2}\, da + \tau_u^{-1}}{\int_0^1 \frac{t(a)}{\rho(a)}\, da}.$$

We summarize these expressions in the proposition below.

Proposition 14. *The equilibrium expressions for information efficiency $\mathcal{I}$ and liquidity $\mathcal{L}$ are given by:*

$$\mathcal{I} = \left(1 + \tau_v \cdot \frac{\int_0^1 \frac{t(a)}{\rho(a)^2}\, da + \tau_u^{-1}}{\left(\int_0^1 \frac{t(a)}{\rho(a)}\, da\right)^2}\right)^{-1}, \qquad \mathcal{L} = \frac{\int_0^1 \frac{t(a)}{\rho(a)^2}\, da + \tau_u^{-1}}{\int_0^1 \frac{t(a)}{\rho(a)}\, da}. \tag{80}$$

We are ready to proceed to comparative statics.

G.5.2 Comparative statics

We assume that absolute risk aversion is a decreasing function of initial wealth. With a slight abuse of notation, we write it as $\rho(W_0(a))$, where $\rho(\cdot)$ is a decreasing function. In addition, we continue to maintain Assumption 2 from the main text and assume that traders are indexed so that $W_0(a)$ is increasing in a.

The comparative statics results remain unchanged from the main part of the paper. The proof, however, requires a slight adjustment to account for the presence of noise traders; the modified argument is presented in Section G.7.

Proposition 15. *Suppose that agent preferences are DARA. Then, there exist thresholds* $0 <$

$a_1^W \leq a_2^W < 1$, *such that for any Robin Hood variation* $W_0^\Delta(a)$ *with* $\underline{a} \leq a_1^W \leq a_2^W \leq \bar{a}$ *we have*

$$\mathcal{I}'(W_0(a))[W_0^\Delta(a)] > 0 \text{ and } \mathcal{I}'(W_0(a))[-W_0^\Delta(a)] < 0;$$
$$\mathcal{L}'(W_0(a))[W_0^\Delta(a)] < 0 \text{ and } \mathcal{L}'(W_0(a))[-W_0^\Delta(a)] > 0.$$

G.6 Comparing Price-Taking Equilibrium, REE, and BNE

Juxtaposing Theorems 4, 5, and 6, we can summarize the equilibrium trading intensities as follows:

$$\beta^{PT}(a) = \frac{t(a)}{\rho(a)}, \quad \beta^{REE}(a) = \frac{t(a)}{\rho(a) + \kappa}, \quad \beta^{BNE}(a) = \frac{t(a)}{\rho(a) + 2\kappa},$$

corresponding to the price-taking equilibrium, competitive Rational Expectations Equilibrium (REE), and Bayesian Nash Equilibrium (BNE), respectively.

Here, $\kappa = 1/\mathcal{L}$ represents the inverse of market liquidity and serves as a measure of price impact. The only difference across the expressions lies in the denominator, which reflects the extent to which traders internalize their influence on the market.

- Price-Taking Equilibrium: Traders behave competitively and ignore both how they affect the informational content of the price and their direct price impact. Hence, no adjustment is made to the denominator.
- Rational Expectations Equilibrium (REE): Traders internalize that they affect the informational content of prices but not their impact on the price level. This introduces a correction term $+\kappa$.
- Bayesian Nash Equilibrium (BNE): Traders account for both how they affect the information in the price and their influence on the price level. This leads to a total adjustment of $+2\kappa$.

Thus, the denominator can be interpreted as $\rho(a) +$ (number of effects internalized) $\cdot\, \kappa$.

G.7 Proofs for Section G

G.7.1 Proof of Theorem 4

Proof of Theorem 4.

Given the strategies in (75), the market-clearing price function takes the form:

$$P^n = k_s^n \left(\sum_i w^n(a_i)\, \Delta s_i \right) + k_u^n\, u,$$

where the constants k_s^n and k_u^n are given by:

$$k_s^n = \frac{\sum_{i=1}^n \beta(a_i) m}{\sum_{i=1}^n \gamma(a_i) m}, \quad k_u^n = \frac{1}{\sum_{i=1}^n \gamma(a_i) m}.$$

The optimal demand of trader i is:

$$x_i^n = \frac{\mathbb{E}[v \mid \Delta s_i, s_p^n] - p}{\rho(a_i)\, \mathrm{Var}[v \mid \Delta s_i, s_p^n]}.$$

We decompose the aggregate demand as follows:

$$\sum_i x_i^n = \sum_i \frac{1}{\rho(a_i)\, \mathrm{Var}[v \mid \Delta s_i, s_p^n]} \left(\mathbb{E}[v \mid \Delta s_i, s_p^n] - \mathbb{E}[v \mid s_p^n] + \underbrace{\mathbb{E}[v \mid s_p^n] - p}_{\text{risk premium}} \right).$$

To compute the first term, we apply the standard Bayesian updating formulas (see Lemma 11) and use the Aggregation Lemma. In the limit $n \to \infty$, this yields:

$$\lim_{n\to\infty} \sum_i \frac{1}{\rho(a_i)\, \mathrm{Var}[v \mid \Delta s_i, s_p^n]} \left(\mathbb{E}[v \mid \Delta s_i, s_p^n] - \mathbb{E}[v \mid s_p^n] \right) = \tag{81}$$
$$\int_0^1 \frac{t(a) - w(a)\,\tau_p}{\rho(a)}\, ds(a) + \int_0^1 \frac{s_p\,\tau_p\,(-t(a)^2 + t(a)w(a)(\tau_p + 3\tau_v) + \tau_p\tau_v w(a)^2)}{\rho(a)\, t(a)\, (\tau_p + \tau_v)}\, da.$$

Here, all quantities without superscript n denote the $n \to \infty$ limit of their discrete counterparts.

To compute the risk premium term, we apply $\mathbb{E}[\cdot \mid s_p]$ to both sides of the market-clearing condition:

$$\lim_{n\to\infty} \sum_i \frac{\mathbb{E}[v \mid s_p^n] - P^n}{\rho(a_i)\,\mathrm{Var}[v \mid \Delta s_i, s_p^n]} = -\mathbb{E}[u \mid s_p].$$

Thus, the risk premium is of order $1/n$ and can be written as:

$$\mathbb{E}[v \mid s_p^n] - p = \frac{1}{n}\psi(s_p) + \text{higher-order terms},$$

where the function $\psi(s_p)$ is determined by market clearing:

$$(\tau_v + \tau_p)\,\psi(s_p) = -\mathbb{E}[u \mid s_p] \int_0^1 1/\rho(a)\,da.$$

Substituting into (84) leads to (78), and the rest of the proof follows the heuristic derivation.

To establish uniqueness of the positive solution κ in (73), note that the equation can be rewritten as:

$$\tau_u^{-1} = \int_0^1 \frac{t(a)\,\gamma(a)}{(\gamma(a)+\kappa)^2\,\kappa}\,da. \tag{82}$$

The right-hand side is a continuous, strictly decreasing function of κ, with limits $+\infty$ as $\kappa \to 0$, and 0 as $\kappa \to \infty$. Therefore, by the Intermediate Value Theorem, a unique positive solution exists. ■

Lemma 11. *We have*

$$E[v|s_p^n] = \frac{\tau_p^n}{\tau_p^n + \tau_v} s_p^n;$$
$$E[v|s_p^n, \Delta s_i] = \frac{t(a_i) - w^n(a_i)\tau_{p,i}^n/(1 - mw^n(a_i))}{\tau_v + \tau_{p,i}^n + t(a_i)m} \Delta s_i + \frac{\tau_{p,i}^n/(1 - mw^n(a_i))}{\tau_v + \tau_{p,i}^n + t(a_i)m} s_p^n;$$
$$\mathrm{Var}[v|s_p, \Delta s_i]^{-1} = \tau_v + \tau_{p,i}^n + t(a_i)m.$$

Here we denoted

$$(\tau_p^n)^{-1} = \sum_j \frac{(w^n(a_j))^2 m}{t(a_j)} + \left(\frac{k_u^n}{k_s^n}\right)^2 \tau_u^{-1} \text{ and } (\tau_{p,i}^n)^{-1} = \sum_{j \neq i} \frac{(w^n(a_j))^2 m}{t(a_j)(1 - mw^n(a_i))^2} + \left(\frac{k_u^n}{k_s^n}\right)^2 \frac{\tau_u^{-1}}{(1 - mw^n(a_i))^2}.$$

Proof of Lemma 11. First, we construct the statistic $s_{p,i}^n$ that is conditionally uncorrelated with Δs_i:

$$s_{p,i}^n = (s_p - w^n(a_i)\Delta s_i)/(1 - mw^n(a_i)) = v + \frac{\sum_{j \neq i} w^n(a_j)\Delta B_j/\sqrt{t(a_j)}}{(1 - mw^n(a_i))} + \frac{k_u^n}{k_s^n} \frac{u}{1 - mw^n(a_i)}.$$

It has precision, $\tau_{p,i}^n$ where $(\tau_{p,i}^n)^{-1} = \sum_{j \neq i} \frac{(w^n(a_j))^2 m}{t(a_j)(1 - mw^n(a_i))^2} + \left(\frac{k_u^n}{k_s^n}\right)^2 \frac{\tau_u^{-1}}{(1 - mw^n(a_i))^2}$. Using the standard formulas, we can write $E[v|s_p^n, \Delta s_i] = E[v|s_{p,i}^n, \Delta s_i] = \frac{t(a_i)\Delta s_i}{\tau_v + \tau_{p,i}^n + t(a_i)m} + \frac{\tau_{p,i}^n s_{p,i}^n}{\tau_v + \tau_{p,i}^n + t(a_i)m}$. Substituting the relationship between s_p and $s_{p,i}^n$ we obtain the stated result. ∎

G.7.2 Proof of Proposition 10

Proof of Proposition 10. First, recall from (67) that

$$s_p = P \cdot \frac{\int_0^1 \gamma(a)\, da}{\int_0^1 \beta(a)\, da}.$$

The price impact λ is defined as the reciprocal of the slope of aggregate demand:

$$\lambda^{-1} = \int_0^1 \gamma(a)\, da.$$

Substituting into the expression for s_p, we get:

$$s_p = \frac{1}{\lambda \int_0^1 \beta(a)\, da} \cdot P.$$

In the large economy limit, prices are weak-form efficient:

$$P = \mathbb{E}[v \mid s_p] = \frac{\tau_p}{\tau_v + \tau_p} \cdot s_p.$$

Substituting the expression for s_p in terms of P, we find:

$$\begin{aligned}\mathbb{E}[v \mid s_p] - P &= \frac{\tau_p}{\tau_v + \tau_p} \cdot s_p - P \\ &= \left(\frac{\tau_p}{\tau_v + \tau_p} \cdot \frac{1}{\lambda \int_0^1 \beta(a)\, da} - 1 \right) P.\end{aligned}$$

Since this expression must be zero for equilibrium prices, we conclude:

$$\frac{\tau_p}{\tau_v + \tau_p} \cdot \frac{1}{\lambda \int_0^1 \beta(a)\, da} = 1.$$

Rewriting this identity, we obtain:

$$\kappa = \frac{\tau_p}{\int_0^1 \beta(a)\, da} = \lambda(\tau_p + \tau_v) = \frac{\lambda}{\mathrm{Var}[v \mid s_p]} = \frac{1}{\mathcal{L}}.$$

The expression for $\mathcal{I}$ follows directly from the definition of information efficiency, while the expression for $\mathcal{L}$ follows from the above identity and the definition of κ given in (70). ■

G.7.3 Proof of Proposition 11

Proof of Proposition 11. Note that the equation (85) that pins down κ in equilibrium can be rewritten as:

$$\kappa\tau_u^{-1} = \int_0^1 \frac{t(a)\,\rho(W_0(a))}{(\rho(W_0(a)) + \kappa)^2}\,da. \tag{83}$$

We now differentiate this equation implicitly with respect to the wealth distribution $W_0(a)$. Specifically, we consider a perturbation of the form $W_0(a) + \epsilon W_0^{\Delta}(a)$, differentiate both sides of (83) with respect to ϵ, assuming $\kappa = \kappa(\epsilon)$, and evaluate the derivative at $\epsilon = 0$, thereby obtaining the Gateaux derivative $\kappa'(W_0(a))[W_0^{\Delta}(a)]$.

This yields:

$$\kappa'(W_0(a))[W_0^{\Delta}(a)] = C_\kappa \int_0^1 \frac{t(a)\,\rho'(W_0(a))}{(\rho(W_0(a)) + \kappa)^2}\left(1 - \frac{2\rho(W_0(a))}{\rho(W_0(a)) + \kappa}\right) W_0^{\Delta}(a)\,da,$$

where

$$C_\kappa = \left(\tau_u^{-1} + 2\int_0^1 \frac{t(a)\,\rho(W_0(a))}{(\kappa + \rho(W_0(a)))^3}\,da\right)^{-1} > 0.$$

is a positive constant.

By Lemma 9, there exists a unique threshold a^* such that

$$1 - \frac{2\rho(W_0(a))}{\rho(W_0(a)) + \kappa} \gtrless 0 \quad \text{if and only if} \quad a \lessgtr a^*.$$

Therefore, for any Robin Hood variation $W_0^{\Delta}(a)$ with $\underline{a} < a_* < \bar{a}$, we have

$$\rho'(W_0(a))\,W_0^{\Delta}(a)\left(1 - \frac{2\rho(W_0(a))}{\rho(W_0(a)) + \kappa}\right) > 0,$$

and hence $\kappa'(W_0(a))[W_0^{\Delta}(a)] > 0$.

We now turn to the comparative statics of $\mathcal{I}$. It suffices to show that

$$\tau_p'(W_0(a))[W_0^\Delta(a)] > 0$$

for a Robin Hood variation with sufficiently low $\underline{a}$ and sufficiently high $\bar{a}$.

Applying the chain rule, we obtain:

$$\tau_p'(W_0(a))[W_0^\Delta(a)] = \tau_p'(W_0(a); \bar{\kappa})[W_0^\Delta(a)] + \frac{\partial \tau_p}{\partial \kappa} \cdot \kappa'(W_0(a))[W_0^\Delta(a)],$$

where the bar in $\tau_p'(W_0(a); \bar{\kappa})$ denotes the derivative of τ_p with respect to $W_0(a)$ while keeping κ fixed.

Differentiating $\tau_p = \kappa \int_0^1 \frac{t(a)}{\rho(a)+\kappa}\, da$, we compute:

$$\tau_p'(W_0(a); \bar{\kappa})[W_0^\Delta(a)] = -\bar{\kappa} \int_0^1 \frac{t(a)\, \rho'(W_0(a))\, W_0^\Delta(a)}{(\bar{\kappa} + \rho(W_0(a)))^2}\, da.$$

By Lemma 12, if the integrand

$$y(a) = -\frac{t(a)\, \rho'(W_0(a))}{(\bar{\kappa} + \rho(W_0(a)))^2}$$

is decreasing in a, then

$$\tau_p'(W_0(a); \bar{\kappa})[W_0^\Delta(a)] > 0.$$

Since $\frac{\partial \tau_p}{\partial \kappa} > 0$ and $\kappa'(W_0(a))[W_0^\Delta(a)] > 0$ by earlier results, the full derivative $\tau_p'(W_0(a))[W_0^\Delta(a)]$ is also positive. This completes the proof of the proposition. ■

Lemma 12. *Let $h^\Delta(a)$ be a Robin Hood variation of a parameter $h(a)$, with thresholds $0 < \underline{a} < \bar{a} < 1$, and suppose that $\int_0^1 h^\Delta(a)\, da = 0$. Let $y(a)$ be a decreasing function on $[0, 1]$. Then:*

$$\int_0^1 y(a)\, h^\Delta(a)\, da > 0.$$

Proof of Lemma 12. Let $a^* \in (\underline{a}, \bar{a})$. Since $y(a)$ is decreasing, the function $y(a) - y(a^*)$ is positive for $a < a^*$ and negative for $a > a^*$. The same sign pattern holds for $h^\Delta(a)$, by the definition of a Robin Hood variation. Hence, the product $(y(a) - y(a^*))h^\Delta(a)$ is non-negative everywhere and strictly positive on a set of positive measure. It follows that:

$$\int_0^1 (y(a) - y(a^*))h^\Delta(a)\, da > 0.$$

Using linearity of the integral, we rewrite the left-hand side:

$$\int_0^1 (y(a) - y(a^*))h^\Delta(a)\, da = \int_0^1 y(a)\, h^\Delta(a)\, da - y(a^*) \int_0^1 h^\Delta(a)\, da.$$

Since $\int_0^1 h^\Delta(a)\, da = 0$ by assumption, the second term vanishes, and we conclude:

$$\int_0^1 y(a)\, h^\Delta(a)\, da > 0,$$

as claimed. ■

G.7.4 Proof of Theorem 5

Proof of Theorem 5.

The proof follows closely that of Theorem 4.

Given the strategies in (75), the market-clearing price function takes the form:

$$P^n = k_s^n \left(\sum_i w^n(a_i)\, \Delta s_i \right) + k_u^n\, u,$$

where the constants k_s^n and k_u^n are given by:

$$k_s^n = \frac{\sum_{i=1}^n \beta(a_i) m}{\sum_{i=1}^n \gamma(a_i) m}, \quad k_u^n = \frac{1}{\sum_{i=1}^n \gamma(a_i) m}.$$

The optimal demand of trader i is:

$$x_i^n = \frac{\mathbb{E}[v \mid \Delta s_i, s_p^n] - p}{\rho(a_i)\operatorname{Var}[v \mid \Delta s_i, s_p^n] + \lambda_i^n}.$$

We decompose the aggregate demand as follows:

$$\sum_i x_i^n = \sum_i \frac{1}{\rho(a_i)\operatorname{Var}[v \mid \Delta s_i, s_p^n] + \lambda_i^n} \left(\mathbb{E}[v \mid \Delta s_i, s_p^n] - \mathbb{E}[v \mid s_p^n] + \underbrace{\mathbb{E}[v \mid s_p^n] - p}_{\text{risk premium}} \right).$$

To compute the first term, we apply the standard Bayesian updating formulas (see Lemma 11) and use the Aggregation Lemma. In the limit $n \to \infty$, this yields:

$$\lim_{n\to\infty} \sum_i \frac{1}{\rho(a_i)\operatorname{Var}[v \mid \Delta s_i, s_p^n] + \lambda_i^n} \left(\mathbb{E}[v \mid \Delta s_i, s_p^n] - \mathbb{E}[v \mid s_p^n]\right) = \tag{84}$$

$$\int_0^1 \frac{t(a) - w(a)\,\tau_p}{\rho(a) + \kappa}\, ds(a) + \int_0^1 \frac{s_p\,\tau_p\,(-t(a)^2 + t(a)w(a)(\tau_p + 3\tau_v) + \tau_p\tau_v w(a)^2)}{(\rho(a)+\kappa)\,t(a)\,(\tau_p + \tau_v)}\, da.$$

Here, all quantities without superscript n denote the $n \to \infty$ limit of their discrete counterparts.

To compute the risk premium term, we apply $\mathbb{E}[\cdot \mid s_p]$ to both sides of the market-clearing condition:

$$\lim_{n\to\infty} \sum_i \frac{\mathbb{E}[v \mid s_p^n] - P^n}{\rho(a_i)\operatorname{Var}[v \mid \Delta s_i, s_p^n] + \lambda_i^n} = -\mathbb{E}[u \mid s_p].$$

Thus, the risk premium is of order $1/n$ and can be written as:

$$\mathbb{E}[v \mid s_p^n] - p = \frac{1}{n}\psi(s_p) + \text{higher-order terms},$$

where the function $\psi(s_p)$ is determined by market clearing:

$$(\tau_v + \tau_p)\,\psi(s_p) = -\mathbb{E}[u \mid s_p] \int_0^1 1/\rho(a)\, da.$$

Substituting into (84) leads to (78), and the rest of the proof follows the heuristic derivation.

To establish uniqueness of the positive solution κ in (73), note that the equation can be rewritten as:

$$\tau_u^{-1} = \int_0^1 \frac{t(a)}{(\rho(a) + 2\kappa)^2} \cdot \left(\frac{\rho(a)}{\kappa} + 1 \right) da. \tag{85}$$

The right-hand side is a continuous, strictly decreasing function of κ, with limits $+\infty$ as $\kappa \to 0$, and 0 as $\kappa \to \infty$. Therefore, by the Intermediate Value Theorem, a unique positive solution exists. ■

G.7.5 Proof of Proposition 13

Proof of Proposition 13. Note that the equation (85), which determines κ in equilibrium, can be rewritten as:

$$\tau_u^{-1} = \int_0^1 \frac{t(a)\,\rho(W_0(a))}{(\rho(W_0(a)) + 2\kappa)^2} \left(\frac{\rho(W_0(a))}{\kappa} + 1 \right) da. \tag{86}$$

We now differentiate this equation implicitly with respect to the wealth distribution $W_0(a)$. Specifically, we consider a perturbation of the form $W_0(a) + \epsilon W_0^{\Delta}(a)$, differentiate both sides of (86) with respect to ϵ, assume $\kappa = \kappa(\epsilon)$, and evaluate the derivative at $\epsilon = 0$. This yields the Gâteaux derivative $\kappa'(W_0(a))[W_0^{\Delta}(a)]$. We assume that $W_0^{\Delta}(a)$ is a Robin Hood variation satisfying $\int_0^1 W_0^{\Delta}(a)\, da = 0$.

Let the right-hand side of equation (86) be denoted by $f(\kappa; W_0(a))$. Then the result of the differentiation can be written as:

$$0 = \frac{\partial f}{\partial \kappa} \cdot \kappa'(W_0(a))[W_0^{\Delta}(a)] + f'(W_0(a), \bar{\kappa})[W_0^{\Delta}(a)], \tag{87}$$

where the bar notation indicates that κ is held fixed while differentiating with respect to $W_0(a)$.

Computing the second term yields:

$$f'(W_0(a),\bar{\kappa})[W_0^{\Delta}(a)] = -\int_0^1 \frac{t(a)\,W_0^{\Delta}(a)\,\rho(W_0(a))\,\rho'(W_0(a))}{\kappa\,(2\kappa+\rho(W_0(a)))^3}\,da.$$

Note that the monotonicity condition

$$a \mapsto -\frac{t(a)\,\rho'(W_0(a))}{(2c+\rho(W_0(a)))^2}$$

being decreasing in a is sufficient to ensure that the integrand

$$a \mapsto -\frac{t(a)\,\rho(W_0(a))\,\rho'(W_0(a))}{(2c+\rho(W_0(a)))^3}$$

is also decreasing in a. Then, by Lemma 12, we have

$$f'(W_0(a),\bar{\kappa})[W_0^{\Delta}(a)] > 0.$$

Since $\partial f/\partial\kappa < 0$, it follows from equation (87) that

$$\kappa'(W_0(a))[W_0^{\Delta}(a)] > 0.$$

We now turn to the comparative statics of $\mathcal{I}$. It suffices to show that

$$\tau_p'(W_0(a))[W_0^{\Delta}(a)] > 0$$

for a Robin Hood variation with sufficiently low $\underline{a}$ and sufficiently high $\bar{a}$.

Applying the chain rule, we obtain:

$$\tau_p'(W_0(a))[W_0^{\Delta}(a)] = \tau_p'(W_0(a);\bar{\kappa})[W_0^{\Delta}(a)] + \frac{\partial \tau_p}{\partial \kappa}\cdot\kappa'(W_0(a))[W_0^{\Delta}(a)],$$

where the bar in $\tau_p'(W_0(a);\bar{\kappa})$ denotes the derivative of τ_p with respect to $W_0(a)$ while keeping κ fixed.

Differentiating $\tau_p = \kappa \int_0^1 \frac{t(a)}{\rho(a)+2\kappa}\, da$, we compute:

$$\tau_p'(W_0(a);\bar{\kappa})[W_0^{\Delta}(a)] = -\bar{\kappa}\int_0^1 \frac{t(a)\,\rho'(W_0(a))\,W_0^{\Delta}(a)}{\left(2\bar{\kappa}+\rho(W_0(a))\right)^2}\, da.$$

By Lemma 12, if the integrand

$$y(a) = -\frac{t(a)\,\rho'(W_0(a))}{\left(2\bar{\kappa}+\rho(W_0(a))\right)^2}$$

is decreasing in a, then the integral is strictly positive for any Robin Hood variation $W_0^{\Delta}(a)$ with thresholds $\underline{a}$ sufficiently small and $\bar{a}$ sufficiently large. Therefore,

$$\tau_p'(W_0(a);\bar{\kappa})[W_0^{\Delta}(a)] > 0.$$

Since $\frac{\partial \tau_p}{\partial \kappa} > 0$ and $\kappa'(W_0(a))[W_0^{\Delta}(a)] > 0$ by earlier results, the full derivative $\tau_p'(W_0(a))[W_0^{\Delta}(a)]$ is also positive. This completes the proof of the proposition. ■

G.7.6 Proof of Proposition 15

Proof of Proposition 15. Differentiating the expressions for information efficiency and liquidity in equation (80), we obtain:

$$\mathcal{I}'(W_0(a))[W_0^{\Delta}(a)] = \frac{2\tau_v}{K_1^2\left(1+\frac{K_2}{K_1^2}\tau_v\right)^2}\int_0^1 \frac{t(a)}{\rho(W_0(a))^2}\,\rho'(W_0(a))\,W_0^{\Delta}(a)\left(\frac{1}{\rho(W_0(a))}-\frac{K_2}{K_1}\right) da,$$

where

$$K_1 \equiv \int_0^1 \frac{t(a)}{\rho(W_0(a))}\, da, \qquad K_2 \equiv \frac{1}{\tau_u} + \int_0^1 \frac{t(a)}{\rho(W_0(a))^2}\, da.$$

Similarly, the derivative of liquidity with respect to a variation in initial wealth is:

$$\mathcal{L}'(W_0(a))[W_0^{\Delta}(a)] = \frac{1}{K_1}\int_0^1 \frac{t(a)}{\rho(W_0(a))^2}\,\rho'(W_0(a))\,W_0^{\Delta}(a)\left(\frac{K_2}{K_1} - \frac{2}{\rho(W_0(a))}\right) da.$$

The remainder of the proof proceeds analogously to that of Proposition 2, invoking Lemma 9 to sign the expressions.

■

H Competitive REE and BNE: General Case

In this section, we show how the CHILE framework accommodates alternative equilibrium concepts—specifically, competitive rational expectations equilibrium (REE) and Bayesian Nash equilibrium (BNE)—under general preferences and payoff structures. The goals are threefold: (i) to demonstrate that the modeling approach extends naturally to these equilibrium concepts; (ii) to show that REE and BNE do not exist without noise, reinforcing the intuition that the small belief perturbations inherent in price-taking equilibrium are what enable existence in the noiseless limit; and (iii) to establish that the information aggregation paradox can arise under alternative equilibrium concepts and is not an artifact of price-taking equilibrium.

The setup is the same as in the main text, except that supply now includes a noise component:

$$\Theta(P) = \bar{\theta} + \theta(P) + u\,\tilde{\theta}(P).$$

Here supply consists of a price-inelastic component $\bar{\theta}$, a price-elastic component $\theta(P)$, and a noisy component $u\,\tilde{\theta}(P)$, where $u \sim N(\bar{u}, \sigma_u^2)$. The mean supply shock $\bar{u}$ is a fixed parameter that is unknown to traders; they hold a prior $\bar{u} \sim N(0, \sigma_{\bar{u}}^2)$. We write $\tau_u^{-1} \equiv \sigma_u^2 + \sigma_{\bar{u}}^2$ for the total ex-ante supply uncertainty as perceived by traders. As we demonstrate below, this supply uncertainty is essential for equilibrium existence.

The equilibrium construction proceeds as in the main text. First, by the Representation Lemma, cumulative demand X takes the form

$$dX = \beta(P, a)\,ds + \delta(P, a)\,da,$$

where β and δ are coefficients to be determined.

Aggregation consistency and optimality link β and δ to the derivatives of optimal demand x^*:

$$\beta(P,a) = x_s^*(P,a;0,0),$$
$$\delta(P,a) = \frac{1}{2t(a)} x_{ss}^*(P,a;0,0) + x_\mu^*(P,a;0,0).$$

The optimal demand x^* solves the utility maximization problem

$$x_i^*(s,P) = \arg\max_x \int_{\mathbb{R}} u(W_0(a_i) + x(V(v) - P),\ a_i)\, f(v,s,P,\mu)\, dv. \tag{88}$$

The key difference from price-taking equilibrium lies in the definition of optimal demand. In both REE and BNE, traders account for the correlation between the noise in their private signal and the noise embedded in the price. Formally, the optimization problem (88) differs from its price-taking counterpart (**??**) in that we no longer assume conditional independence of the signal and the price when decomposing the density $f_{v|\Delta s_i, \mathbf{P}_i^n}(v,s,P)$.

The central object that allows us to isolate the price information uncorrelated with a given trader's signal is the *residual demand distribution*. Following Wilson (1979), for a given price level P—treated as a parameter (i.e., the input to the demand schedule)—define

$$H(x,v,P,\mu) \equiv \mathbb{P}\left(\sum_{j \neq i} x_j^*(s_j,P,\mu) < x \;\middle|\; v \right). \tag{89}$$

The parameter μ captures the dependence of residual demand on the number of traders. The distribution H plays a role analogous to the price belief $g(v,P,\mu)$ in the construction of price-taking equilibrium. We require that $H(x,v,P,\mu)$ converge to $H(x,v,P,0)$ pointwise in v for any fixed x and P as $\mu \to 0$. By the representation result, the limiting distribution is

$$H(x,v,P,0) = \mathbb{P}\left(\int_0^1 \beta(a,P)\, ds(a) + \int_0^1 \delta(a,P)\, da < x \;\middle|\; v \right). \tag{90}$$

The distribution H is related to the distribution of the market-clearing price $\tilde{P}$ as follows. Because $\tilde{P} < P$ if and only if cumulative demand at P falls short of supply, conditioning on the realized supply shock u,

$$\mathbb{P}(\tilde{P} < P \mid s_i, v, u) = \mathbb{P}\left(\sum_{j \neq i} x_j^*(s_j, P) < \Theta(P) - x_i^*(P, s_i) \,\middle|\, v,\, u\right) = H(\Theta(P) - x_i^*(P, s_i),\ v,\ P,\ \mu).$$

The corresponding density is

$$h(P;\, x_i^*, v, u, \mu) = \frac{\partial}{\partial P} H(\Theta(P) - x_i^*,\ v,\ P,\ \mu).$$

We can then decompose the joint density as

$$f(v, s_i, P, u, \mu) = \underbrace{h(P;\, x_i^*(P, s_i), v, u, \mu)}_{\text{density of } P|s_i,v,u} \cdot \underbrace{\exp\big(t(a_i)\big(s_i v - \tfrac{\mu}{2} v^2\big)\big)}_{\propto \text{ density of } s_i|v} \cdot \underbrace{f(v)}_{\text{density of } v} \cdot \underbrace{f_u(u)}_{\text{density of } u}. \quad (91)$$

Dropping terms that do not depend on the choice variable, the optimal demand can be formulated as follows.

REE. In competitive REE, each trader maximizes expected utility taking the equilibrium price distribution as given. The optimal demand solves

$$x^*(P, a;\, s, \mu) = \arg\max_y \int_{\mathbb{R}} \int_{\mathbb{R}} u(W_0(a) + y(V(v) - P),\, a) \times\ h(P;\, x^*(\cdot), v, u, \mu)\, \exp\big(t(a)\big(sv - \tfrac{\mu}{2} v^2\big)\big)\, f(v)\, f_u(u)\, dv\, du. \quad (92)$$

The trader correctly recognizes that the equilibrium price distribution is determined by the equilibrium demand x^*, thereby accounting for how her information is incorporated into the price. However, she treats the density h as invariant to the off-equilibrium demand choice y—that is, she ignores the price impact of her own trade.

BNE. In BNE, each trader additionally internalizes that her demand choice affects the market-clearing price and hence the density h. The optimal demand schedule solves

$$x^*(P, a;\, s, \mu) = \arg\max_{y(\cdot)} \int_{\mathbb{R}}\int_{\mathbb{R}} u(W_0(a) + y(P)(V(v) - P),\, a)$$
$$\times\ h(P;\, y(\cdot), v, u, \mu)\, \exp\big(t(a)\big(sv - \tfrac{\mu}{2}\, v^2\big)\big)\, f(v)\, f_u(u)\, dv\, du. \quad (93)$$

The key distinction from REE is that the density h now depends on the choice variable $y(\cdot)$ rather than on the equilibrium demand x^*, reflecting the trader's internalization of her price impact.

Before formally defining the equilibrium, we state the technical conditions required for the equilibrium construction.

Assumption 5 (Technical restrictions on the primitives)**.** *The following technical conditions are satisfied:*

1. *There exist* $q \geq 0$ *and* $C_u < \infty$ *such that for all* $w \in \mathbb{R}$, $|u'(w,a)| + |u''(w,a)| + |u'''(w,a)| \leq C_u(1 + |w|^q)$.

2. $V(v)$ *satisfies* $\lim_{v\to\pm\infty} \frac{\ln|V(v)|}{v^2} \leq 0$.

3. *There exist* $\varepsilon > 0$ *and constants* $A, k > 0$ *such that for all* $v, u \in \mathbb{R}$,

$$\sup_{|\mu|\leq\varepsilon} |h(P;\, x, v, u, \mu)| \leq Ae^{-k(v^2+u^2)}.$$

Moreover, denoting partial derivatives by subscripts (e.g., $h_x = \partial h/\partial x$*), the same bounds hold for the derivatives* h_l *and* H_l

$$\sup_{|\mu|\leq\varepsilon} |h_l(P;\, x, v, u, \mu)| \leq Ae^{-k(v^2+u^2)}, \quad \text{for } l \in \{x, xx, \mu, P, Px\};$$

$$\sup_{|\mu|\leq\varepsilon} |H_l(x, v, P, \mu)| \leq M(P)\, e^{-k(v^2+u^2)} \quad \text{for } l \in \{x, xx, P, Px\}.$$

4. *The density $h(P;\, x, v, u, \mu)$ is twice continuously differentiable with respect to x and continuously differentiable with respect to (μ, P), with a continuous mixed partial derivative h_{Px}, in a neighborhood of $x = 0, \mu = 0$. In addition, $H(x, v, P, \mu)$ is twice continuously differentiable in x and continuously differentiable in (P, μ), with continuous mixed partials needed above, e.g. H_{Px}), in a neighborhood of of $x = 0, \mu = 0$.*

The equilibrium construction requires differentiating the first-order conditions for optimality and evaluating the resulting expressions in the large-economy limit ($\mu \to 0,\ s \to 0$). Part 4 ensures that the equilibrium objects—in particular, the residual demand density h—are smooth enough for these derivatives to be well defined. Parts 1–3 provide the growth and integrability bounds needed to invoke the Dominated Convergence Theorem, allowing us to pass limits and derivatives under the integral sign.

We now state the equilibrium definition formally.

Definition 8. *The cumulative demand $X(a, P)$ constitutes a competitive REE (respectively, a BNE) if it is a limit of cumulative demands constructed from $x^*(P, a;\, s, \mu)$, where x^* satisfies the optimality condition (92) for the REE (respectively, (93) for the BNE), the distribution $H(x, v, P, \mu)$ satisfies the technical conditions in Assumption 5, and $H(x, v, P, \mu)$ converges pointwise in v to $H(x, v, P, 0)$ given in (90) for any fixed x and P.*

Before deriving competitive REE and BNE equilibria separately, we define the market quality objects.

H.1 Market quality

Our definition of informational efficiency follows the main text:

$$\mathcal{I} = 1 - \frac{\mathbb{V}\mathrm{ar}(v|P)}{\mathbb{V}\mathrm{ar}(v)}.$$

Because $\bar{u}$ is a fixed parameter of the data-generating process, the conditional variance $\mathbb{V}\text{ar}(v|P)$ is evaluated under the actual DGP, in which the noise variance of supply is σ_u^2 (the variance of u around its true mean $\bar{u}$) rather than the traders' perceived variance $\tau_u^{-1} = \sigma_u^2 + \sigma_{\bar{u}}^2$. Thus, while the equilibrium trading behavior—and in particular the fixed-point system for κ—is governed by the perceived noise τ_u^{-1}, the informational efficiency $\mathcal{I}$ is determined by the actual noise σ_u^2. We elaborate on this distinction in Section H.5.

Similarly, we define liquidity based on how much price-inelastic supply shocks can move prices. We assume that these shocks are unexpected, in that demand coefficients do not adjust to changes in $\bar{\theta}$. The corresponding market-clearing price is denoted $\mathcal{P}(\bar{\theta})$. We use three interrelated measures.

Price impact is

$$\lambda(P) = -\frac{\partial \mathcal{P}(\bar{\theta})}{\partial \bar{\theta}};$$

The price impact per unit of risk is

$$\kappa = \frac{\lambda(P)}{\mathbb{V}\text{ar}(V|P)};$$

And liquidity is

$$\mathcal{L} = 1/\kappa(P).$$

H.2 Competitive REE

We start with a formal derivation; a heuristic derivation is in Section H.3. We proceed as in the main text, deriving the coefficients $\beta(P, a)$ and $\delta(P, a)$ from the Aggregation Lemma and the first-order conditions for optimal demand, evaluated in the large-economy limit. We then impose market clearing to characterize the equilibrium.

H.2.1 Deriving $\beta(P, a)$

By Aggregation Lemma, the trading intensity equals the signal sensitivity of optimal demand evaluated in the large-economy limit: $\beta(P, a) = x_s(P, a; 0, 0)$. Computing this derivative requires differentiating the first-order condition (92) with respect to s and evaluating at $\mu = 0$, $s = 0$. Proceeding this way yields the following.

Lemma 13. *Suppose that Assumption 5 is satisfied and that $0 < \lambda(P) < \infty$ for every P. Then:*

$$\beta(P, a) = \frac{t(a)\,\mathrm{Cov}(V(v), v \mid P)}{\rho(a)\,\mathrm{Var}(V(v) \mid P) + \lambda(P)}.$$

H.2.2 Equilibrium

The coefficient $\beta(P, a)$ is the central object for the equilibrium characterization: it determines both market quality (informational efficiency and liquidity) and the fixed-point system that pins down the aggregate quantities κ and τ_p. The companion coefficient $\delta(P, a)$, which governs the deterministic component of demand, is more involved—reflecting prudence, skewness, and the price-sensitivity of Kyle's lambda—but does not affect the fixed-point system. Its characterization is stated in Lemma 15 and proved in Section H.6. The coefficient $\beta(P, a)$ is the central object for the equilibrium characterization: it determines both market quality (informational efficiency and liquidity) and the fixed-point system that pins down the aggregate quantities κ and τ_p. The companion coefficient $\delta(P, a)$, which governs the deterministic component of demand, is more involved—reflecting prudence, the conditional third central moment ("skewness"), and the price-sensitivity of Kyle's lambda—but does not affect the fixed-point system. Its characterization is stated in Lemma 15 and proved in Section H.6.

The key challenge is that $\beta(P, a)$ depends on the endogenous price informativeness τ_p and Kyle's lambda $\lambda(P)$, which must be determined in equilibrium. We introduce the following assumption to ensure tractability.

Assumption 6 (Noise Calibration)**.** *The noise supply function $\tilde{\theta}(P)$ satisfies*

$$\frac{\mathbb{E}[V'(v) \mid P]}{\mathrm{Var}(V(v) \mid P)\, \tilde{\theta}(P)} = \xi \tag{94}$$

for some $\xi > 0$ that does not depend on P.[55]

While Assumption 6 may appear restrictive, it is in fact quite natural, as the following two leading cases illustrate. For linear payoffs ($V(v) = v$), we have $\mathbb{E}[V'|P] = 1$ and $\mathrm{Var}(V|P) = \tau^{-1}$, both independent of P, so the assumption is automatically satisfied for any constant $\tilde{\theta}$—the standard normally distributed price-inelastic noisy supply specification in the literature. For log-normal payoffs ($V(v) = e^v$), we have $V'(v) = V(v)$, so $\mathbb{E}[V'|P] = P$ and $\mathrm{Var}(V|P) = P^2(e^{1/\tau} - 1)$. Here $1/\tau = \mathbb{V}\mathrm{ar}(v|P)$. The assumption then requires $\tilde{\theta}(P) \propto 1/P$, which means that the *dollar* noise supply $P \cdot u\, \tilde{\theta}(P)$ does not depend on P—i.e., noise traders submit normally distributed price-inelastic dollar demand.

With this assumption at hand, we are ready to characterize the equilibrium.

Theorem 7. *Suppose that Assumptions 5 and 6 are satisfied. Then there exists an equilibrium with the following properties.*

The equilibrium price function has the representation $P = \mathcal{P}(s_p)$, where

$$\mathcal{P}(x) = \int V\left(\frac{\tau_p}{\tau}\, x + \frac{z}{\sqrt{\tau}}\right) d\Phi(z), \tag{95}$$

Φ denotes the standard normal cumulative distribution function, $\tau = \tau_v + \tau_p$ is the posterior precision of v, and s_p is the equilibrium sufficient statistic related to the price by $s_p = \mathcal{P}^{-1}(P)$.

[55] The conditional moments $\mathbb{E}[V'(v) \mid P]$ and $\mathrm{Var}(V(v) \mid P)$ are determined by τ_p and P, since $v \mid P \sim N(\frac{\tau_p}{\tau}\mathcal{P}^{-1}(P), \tau^{-1})$ with $\tau = \tau_v + \tau_p$. The assumption eliminates the dependence on P, so ξ can depend on τ_p but on no other endogenous quantity. When $\xi = \xi(\tau_p)$, the fixed-point system (98)–(99) must be solved taking this dependence into account.

The sufficient statistic has the representation

$$s_p = \int_0^1 \omega(a)\, ds(a) - \frac{u\,\tilde{\theta}(P)}{\int_0^1 \beta(a,P)\, da},$$

where the weighting function is

$$\omega(a) = \frac{t(a)/(\rho(a)+\kappa)}{\int_0^1 t(b)/(\rho(b)+\kappa)\, db}.$$

The precision of s_p is given by

$$\tau_p = \frac{\left(\int_0^1 \frac{t(a)}{\rho(a)+\kappa}\, da\right)^2}{\int_0^1 \frac{t(a)}{(\rho(a)+\kappa)^2}\, da + \frac{\tau^2}{\xi^2 \tau_u}}. \tag{96}$$

The equilibrium cumulative demand has the representation $dX(a) = \beta(a,P)\, ds(a) + \delta(a,P)\, da$, where

$$\beta(a,P) = \frac{t(a)}{\rho(a)+\kappa} \cdot \frac{\mathbb{E}[V'(v) \mid P]}{\tau \operatorname{Var}(V(v) \mid P)}, \tag{97}$$

and $\delta(a,P)$ is given by Lemma 15, with $D(P,a) = (\rho(a)+\kappa)\operatorname{Var}(V(v) \mid P)$.

The equilibrium is fully determined by the aggregate quantit κ and τ_p, which solve the system

$$\kappa = \frac{\tau_p}{\int_0^1 \frac{t(a)}{\rho(a)+\kappa}\, da}, \tag{98}$$

$$\tau_p = \frac{\left(\int_0^1 \frac{t(a)}{\rho(a)+\kappa}\, da\right)^2}{\int_0^1 \frac{t(a)}{(\rho(a)+\kappa)^2}\, da + \frac{(\tau_v+\tau_p)^2}{\xi^2 \tau_u}}. \tag{99}$$

Substituting (98) into (99), the system reduces to a single fixed-point equation for κ:

$$\kappa = \frac{\int_0^1 \frac{t(a)}{\rho(a)+\kappa}\, da}{\int_0^1 \frac{t(a)}{(\rho(a)+\kappa)^2}\, da + \frac{\left(\tau_v + \kappa \int_0^1 \frac{t(a)}{\rho(a)+\kappa}\, da\right)^2}{\xi^2 \tau_u}}. \tag{100}$$

The next result shows that noise is essential for equilibrium existence.

Proposition 16 (Non-existence without noise)**.** *Suppose that Assumptions 5 and 6 hold and that* $1/\tau_u = 0$ *(i.e., there is no noise in supply). Then no competitive REE exists.*

The result above can be understood as follows. Without noise, for any realization of the price, the environment reduces to one in which the no-trade theorem applies: each trader's demand must be zero. This is consistent only if traders do not use their private information when trading, i.e., $\beta(a, P) = 0$ for all a. But $\beta(a, P) = 0$ for all a cannot be an equilibrium either, since in that case the price conveys no additional information and traders would find it optimal to trade on their signals.

This observation also illuminates the existence result for the price-taking equilibrium. The key difference is that price-taking traders make small "mistakes" by ignoring the correlation between the noise in their own signal and the noise embedded in the price. These mistakes aggregate across the continuum of traders, generating a sufficient amount of endogenous noise for the equilibrium to exist. Crucially, it is these small mistakes in *inference*—not the negligible price impact of each trader—that sustain the equilibrium.

H.2.3 Log-normal payoffs

We now specialize the general characterization in Theorem 7 to the leading case of log-normal payoffs, $V(v) = e^v$. We assume that noise traders submit price-inelastic dollar demand, so that

$$\tilde{\theta}(P) = \frac{1}{P}.$$

The dollar noise supply is then $P \cdot u\,\tilde{\theta}(P) = u$, which does not depend on P.

Verifying the noise calibration. For $V(v) = e^v$, we have $V'(v) = e^v = V(v)$, so $\mathbb{E}[V'(v) \mid P] = \mathbb{E}[V(v) \mid P] = P$. The conditional variance of the payoff is

$$\operatorname{Var}(V(v) \mid P) = P^2(e^{1/\tau} - 1),$$

where $\tau = \tau_v + \tau_p$ and we use the log-normal variance formula. Substituting into the noise calibration condition (94):

$$\xi = \frac{\mathbb{E}[V'(v) \mid P]}{\operatorname{Var}(V(v) \mid P)\, \tilde{\theta}(P)} = \frac{P}{P^2(e^{1/\tau} - 1) \cdot 1/P} = \frac{1}{e^{1/\tau} - 1}.$$

Since $\tau = \tau_v + \tau_p$, the parameter ξ depends on the endogenous τ_p but not on P, confirming that Assumption 6 is satisfied.

Equilibrium characterization. With the noise calibration verified, we can specialize the general characterization of Theorem 7 to obtain closed-form expressions for all equilibrium objects.

Corollary 4 (Log-normal payoffs)**.** *Suppose that $V(v) = e^v$, that Assumption 5 holds, and that $\tilde{\theta}(P) = 1/P$. Then the equilibrium of Theorem 7 has the following explicit form.*

1. ***Price function.*** *The price function is log-linear in the sufficient statistic:*

$$P = \mathcal{P}(s_p) = \exp\left(\frac{\tau_p}{\tau}\, s_p + \frac{1}{2\tau}\right),$$

so that $\ln P = \frac{\tau_p}{\tau}\, s_p + \frac{1}{2\tau}$*, or equivalently* $s_p = \frac{\tau}{\tau_p}\big(\ln P - \frac{1}{2\tau}\big)$.

2. ***Trading intensity.*** *The trading intensity is*

$$\beta(a, P) = \frac{t(a)}{\rho(a) + \kappa} \cdot \frac{1}{\tau\, P\, (e^{1/\tau} - 1)}. \tag{101}$$

The dollar trading intensity $P\, \beta(a, P) = \frac{t(a)}{(\rho(a)+\kappa)\, \tau\, (e^{1/\tau}-1)}$ *is independent of* P.

3. ***Price impact.*** *Kyle's lambda is* $\lambda(P) = \kappa\, P^2(e^{1/\tau} - 1)$*, which is proportional to* P^2*.*

4. ***Price informativeness.*** *The precision of the sufficient statistic is*

$$\tau_p = \frac{A(\kappa)^2}{B(\kappa) + \dfrac{\tau^2\,(e^{1/\tau} - 1)^2}{\tau_u}}, \tag{102}$$

where $A(\kappa) = \int_0^1 \frac{t(a)}{\rho(a)+\kappa}\, da$ *and* $B(\kappa) = \int_0^1 \frac{t(a)}{(\rho(a)+\kappa)^2}\, da$*.*

5. ***Equilibrium system.*** *The aggregate quantities* κ *and* τ_p *are determined by* $\kappa = \tau_p/A(\kappa)$ *together with* (102)*, where* $\tau = \tau_v + \tau_p$*. Substituting yields a single fixed-point equation for* κ*:*

$$\kappa = \frac{A(\kappa)}{B(\kappa) + \dfrac{(\tau_v + \kappa\, A(\kappa))^2\,(e^{1/(\tau_v + \kappa\, A(\kappa))} - 1)^2}{\tau_u}}. \tag{103}$$

H.2.4 Normal payoffs

We now specialize the general characterization in Theorem 7 to the benchmark case of normal payoffs, $V(v) = v$. This is the standard specification in the noisy rational expectations literature. We assume that noise traders submit a price-inelastic quantity, so that

$$\tilde{\theta}(P) = 1.$$

Verifying the noise calibration. For $V(v) = v$, we have $V'(v) = 1$, so $\mathbb{E}[V'(v) \mid P] = 1$. The conditional variance of the payoff is $\mathrm{Var}(V(v) \mid P) = \mathrm{Var}(v \mid P) = 1/\tau$, where $\tau = \tau_v + \tau_p$. Substituting into the noise calibration condition (94):

$$\xi = \frac{\mathbb{E}[V'(v) \mid P]}{\mathrm{Var}(V(v) \mid P)\,\tilde{\theta}(P)} = \frac{1}{1/\tau \cdot 1} = \tau.$$

Since $\tau = \tau_v + \tau_p$, the parameter ξ depends on the endogenous τ_p but not on P, confirming that Assumption 6 is satisfied.

Equilibrium characterization.

Corollary 5 (Normal payoffs, REE)**.** *Suppose that $V(v) = v$, that Assumption 5 holds, and that $\tilde{\theta}(P) = 1$. Then the equilibrium of Theorem 7 has the following explicit form.*

1. ***Price function.*** *The price function is linear in the sufficient statistic:*

$$P = \mathcal{P}(s_p) = \frac{\tau_p}{\tau}\, s_p,$$

   so that $s_p = \frac{\tau}{\tau_p}\, P$.

2. ***Trading intensity.*** *The trading intensity is*

$$\beta(a, P) = \frac{t(a)}{\rho(a) + \kappa},$$

   and is independent of P.

3. ***Price impact.*** *Kyle's lambda is $\lambda(P) = \kappa/\tau$, which is constant in P.*

4. ***Price informativeness.*** *The precision of the sufficient statistic is*

$$\tau_p = \frac{A(\kappa)^2}{B(\kappa) + \tau_u^{-1}}, \tag{104}$$

   where $A(\kappa) = \int_0^1 \frac{t(a)}{\rho(a)+\kappa}\, da$ and $B(\kappa) = \int_0^1 \frac{t(a)}{(\rho(a)+\kappa)^2}\, da$.

5. ***Equilibrium system.*** *The aggregate quantities κ and τ_p are determined by $\kappa = \tau_p/A(\kappa)$ together with (104). Substituting yields a single fixed-point equation for κ:*

$$\kappa = \frac{A(\kappa)}{B(\kappa) + \tau_u^{-1}}. \tag{105}$$

The equilibrium system (105) is identical to equation (73) in Theorem 4, confirming that the general framework recovers the standard CARA-normal competitive REE of Section G as a special case.

H.3 Heuristic derivation of equilibrium

Following the heuristic approach used for the price-taking case, one can obtain:

$$\beta(a,P)\,ds(a) = \frac{\mathbb{E}[V(v) - P \mid ds(a), s_p]}{\rho(a)\,\mathrm{Var}(V(v) \mid P)} + \cdots \tag{106}$$

where s_p denotes the sufficient statistic for the information conveyed by the price and "$\cdots$" denotes terms that do not depend on $ds(a)$.

To pin down $\beta(a,P)$, we need to separate out the $ds(a)$ term in $\mathbb{E}[V(v) - P \mid ds(a), s_p]$. We proceed in two steps: first deriving $\mathbb{E}[v \mid ds(a), s_p]$, and then passing to $V(v)$.

Step 1: Linear projection of v. We rewrite the conditional expectation as:

$$\mathbb{E}[v \mid ds(a), s_p] = \mathbb{E}[v \mid ds(a),\ s_p - \omega(a)\,ds(a)],$$

where $\omega(a) = \beta(a,P)/\int_0^1 \beta(b,P)\,db$. Since $ds(a)$ and $s_p - \omega(a)\,ds(a)$ are conditionally independent given v, and $v \mid s_p$ is normally distributed, the standard linear projection formula gives:

$$\begin{aligned}\mathbb{E}[v \mid ds(a),\ s_p - \omega(a)\,ds(a)] &= \frac{t(a)}{\tau}\,ds(a) + \frac{\tau_p}{\tau}(s_p - \omega(a)\,ds(a)) \\ &= \frac{t(a) - \tau_p\,\omega(a)}{\tau}\,ds(a) + \cdots,\end{aligned}$$

where $\tau = \tau_v + \tau_p$ and "$\cdots$" denotes terms that do not depend on $ds(a)$.

Step 2: From v to $V(v)$ via Stein's lemma. Since adding one infinitesimal signal $ds(a)$ does not change the posterior variance of v (which remains $1/\tau$ to leading order), the effect on

$\mathbb{E}[V(v)]$ is captured entirely by the shift in the posterior mean. By Stein's lemma,

$$\text{Cov}(V(v), v \mid s_p) = \frac{\mathbb{E}[V'(v) \mid s_p]}{\tau},$$

so the coefficient of $ds(a)$ in $\mathbb{E}[V(v) \mid ds(a), s_p]$ is:

$$\mathbb{E}[V'(v) \mid s_p] \cdot \frac{t(a) - \tau_p\, \omega(a)}{\tau}.$$

Matching the coefficients of $ds(a)$ in (106):

$$\beta(a, P) = \frac{\mathbb{E}[V'(v) \mid P]}{\tau\, \rho(a)\, \text{Var}(V(v) \mid P)} \left(t(a) - \tau_p\, \omega(a)\right). \tag{107}$$

Step 3: Solving for $\beta(a, P)$. Since $\omega(a) = \beta(a, P)/\int_0^1 \beta(b, P)\, db$ and the common factor $\eta(P) \equiv \mathbb{E}[V'(v) \mid P]/(\tau\, \text{Var}(V(v) \mid P))$ cancels in the ratio, we define $g(a) = \beta(a, P)/\eta(P)$ so that $\omega(a) = g(a)/\int_0^1 g(b)\, db$. Then (107) simplifies to:

$$g(a) = \frac{1}{\rho(a)} \left(t(a) - \tau_p \cdot \frac{g(a)}{\int_0^1 g(b)\, db} \right).$$

To solve for $g(a)$, we introduce the shorthand:

$$\kappa \equiv \frac{\tau_p}{\int_0^1 g(b)\, db}.$$

Then:

$$g(a) = \frac{1}{\rho(a)}(t(a) - \kappa\, g(a)) \quad \Longleftrightarrow \quad g(a) = \frac{t(a)}{\rho(a) + \kappa}.$$

Thus, once κ is determined, $\beta(a, P)$ is fully characterized:

$$\beta(a, P) = \frac{t(a)}{\rho(a) + \kappa} \cdot \frac{\mathbb{E}[V'(v) \mid P]}{\tau\, \text{Var}(V(v) \mid P)},$$

which is equation (97).

Step 4: Computing τ_p and the role of noise calibration. The sufficient statistic has the representation:

$$s_p = v + \int_0^1 \frac{\omega(a)}{\sqrt{t(a)}}\, dB(a) - \frac{u\,\tilde{\theta}(P)}{\int_0^1 \beta(a,P)\, da}.$$

By Itô isometry, the inverse precision of s_p is:

$$\tau_p^{-1} = \int_0^1 \frac{\omega(a)^2}{t(a)}\, da + \frac{\tilde{\theta}(P)^2}{\tau_u \left(\int_0^1 \beta(a,P)\, da\right)^2}.$$

Substituting $\omega(a) = \beta(a,P)/\int_0^1 \beta(b,P)\, db$ and using $\beta(a,P) = \eta(P) \cdot t(a)/(\rho(a)+\kappa)$, the $\eta(P)$ factors cancel in the first term. Under Assumption 6, $\tilde{\theta}(P) = (\tau/\xi)\,\eta(P)$, so $\tilde{\theta}(P)/\eta(P) = \tau/\xi$ is P-independent, and the $\eta(P)$ factors cancel in the second term as well. We obtain:

$$\tau_p = \frac{\left(\int_0^1 \frac{t(a)}{\rho(a)+\kappa}\, da\right)^2}{\int_0^1 \frac{t(a)}{(\rho(a)+\kappa)^2}\, da + \frac{\tau^2}{\xi^2\,\tau_u}},$$

which is equation (96).

Step 5: Fixed-point equation for κ. From the definition $\kappa = \tau_p/\int_0^1 g(b)\, db = \tau_p/\int_0^1 \frac{t(a)}{\rho(a)+\kappa}\, da$ and the expression for τ_p above:

$$\kappa = \frac{\int_0^1 \frac{t(a)}{\rho(a)+\kappa}\, da}{\int_0^1 \frac{t(a)}{(\rho(a)+\kappa)^2}\, da + \frac{\tau^2}{\xi^2\,\tau_u}}.$$

Since $\tau = \tau_v + \tau_p = \tau_v + \kappa \int_0^1 \frac{t(a)}{\rho(a)+\kappa}\, da$, this yields the single fixed-point equation (100) for κ. Once κ is determined, τ_p, $\beta(a,P)$, and the price function $P = \mathcal{P}(s_p)$ are all determined as in Theorem 7.

H.4 BNE

We start with a formal derivation; a heuristic derivation is in Section H.4.6. We now derive the Bayesian Nash equilibrium for the general case, following the structure of the competitive REE analysis above. The key difference is that each trader internalizes the effect of her demand on the market-clearing price, as formalized in the optimization problem (93).

H.4.1 First-order condition

The optimization problem (93) is a calculus of variations problem: the density $h(P;\, y(\cdot), v, u, \mu)$ depends on both $y(P)$ and $y'(P)$, since the total P-derivative of the residual demand CDF involves the slope of the demand schedule.[56] Consequently, one cannot optimize pointwise in $y(P)$; the first-order condition must be derived via the Euler–Lagrange equation.

Euler condition (finite μ). Following Wilson (1979), at the equilibrium schedule $y(\cdot) = x^*(\cdot)$, the Euler condition reduces to the algebraic condition:

$$\int_{\mathbb{R}}\int_{\mathbb{R}} u'(W_0(a)+x^*(V(v)-P),\, a)\left[(V(v)-P)\, H_P(\Theta(P)-x^*,\, v,\, P,\, \mu) - x^*\, H_x(\Theta(P)-x^*,\, v,\, P,\, \mu)\right] \times \mathcal{E}(v,s,\mu)\, f(v)\, f_u(u)\, dv\, du = 0, \quad (108)$$

where $\mathcal{E}(v,s,\mu) = \exp\!\big(t(a)(sv - \frac{\mu}{2}v^2)\big)$. Here, $H_P(\Theta(P)-x^*,\, v,\, P,\, \mu)$ is the partial derivative of the residual demand CDF H, with respect to P, holding the demand level y fixed, and $H_x = H_x(\Theta(P)-x^*, v, P, \mu)$ is the density of the residual demand at the market-clearing point.

The Euler condition (108) is valid for any μ and does not require additive separability of the residual demand in price and noise. Compared with the competitive REE first-order condition, the key new term is $-x^*\, H_x$, which captures the cost of price impact: the trader's demand

[56] Explicitly, $h(P;\, y(\cdot), v, u, \mu) = H_x(\Theta(P) - y(P), v, P, \mu)(\Theta'(P) - y'(P)) + H_P(\Theta(P) - y(P), v, P, \mu)$, where H_x and H_P denote partial derivatives of the residual demand CDF (89).

displaces probability mass in the residual demand distribution, raising the clearing price.

H.4.2 Deriving $\beta(P, a)$

By the Aggregation Lemma, $\beta(P, a) = x_s(P, a;\, 0, 0)$. As in the competitive REE, we compute this derivative by differentiating the first-order condition (108) with respect to s and applying the Implicit Function Theorem at the limit point $s = 0$, $\mu = 0$, $x^* = 0$.

Lemma 14. *Suppose that Assumptions 5 and 6 are satisfied and that $0 < \lambda(P) < \infty$ for every P. Then:*

$$\beta(P, a) = \frac{t(a)\ \mathrm{Cov}(V(v), v \mid P)}{\rho(a)\ \mathrm{Var}(V(v) \mid P) + 2\lambda(P)}.$$

The only difference from the REE expression (Lemma 13) is the factor of 2 multiplying $\lambda(P)$ in the denominator. In the proof (see below), the two copies of $\lambda(P)$ arise from distinct terms in Wilson's Euler condition (108): the first from the density derivative h_{x^*} (the inference channel, identical to REE), and the second from the residual demand density H_x (the trader's internalization of price impact, new in BNE).

H.4.3 Equilibrium

As in the competitive REE, the coefficient $\beta(P, a)$ fully determines market quality and the fixed-point system for κ and τ_p. The coefficient $\delta(P, a)$ enters only through the market-clearing condition; its characterization is stated in Lemma 16 and proved in Section H.6.

We now characterize the BNE equilibrium. The structure parallels the competitive REE (Theorem 7).

Theorem 8. *Suppose that Assumptions 5 and 6 are satisfied. Then there exists an equilibrium with the following properties.*

The equilibrium price function has the same representation as in Theorem 7: $P = \mathcal{P}(s_p)$, *where*

$$\mathcal{P}(x) = \int V\left(\frac{\tau_p}{\tau}\,x + \frac{z}{\sqrt{\tau}}\right) d\Phi(z), \tag{109}$$

$\tau = \tau_v + \tau_p$, *and* $s_p = \mathcal{P}^{-1}(P)$.

The sufficient statistic is

$$s_p = \int_0^1 \omega(a)\,ds(a) - \frac{u\,\tilde{\theta}(P)}{\int_0^1 \beta(a,P)\,da},$$

where the weighting function is

$$\omega(a) = \frac{t(a)/(\rho(a)+2\kappa)}{\int_0^1 t(b)/(\rho(b)+2\kappa)\,db}.$$

The precision of s_p *is*

$$\tau_p = \frac{\left(\int_0^1 \frac{t(a)}{\rho(a)+2\kappa}\,da\right)^2}{\int_0^1 \frac{t(a)}{(\rho(a)+2\kappa)^2}\,da + \frac{\tau^2}{\xi^2\tau_u}}. \tag{110}$$

The trading intensity is

$$\beta(a,P) = \frac{t(a)}{\rho(a)+2\kappa}\cdot\frac{\mathbb{E}[V'(v)\mid P]}{\tau\ \mathrm{Var}(V(v)\mid P)}, \tag{111}$$

and $\delta(a,P)$ *is given by Lemma 16, with* $D(P,a) = (\rho(a)+2\kappa)\ \mathrm{Var}(V(v)\mid P)$.

Here, $\kappa = \lambda(P)/\mathrm{Var}(V(v)\mid P) \geq 0$ *denotes the price impact ratio, where* $\lambda(P) = -\frac{\partial}{\partial\theta}\mathbb{E}[V(v)\mid P]$ *is Kyle's lambda. Under the assumptions,* κ *does not depend on* P.

The equilibrium is fully determined by the aggregate quantities κ *and* τ_p*, which solve the* *system*

$$\kappa = \frac{\tau_p}{\int_0^1 \frac{t(a)}{\rho(a)+2\kappa}\, da}, \tag{112}$$

$$\tau_p = \frac{\left(\int_0^1 \frac{t(a)}{\rho(a)+2\kappa}\, da\right)^2}{\int_0^1 \frac{t(a)}{(\rho(a)+2\kappa)^2}\, da + \frac{(\tau_v+\tau_p)^2}{\xi^2\tau_u}}. \tag{113}$$

Substituting (112) *into* (113)*, the system reduces to a single fixed-point equation for* κ*:*

$$\kappa = \frac{\int_0^1 \frac{t(a)}{\rho(a)+2\kappa}\, da}{\int_0^1 \frac{t(a)}{(\rho(a)+2\kappa)^2}\, da + \frac{\left(\tau_v+\kappa\int_0^1 \frac{t(a)}{\rho(a)+2\kappa}\, da\right)^2}{\xi^2\tau_u}}. \tag{114}$$

Every expression in the BNE equilibrium is obtained from the corresponding REE expression (Theorem 7) by replacing $\rho(a)+\kappa$ with $\rho(a)+2\kappa$. This clean mapping extends the relationship between REE and BNE established in the CARA-normal model (Section G.3) to the general setting.

Proposition 17 (Non-existence without noise)**.** *Suppose that Assumptions 5 and 6 hold and that* $1/\tau_u = 0$*. Then no BNE exists.*

Proof. Setting $1/\tau_u = 0$ in (112)–(113) and proceeding as in the proof of Proposition 16, the fixed-point equation requires $A(\kappa) = \kappa\, B(\kappa)$ where $A(\kappa) = \int_0^1 \frac{t(a)}{\rho(a)+2\kappa}\, da$ and $B(\kappa) = \int_0^1 \frac{t(a)}{(\rho(a)+2\kappa)^2}\, da$. However,

$$A(\kappa) - \kappa\, B(\kappa) = \int_0^1 \frac{t(a)(\rho(a)+\kappa)}{(\rho(a)+2\kappa)^2}\, da > 0$$

for all $\kappa \geq 0$, so no solution exists. ■

H.4.4 Log-normal payoffs

Specializing to $V(v) = e^v$ and $\tilde{\theta}(P) = 1/P$, Theorem 8 yields the following closed-form characterization.

Corollary 6 (Log-normal payoffs, BNE)**.** *Suppose that $V(v) = e^v$, that Assumption 5 holds, and that $\tilde{\theta}(P) = 1/P$. Then the equilibrium of Theorem 8 has the following explicit form.*

1. ***Price function.*** *The price function is the same as in REE:*

$$P = \mathcal{P}(s_p) = \exp\left(\frac{\tau_p}{\tau}\, s_p + \frac{1}{2\tau}\right).$$

2. ***Trading intensity.*** *The trading intensity is*

$$\beta(a, P) = \frac{t(a)}{\rho(a) + 2\kappa} \cdot \frac{1}{\tau\, P\, (e^{1/\tau} - 1)}.$$

The dollar trading intensity $P\,\beta(a, P) = \frac{t(a)}{(\rho(a)+2\kappa)\,\tau\,(e^{1/\tau}-1)}$ is independent of P.

3. ***Price impact.*** *Kyle's lambda is $\lambda(P) = \kappa\, P^2(e^{1/\tau} - 1)$, proportional to P^2.*

4. ***Price informativeness.*** *The precision of the sufficient statistic is*

$$\tau_p = \frac{A(\kappa)^2}{B(\kappa) + \dfrac{\tau^2\,(e^{1/\tau} - 1)^2}{\tau_u}}, \tag{115}$$

where $A(\kappa) = \int_0^1 \frac{t(a)}{\rho(a)+2\kappa}\, da$ and $B(\kappa) = \int_0^1 \frac{t(a)}{(\rho(a)+2\kappa)^2}\, da$.

5. ***Equilibrium system.*** *The aggregate quantities κ and τ_p are determined by $\kappa = \tau_p / A(\kappa)$ together with (115), where $\tau = \tau_v + \tau_p$. Substituting yields a single fixed-point equation for κ:*

$$\kappa = \frac{A(\kappa)}{B(\kappa) + \dfrac{(\tau_v + \kappa\, A(\kappa))^2\,(e^{1/(\tau_v+\kappa\, A(\kappa))} - 1)^2}{\tau_u}}.$$

Proof. The proof is identical to that of Corollary 4, with $\rho(a) + \kappa$ replaced by $\rho(a) + 2\kappa$ throughout. ■

H.4.5 Normal payoffs

Specializing to $V(v) = v$ and $\tilde{\theta}(P) = 1$, Theorem 8 yields the following closed-form characterization.

Corollary 7 (Normal payoffs, BNE)**.** *Suppose that $V(v) = v$, that Assumption 5 holds, and that $\tilde{\theta}(P) = 1$. Then the equilibrium of Theorem 8 has the following explicit form.*

1. ***Price function.*** *The price function is linear:*

$$P = \mathcal{P}(s_p) = \frac{\tau_p}{\tau}\, s_p.$$

2. ***Trading intensity.*** *The trading intensity is*

$$\beta(a, P) = \frac{t(a)}{\rho(a) + 2\kappa},$$

   and is independent of P.

3. ***Price impact.*** *Kyle's lambda is $\lambda(P) = \kappa/\tau$, constant in P.*

4. ***Price informativeness.*** *The precision of the sufficient statistic is*

$$\tau_p = \frac{A(\kappa)^2}{B(\kappa) + \tau_u^{-1}}, \tag{116}$$

   where $A(\kappa) = \int_0^1 \frac{t(a)}{\rho(a)+2\kappa}\, da$ and $B(\kappa) = \int_0^1 \frac{t(a)}{(\rho(a)+2\kappa)^2}\, da$.

5. ***Equilibrium system.*** *The aggregate quantities κ and τ_p are determined by $\kappa = \tau_p / A(\kappa)$*

together with (116). Substituting yields a single fixed-point equation for κ:

$$\kappa = \frac{A(\kappa)}{B(\kappa) + \tau_u^{-1}}. \tag{117}$$

The equilibrium system (117) is identical to equation (79) in Theorem 5, confirming that the general framework recovers the standard CARA-normal BNE of Section G as a special case. As in that section, the only difference between the REE and BNE equilibria is the replacement of $\rho(a) + \kappa$ by $\rho(a) + 2\kappa$.

H.4.6 Heuristic derivation of the BNE

Following the heuristic approach used for the competitive REE (Section H.3), we derive the BNE trading intensity and equilibrium system. The starting point is the limit form (**??**) of the Euler condition, which—under Assumption 6—reduces to pointwise optimization along the residual supply curve. The only modification to the REE heuristic is the inclusion of the price-impact cost $\lambda(P)$ in the first-order condition.

The BNE analog of equation (106) is:

$$\beta(a, P)\, ds(a) = \frac{\mathbb{E}[V(v) - P \mid ds(a), s_p]}{(\rho(a) + \kappa)\ \mathrm{Var}(V(v) \mid P)} + \cdots \tag{118}$$

where $\kappa = \lambda(P)/\mathrm{Var}(V(v) \mid P)$ and the "$\cdots$" denotes terms independent of $ds(a)$. The denominator now includes κ from the price impact in addition to $\rho(a)$ from risk aversion.

Step 1: Linear projection of v. Proceeding exactly as in Step 1 of Section H.3, the coefficient of $ds(a)$ in $\mathbb{E}[v \mid ds(a), s_p]$ is:

$$\frac{t(a) - \tau_p\, \omega(a)}{\tau},$$

where $\omega(a) = \beta(a, P)/\int_0^1 \beta(b, P)\, db$.

Step 2: From v to $V(v)$ via Stein's lemma. Exactly as in Step 2, the coefficient of $ds(a)$ in $\mathbb{E}[V(v) \mid ds(a), s_p]$ is:

$$\mathbb{E}[V'(v) \mid s_p] \cdot \frac{t(a) - \tau_p\, \omega(a)}{\tau}.$$

Step 3: Solving for $\beta(a, P)$. Matching the coefficients of $ds(a)$ in (118):

$$\beta(a, P) = \frac{\mathbb{E}[V'(v) \mid P]}{\tau\, (\rho(a) + \kappa)\ \mathrm{Var}(V(v) \mid P)}\, (t(a) - \tau_p\, \omega(a)).$$

Defining $\eta(P) \equiv \mathbb{E}[V'(v) \mid P]/(\tau\ \mathrm{Var}(V(v) \mid P))$ and $g(a) = \beta(a, P)/\eta(P)$, we obtain (as in Step 3 of Section H.3):

$$g(a) = \frac{1}{\rho(a) + \kappa}\, (t(a) - \kappa\, g(a)) \quad \iff \quad g(a) = \frac{t(a)}{\rho(a) + 2\kappa}.$$

Here, the left-hand side has $\rho(a) + \kappa$ in the denominator (from the FOC, which includes both risk aversion and price impact), and the right-hand side produces $\rho(a)+2\kappa$ after absorbing the additional κ from the inference channel (via $\omega(a)$). This is the source of the factor 2: one κ from price impact, one from inference.

Thus:

$$\beta(a, P) = \frac{t(a)}{\rho(a) + 2\kappa} \cdot \frac{\mathbb{E}[V'(v) \mid P]}{\tau\ \mathrm{Var}(V(v) \mid P)},$$

which is equation (111).

Step 4: Computing τ_p. The sufficient statistic and its precision are computed identically to Steps 4–5 of Section H.3, with $\rho(a) + \kappa$ replaced by $\rho(a) + 2\kappa$. Under Assumption 6, the $\eta(P)$ factors cancel as before, yielding

$$\tau_p = \frac{\left(\int_0^1 \frac{t(a)}{\rho(a)+2\kappa}\, da\right)^2}{\int_0^1 \frac{t(a)}{(\rho(a)+2\kappa)^2}\, da + \frac{\tau^2}{\xi^2\, \tau_u}},$$

which is equation (110).

Step 5: Fixed-point equation for κ. From $\kappa = \tau_p / \int_0^1 g(b)\, db = \tau_p / \int_0^1 \frac{t(a)}{\rho(a)+2\kappa}\, da$ and the expression for τ_p above:

$$\kappa = \frac{\int_0^1 \frac{t(a)}{\rho(a)+2\kappa}\, da}{\int_0^1 \frac{t(a)}{(\rho(a)+2\kappa)^2}\, da + \frac{\tau^2}{\xi^2\, \tau_u}}.$$

Since $\tau = \tau_v + \tau_p = \tau_v + \kappa \int_0^1 \frac{t(a)}{\rho(a)+2\kappa}\, da$, this yields the single fixed-point equation (114) for κ.

H.5 Price informativeness in liquid markets

The main complication in extending the comparative statics of Propositions 2 and 3 to REE and BNE is that the price-impact parameter κ is itself an equilibrium object. Changes in primitives—the distributions of $t(a)$ and $W_0(a)$—affect trading intensities not only directly, through $\rho(a)$ and $t(a)$, but also indirectly through κ. We now show that in sufficiently liquid markets this indirect channel vanishes, and the comparative statics from the price-taking benchmark carry over to both REE and BNE. Because we take the market to its perfectly liquid limit ($\kappa \to 0$), liquidity itself becomes trivially perfect in this limit; accordingly, we focus exclusively on price informativeness.

We parameterize market liquidity through the traders' uncertainty about mean supply, $\sigma_{\bar{u}}^2$. When $\sigma_{\bar{u}}^2$ is large, traders attribute substantial noise to the price, which depresses price impact and makes markets liquid.

Proposition 18 (Liquid-market limit)**.** *Suppose that Assumptions 5 and 6 hold. In both the REE and BNE equilibria, as $\sigma_{\bar{u}}^2 \to \infty$ (with σ_u^2 fixed):*

1. *$\kappa \to 0$ and $\mathcal{L} \to \infty$.*

2. *The actual precision of the price signal converges to*

$$\tau_p^* \;\equiv\; \frac{A(0)^2}{B(0) + c_0\,\sigma_u^2}, \tag{119}$$

where $A(0) = \int_0^1 \frac{t(a)}{\rho(a)}\,da$, $B(0) = \int_0^1 \frac{t(a)}{\rho(a)^2}\,da$, *and* $c_0 = \tau_v^2/\xi_0^2 > 0$ *with* ξ_0 *the noise-calibration parameter evaluated in the uninformative-price limit* ($\tau_p = 0$)*.*[57] *Informational efficiency converges to* $\mathcal{I}^* = \tau_p^*/(\tau_v + \tau_p^*)$.

The limiting precision τ_p^* has the same functional dependence on $t(a)$ and $\rho(a)$ as the price-taking precision $\tau_p^{PT} = A(0)^2/B(0)$: the additional term $c_0\,\sigma_u^2$ in the denominator is a positive constant that does not depend on $t(a)$ or $\rho(a)$. Consequently, the Gateaux derivatives of τ_p^*—and hence of $\mathcal{I}^*$—with respect to $t(a)$ and $W_0(a)$ have the same signs as in the price-taking equilibrium. By continuity, these signs are preserved for all $\sigma_{\bar{u}}^2$ sufficiently large.

Corollary 8 (Information efficiency in REE and BNE)**.** *Suppose that trader preferences are DARA and that Assumptions 5 and 6 hold. For all* $\sigma_{\bar{u}}^2$ *sufficiently large:*

1. *(Wealth inequality.) The comparative statics of Proposition 2 hold in both REE and BNE: there exist thresholds such that a Robin Hood transfer in wealth from the sufficiently rich to the sufficiently poor improves informational efficiency.*

2. *(Information inequality.) The comparative statics of Proposition 3 hold in both REE and BNE: a Robin Hood transfer in information precision from the rich to the poor improves informational efficiency.*

3. *(Information-aggregation paradox.) The paradox of Corollary 1 arises in both REE and BNE: weakly increasing the precision of all traders can reduce informational efficiency.*

Intuitively, when $\sigma_{\bar{u}}^2$ is large, traders attribute most of the price variation to supply noise, making the price nearly uninformative from their perspective. This drives κ to zero, eliminating

[57] For normal payoffs ($V(v) = v$), $c_0 = 1$. For log-normal payoffs ($V(v) = e^v$), $c_0 = \tau_v^2(e^{1/\tau_v} - 1)^2$.

the wedge between the three equilibrium concepts. As the trading intensities

$$\beta^{PT}(a) \propto \frac{t(a)}{\rho(a)}, \qquad \beta^{REE}(a) \propto \frac{t(a)}{\rho(a)+\kappa}, \qquad \beta^{BNE}(a) \propto \frac{t(a)}{\rho(a)+2\kappa}$$

make clear, when $\kappa \to 0$ all three coincide. The direct effects of changing $t(a)$ or $W_0(a)$ on market quality—which are the novel economic forces identified in the main text—therefore dominate, and the information-aggregation paradox carries over to REE and BNE.

H.6 Proofs for Section H

Note on the u integration. Throughout the proofs below, the first-order condition G is a double integral over (v, u), reflecting the conditioning on the supply shock u in the density decomposition (91). In the limit $s = 0$, $\mu = 0$, $x^* = 0$, the density $h(P; 0, v, u, 0)$ equals the conditional density $f(P \mid v, u)$, and $h\, f(v)\, f_u(u) = f(v, u \mid P)\, f(P)$ by Bayes' rule. Since all payoff-relevant integrands depend on u only through h and its derivatives, integrating over u yields the marginal density $f(v \mid P)$. We therefore write all such intermediate expressions as single integrals over v, with $f(P)$ denoting the unconditional price density. In the BNE proofs, the residual-demand quantities $H_x(\Theta(P), v, P, 0)$ and $H_{xx}(\Theta(P), v, P, 0)$ additionally depend on u through the supply $\Theta(P)$; those terms are handled by averaging against $f_u(u)$ explicitly (see Eq. (124) and its proof).

H.6.1 Proof of Lemma 13

Proof. The optimal demand x^* satisfies the first-order condition (FOC) of the maximization problem:

$$G(x^*, s) \equiv \int_{\mathbb{R}}\int_{\mathbb{R}} (V(v) - P)u'\left(W_0(a) + x^*(V(v) - P)\right) \Psi(v, P, s, x^*, u)\, f(v)\, f_u(u)\, dv\, du = 0$$

where for this proof only we denote the density by:

$$\Psi(P; s, x^*, v, u, \mu) = h(P; x^*, v, u, \mu) \exp\left(t(a)\left(sv - \frac{\mu}{2}v^2\right)\right).$$

The FOC is both necessary and sufficient due to the concavity of the utility function.

By the Implicit Function Theorem, the sensitivity $\beta(P, a)$ at the limit $(s = 0, \mu = 0, x^* = 0)$ is:

$$\beta(P, a) = \frac{\partial x^*}{\partial s} = -\frac{\partial G/\partial s}{\partial G/\partial x^*}.$$

Assumption 5, parts 1–3 imply that the derivatives can be passed under the integral sign, which together with part 4 implies that G is continuously differentiable at $x^* = 0$ and $s = 0$. Part 4 further ensures that $\partial G/\partial x^* \neq 0$, and part 2 guarantees that it is finite. Together, these conditions allow us to invoke the Implicit Function Theorem.

Step 1: The Numerator ($\partial G/\partial s$). Differentiating G with respect to s and evaluating at the limit (where the exponential term is 1):

$$\frac{\partial G}{\partial s} = \int_{\mathbb{R}}\int_{\mathbb{R}} (V(v) - P)u'(W_0)\left[h \cdot \frac{\partial}{\partial s}\exp\left(t(a)sv\right)\right] f(v)\, f_u(u)\, dv\, du$$

At $s = 0$, the derivative of the exponential is $t(a)v$. In the limit, h becomes the conditional density $f(P \mid v, u)$. By the convention above, integrating against $f_u(u)$ and applying Bayes' rule $f(P \mid v, u)\, f(v)\, f_u(u) = f(v, u \mid P)\, f(P)$ reduces the expression to a single integral over v:

$$f(P)\frac{\partial G}{\partial s} = u'(W_0)t(a)\int_{\mathbb{R}} (V(v) - P)v\, f(v|P)\, dv.$$

Here, $f(P)$ denotes the density of P in the limit economy. Using the condition $E[V(v)|P] = P$, the integral represents the covariance:

$$f(P)\frac{\partial G}{\partial s} = u'(W_0)t(a)\,\mathrm{Cov}(V(v), v \mid P).$$

We can pass the derivative under the integral and take the limits due to Assumption 5, parts 1–3.

Step 2: The Denominator ($\partial G/\partial x^*$). Differentiating G with respect to x^* involves both the utility function and the density h. Evaluating at the limit where h becomes $f(P \mid v, u)$ and reducing via the convention above:

$$f(P)\frac{\partial G}{\partial x^*} = u''(W_0)\int_{\mathbb{R}}(V(v) - P)^2 f(v|P)\,dv + u'(W_0)\int_{\mathbb{R}}(V(v) - P)\frac{\partial h}{\partial x^*}\,f(v)\,dv.$$

Taking into account that $E[V(v)|P] = P$, the first integral becomes $\text{Var}(V(v) \mid P)$. The second integral involves the sensitivity of the conditional expectation to the demand x^*. The x^* enters via the conditioning event in the residual demand distribution: $\mathcal{F} = \sigma(\int_0^1 \beta(P, a)ds(a) + \int_0^1 \delta(P, a)da + x^* = \bar{\theta} + \theta(P) + u\tilde{\theta}(P))$. Using the relationship

$$\frac{\partial}{\partial x^*}E[V(v) \mid \mathcal{F}] = -\frac{\partial}{\partial \bar{\theta}}E[V(v) \mid \mathcal{F}] = -\lambda(P),$$

we have:

$$\int_{\mathbb{R}}(V(v) - P)\frac{\partial h}{\partial x^*}\,f(v)\,dv = f(P)\frac{\partial}{\partial x^*}E[V(v) - P \mid P] = -f(P)\lambda(P).$$

This simplifies to:

$$f(P)\frac{\partial G}{\partial x^*} = u''(W_0)\,\text{Var}(V(v) \mid P) - u'(W_0)\lambda(P).$$

As in the previous step, we can pass the derivative under the integral and take the limits due to Assumption 5.

Step 3: Combining Terms. Substituting the numerator and denominator into the implicit function formula:

$$\beta(P,a) = -\frac{u'(W_0)t(a)\,\mathrm{Cov}(V(v),v \mid P)}{u''(W_0)\,\mathrm{Var}(V(v)\mid P) - u'(W_0)\lambda(P)}.$$

Dividing the numerator and denominator by $u'(W_0)$ and using $\rho(a) = -u''(W_0)/u'(W_0)$:

$$\beta(P,a) = \frac{t(a)\,\mathrm{Cov}(V(v),v \mid P)}{\rho(a)\,\mathrm{Var}(V(v)\mid P) + \lambda(P)}.$$

■

H.6.2 Statement and proof of Lemma 15

By the Aggregation Lemma, the coefficient $\delta(P,a)$ combines the curvature of optimal demand in the signal (x_{ss}) and the sensitivity to market size (x_μ):

$$\delta(P,a) = \frac{1}{2t(a)}x_{ss}(P,a;0,0) + x_\mu(P,a;0,0).$$

Lemma 15. *Suppose that Assumption 5 is satisfied and that $0 < \lambda(P) < \infty$ for every P. Let $\Sigma_{XY}(P) = \mathrm{Cov}(X,Y \mid P)$. Then:*

$$\delta(P,a) = \frac{1}{D(P,a)}\Bigg(\psi(P) + \beta(P,a)\left[-\lambda(P)\Sigma'_{Vv}(P) - \rho(a)\mathbb{E}\big[(V-P)^2 v \mid P\big] - \lambda(P)\mathbb{E}[v \mid P]\right] \\ + \frac{\beta(P,a)^2}{2t(a)}\left[\pi(a)\rho(a)\,\mathrm{Skew}(V\mid P) + 2\rho(a)\lambda(P)\Sigma'_{VV}(P) + \lambda(P)\lambda'(P)\right]\Bigg),$$

where $D(P,a) = \rho(a)\Sigma_{VV}(P) + \lambda(P)$, $\pi(a) = -u'''(W_0)/u''(W_0)$, and $\psi(P) = \frac{\partial}{\partial\mu}E[V-P \mid P]$.

Proof. We follow steps similar to the computation of $\beta(P,a)$. The optimal demand x^* satisfies the first-order condition $G(x^*,s,\mu) = 0$. From the derivation of $\beta(P,a)$, we know that $f(P)G_x = -u'(W_0)D(P,a)$.

Step 1: Computing x_μ. By the Implicit Function Theorem, $x_\mu = -G_\mu/G_x$. Differentiating G with respect to μ at the limit $(s = 0, \mu = 0)$ yields:

$$G_\mu = \int_{\mathbb{R}}\int_{\mathbb{R}} (V - P)u'(W_0)\left[\frac{\partial h}{\partial \mu} - h\frac{t(a)v^2}{2}\right] f(v)\, f_u(u)\, dv\, du.$$

Using $hf(v) = f(P)f(v|P)$ and $E[V|P] = P$, the second term is proportional to $\mathrm{Cov}(V, v^2 \mid P)$. Defining $\psi(P) \equiv \frac{1}{f(P)}\int(V - P)\frac{\partial h}{\partial \mu} f(v)\, dv$, we have:

$$f(P)G_\mu = u'(W_0)f(P)\left[\psi(P) - \frac{t(a)}{2}\mathrm{Cov}(V, v^2 \mid P)\right].$$

Thus,

$$x_\mu = \frac{\psi(P) - \frac{t(a)}{2}\mathrm{Cov}(V, v^2 \mid P)}{D(P, a)}.$$

We can pass the derivative under the integral and take the limits due to Assumption 5.

Step 2: Computing x_{ss}. Using the second derivative of the implicit function, we have $x_{ss} = -\frac{1}{G_x}\left[G_{ss} + 2\beta G_{xs} + \beta^2 G_{xx}\right]$. Let $\mathcal{E} \equiv \exp\left(t(a)\left(sv - \frac{\mu}{2}v^2\right)\right)$ denote the exponential part of the density. At the limit $s = 0, \mu = 0$, we have $\mathcal{E} = 1$, $\frac{\partial \mathcal{E}}{\partial s} = t(a)v$, and $\frac{\partial^2 \mathcal{E}}{\partial s^2} = t(a)^2 v^2$.

The term G_{ss} is given by:

$$f(P)G_{ss} = u'(W_0)\int(V - P)f(v|P)f(P)\frac{\partial^2 \mathcal{E}}{\partial s^2}\, dv = u'(W_0)f(P)t(a)^2\,\mathrm{Cov}(V, v^2 \mid P).$$

Its contribution to $\delta(P, a)$ via $\frac{1}{2t(a)}x_{ss}$ is:

$$\frac{1}{2t(a)}\frac{G_{ss}}{-G_x} = \frac{t(a)}{2D(P, a)}\mathrm{Cov}(V, v^2 \mid P).$$

This term exactly cancels the $-\frac{t(a)}{2}\mathrm{Cov}(V, v^2 \mid P)$ term from x_μ.

The term G_{xs} is:

$$G_{xs} = \int_{\mathbb{R}}\int_{\mathbb{R}} \frac{\partial}{\partial x^*}\left[(V-P)u'h\right] t(a) v f(v)\, f_u(u)\, dv\, du.$$

Expanding the derivative and using $\rho(a) = -u''/u'$:

$$f(P)G_{xs} = u'(W_0)f(P)t(a)\left[-\rho(a)\mathbb{E}\big[(V-P)^2 v \mid P\big] + \frac{\partial}{\partial x^*}E[(V-P)v \mid P]\right].$$

The term G_{xx} is:

$$G_{xx} = \int_{\mathbb{R}}\int_{\mathbb{R}} \frac{\partial^2}{\partial (x^*)^2}\left[(V-P)u'h\right] f(v)\, f_u(u)\, dv\, du.$$

Expanding the second derivative and using $\pi(a) = -u'''/u''$ (so $-u''' = \pi(a)u'' = -\pi(a)\rho(a)u'$):

$$f(P)G_{xx} = u'(W_0)f(P)\Big[\pi(a)\rho(a)\,\mathrm{Skew}(V \mid P) \\ - 2\rho(a)\frac{\partial}{\partial x^*}E[(V-P)^2 \mid P] + \frac{\partial^2}{\partial (x^*)^2}E[V-P \mid P]\Big].$$

The technical conditions in Assumption 5 ensure that we can differentiate under the integral sign and evaluate the limits for all second-order terms.

We now simplify the derivative terms. Recall that the conditioning event in the residual demand distribution h depends on x^*: $\mathcal{F} = \sigma(\int_0^1 \beta(P,a)ds(a) + \int_0^1 \delta(P,a)da + x^* = \bar{\theta} + \theta(P) + u\tilde{\theta}(P))$. Denoting the market clearing price corresponding to inelastic supply $\bar{\theta}$ by $\mathcal{P}(\bar{\theta})$, we can write

$$\frac{\partial}{\partial x^*}E[Y \mid \mathcal{F}] = -\frac{\partial}{\partial \bar{\theta}}E[Y \mid \mathcal{F}] = -\frac{\partial}{\partial \bar{\theta}}E[Y \mid \mathcal{P}(\bar{\theta})].$$

In the last transition we noted that for $x^* = 0$ the conditioning set $\mathcal{F}$ becomes $\sigma(\mathcal{P}(\bar{\theta}))$. Using the chain rule, and noting that $\mathcal{P}(\bar{\theta})' = \lambda(\mathcal{P}(\bar{\theta}))$ we derive that the partial derivative with respect to x^* satisfies the operator relationship for conditional moments whose integrand does not depend explicitly on P:

$$\frac{\partial}{\partial x^*}E[Y \mid P] = -\lambda(P)\frac{\partial}{\partial P}E[Y \mid P].$$

Using this relationship, we can express the terms as derivatives with respect to P. First, using $E[(V-P)^2 \mid P] = \Sigma_{VV}(P)$, we have:

$$\frac{\partial}{\partial x^*}E[(V-P)^2 \mid P] = -\lambda(P)\Sigma'_{VV}(P).$$

Second, writing $(V-P)v = Vv - Pv$, the first term depends on P only through the conditioning event, while the second contributes an explicit P-derivative. Hence

$$\frac{\partial}{\partial x^*}E[(V-P)v \mid P] = -\lambda(P)\Sigma'_{Vv}(P) - \lambda(P)\mathbb{E}[v \mid P].$$

Second, for the second derivative, we have:

$$\frac{\partial^2}{\partial (x^*)^2}E[V-P \mid P] = \lambda(P)\lambda'(P).$$

Step 3: Combining Terms. Substituting these expressions into the terms for G_{xs} and G_{xx}, and combining the surviving terms yields the expression in the lemma. ■

H.6.3 Proof of Theorem 7

Proof. The proof proceeds in five steps. Steps 1–2 establish the demand coefficients using Lemmas 13 and 15. Step 3 derives the price informativeness from the market clearing condition. Step 4 shows that, under Assumption 6, τ_p and κ are price-independent. Step 5 derives the price function via the efficiency condition and closes the system.

Step 1: Demand coefficients from Lemmas 13 and 15. By Lemma 13, the trading intensity is

$$\beta(P,a) = \frac{t(a)\ \mathrm{Cov}(V(v), v \mid P)}{\rho(a)\,\mathrm{Var}(V(v) \mid P) + \lambda(P)}. \tag{120}$$

Since $v \mid P$ is normally distributed, Stein's Lemma gives

$$\mathrm{Cov}(V(v), v \mid P) = \mathrm{Var}(v \mid P) \cdot \mathbb{E}[V'(v) \mid P] = \frac{1}{\tau}\,\mathbb{E}[V'(v) \mid P].$$

Defining the price impact ratio $\kappa(P) \equiv \lambda(P)/\mathrm{Var}(V(v) \mid P)$, we can rewrite (120) as

$$\beta(P,a) = \eta(P) \cdot \frac{t(a)}{\rho(a) + \kappa(P)}, \qquad \text{where } \eta(P) \equiv \frac{\mathbb{E}[V'(v) \mid P]}{\tau\,\mathrm{Var}(V(v) \mid P)}. \tag{121}$$

The coefficient $\delta(P,a)$ is given by Lemma 15.

Step 2: Market clearing and the sufficient statistic. The market clearing condition reads

$$\int_0^1 \beta(a,P)\,ds(a) + \int_0^1 \delta(a,P)\,da = \bar{\theta} + \theta(P) + u\,\tilde{\theta}(P). \tag{122}$$

Using $ds(a) = v\,da + dB(a)/\sqrt{t(a)}$ and rearranging, we define the sufficient statistic

$$s_p \equiv \frac{\int_0^1 \beta(a,P)\,ds(a) - u\,\tilde{\theta}(P)}{\int_0^1 \beta(a,P)\,da} = v + \int_0^1 \frac{\omega(a)}{\sqrt{t(a)}}\,dB(a) - \frac{u\,\tilde{\theta}(P)}{\int_0^1 \beta(a,P)\,da},$$

where $\omega(a) = \beta(a,P)/\int_0^1 \beta(b,P)\,db$. The two noise terms are independent, so the precision of s_p as a signal about v is

$$\tau_p^{-1} = \frac{\int_0^1 \frac{\beta(a,P)^2}{t(a)}\,da + \frac{\tilde{\theta}(P)^2}{\tau_u}}{\left(\int_0^1 \beta(a,P)\,da\right)^2}. \tag{123}$$

The sufficient statistic s_p is equivalently recovered from the price: using (122),

$$s_p = \frac{\bar{\theta} + \theta(P) - \int_0^1 \delta(a,P)\,da}{\int_0^1 \beta(a,P)\,da} \equiv h(P).$$

Step 3: Price-independence of τ_p and κ under Assumption 6. From (121), $\beta(a,P) =$

$\eta(P) \cdot t(a)/(\rho(a) + \kappa(P))$. Substituting into (123) and letting $A \equiv \int_0^1 \frac{t(a)}{\rho(a)+\kappa}\, da$ and $B \equiv \int_0^1 \frac{t(a)}{(\rho(a)+\kappa)^2}\, da$, we obtain

$$\tau_p^{-1} = \frac{\eta(P)^2\, B + \tilde{\theta}(P)^2/\tau_u}{\eta(P)^2\, A^2} = \frac{B}{A^2} + \frac{\tilde{\theta}(P)^2}{\tau_u\, \eta(P)^2\, A^2}.$$

The first term is P-independent (given κ). The second term is P-independent if and only if $\tilde{\theta}(P)/\eta(P)$ does not depend on P. Under Assumption 6,

$$\tilde{\theta}(P) = \frac{\mathbb{E}[V'(v) \mid P]}{\xi\, \mathrm{Var}(V(v) \mid P)} = \frac{\tau}{\xi}\, \eta(P),$$

so $\tilde{\theta}(P)/\eta(P) = \tau/\xi$, which is indeed P-independent. Thus,

$$\tau_p = \frac{A^2}{B + \tau^2/(\xi^2 \tau_u)},$$

which does not depend on P.

We next show that κ is also P-independent. The market-clearing price $\mathcal{P}(s_p, \bar{\theta})$ is determined implicitly by

$$s_p = \frac{\bar{\theta} + \theta(P) - \int_0^1 \delta(a, P)\, da}{\int_0^1 \beta(a, P)\, da}.$$

Standard differentiation yields

$$\lambda(P) = \frac{\frac{\tau_p}{\tau}\, \mathbb{E}[V'(v) \mid P]}{\int_0^1 \beta(a, P)\, da}.$$

Substituting $\int_0^1 \beta\, da = \eta(P)\, A$ and $\lambda = \kappa\, \mathrm{Var}(V|P)$, and using $\eta(P) = \mathbb{E}[V'|P]/(\tau\, \mathrm{Var}(V|P))$, we obtain

$$\kappa = \frac{\tau_p}{A},$$

which does not depend on P.

Step 4: Price function via the efficiency condition. Since τ_p is P-independent, $v \mid s_p \sim N\big(\frac{\tau_p}{\tau}\, s_p,\ \tau^{-1}\big)$. The efficiency condition $\mathbb{E}[V(v) \mid P] = P$ (up to the infinitesimal risk premium) gives

$$P = \mathbb{E}\left[V\left(\frac{\tau_p}{\tau}\, s_p + \frac{z}{\sqrt{\tau}}\right)\right] = \int V\left(\frac{\tau_p}{\tau}\, s_p + \frac{z}{\sqrt{\tau}}\right) d\Phi(z) = \mathcal{P}(s_p),$$

where $z = \sqrt{\tau}\left(v - \frac{\tau_p}{\tau}\, s_p\right)$ has a standard normal distribution.

Step 5: Closing the system. The equilibrium is determined once κ and τ_p are known. From Steps 3 and 4, these satisfy the system

$$\kappa = \frac{\tau_p}{A(\kappa)},$$
$$\tau_p = \frac{A(\kappa)^2}{B(\kappa) + (\tau_v + \tau_p)^2/(\xi^2 \tau_u)},$$

where $A(\kappa) = \int_0^1 \frac{t(a)}{\rho(a)+\kappa}\, da$ and $B(\kappa) = \int_0^1 \frac{t(a)}{(\rho(a)+\kappa)^2}\, da$. ■

H.6.4 Proof of Proposition 16

Proof. Setting $1/\tau_u = 0$ in the fixed-point system (98)–(99) reduces it to

$$\kappa = \frac{\tau_p}{A(\kappa)}, \qquad \tau_p = \frac{A(\kappa)^2}{B(\kappa)},$$

where $A(\kappa) \equiv \int_0^1 \frac{t(a)}{\rho(a)+\kappa}\, da$ and $B(\kappa) \equiv \int_0^1 \frac{t(a)}{(\rho(a)+\kappa)^2}\, da$. Substituting the second equation into the first yields the requirement $\kappa = A(\kappa)/B(\kappa)$, or equivalently, $A(\kappa) - \kappa\, B(\kappa) = 0$. However,

$$A(\kappa) - \kappa\, B(\kappa) = \int_0^1 t(a)\left[\frac{1}{\rho(a)+\kappa} - \frac{\kappa}{(\rho(a)+\kappa)^2}\right] da = \int_0^1 \frac{t(a)\,\rho(a)}{(\rho(a)+\kappa)^2}\, da > 0$$

for all $\kappa \geq 0$, since $t(a) > 0$ and $\rho(a) > 0$. Therefore $A(\kappa)/B(\kappa) > \kappa$ for every $\kappa \geq 0$, and the fixed-point equation has no solution. ■

H.6.5 Proof of Corollary 4

Proof. We specialize each component of Theorem 7 to $V(v) = e^v$ and $\tilde{\theta}(P) = 1/P$.

Item 1. Since $v \mid s_p \sim N\big(\frac{\tau_p}{\tau} s_p,\, \tau^{-1}\big)$, the price function (95) becomes

$$\mathcal{P}(s_p) = \int \exp\!\left(\frac{\tau_p}{\tau}\, s_p + \frac{z}{\sqrt{\tau}}\right) d\Phi(z) = \exp\!\left(\frac{\tau_p}{\tau}\, s_p + \frac{1}{2\tau}\right),$$

using the moment-generating function of the standard normal.

Item 2. The common factor from (121) is

$$\eta(P) = \frac{\mathbb{E}[V'(v) \mid P]}{\tau\, \mathrm{Var}(V(v) \mid P)} = \frac{P}{\tau\, P^2 (e^{1/\tau} - 1)} = \frac{1}{\tau\, P\, (e^{1/\tau} - 1)}.$$

Substituting into (97) gives (101). Multiplying by P cancels the price dependence.

Item 3. From $\kappa = \lambda(P)/\mathrm{Var}(V(v) \mid P)$ and $\mathrm{Var}(V(v) \mid P) = P^2(e^{1/\tau} - 1)$, we obtain $\lambda(P) = \kappa\, P^2 (e^{1/\tau} - 1)$.

Item 4. The general formula (99) gives $\tau_p = A(\kappa)^2/(B(\kappa) + \tau^2/(\xi^2 \tau_u))$. Substituting $\xi = 1/(e^{1/\tau} - 1)$:

$$\frac{\tau^2}{\xi^2\, \tau_u} = \frac{\tau^2\, (e^{1/\tau} - 1)^2}{\tau_u},$$

which yields (102).

Item 5. Using $\kappa = \tau_p/A(\kappa)$ to eliminate τ_p and $\tau = \tau_v + \kappa\, A(\kappa)$ to express τ in terms of κ, equation (102) reduces to the single equation (103). ∎

H.6.6 Proof of Corollary 5

Proof. We specialize each component of Theorem 7 to $V(v) = v$ and $\tilde{\theta}(P) = 1$.

For $V(v) = v$, the relevant conditional moments are $\mathbb{E}[V'(v) \mid P] = 1$, $\mathrm{Var}(V(v) \mid P) = 1/\tau$, and $\mathrm{Cov}(V(v), v \mid P) = 1/\tau$, where $\tau = \tau_v + \tau_p$.

Item 1. Since $v \mid s_p \sim N\big(\frac{\tau_p}{\tau} s_p,\, \tau^{-1}\big)$, the price function (95) becomes

$$\mathcal{P}(s_p) = \int \left(\frac{\tau_p}{\tau}\, s_p + \frac{z}{\sqrt{\tau}} \right) d\Phi(z) = \frac{\tau_p}{\tau}\, s_p.$$

Item 2. The common factor from (121) is $\eta(P) = 1/(\tau \cdot 1/\tau) = 1$. Substituting into (97) gives $\beta(a, P) = t(a)/(\rho(a) + \kappa)$, which is independent of P.

Item 3. From $\kappa = \lambda(P)/\mathrm{Var}(V(v) \mid P)$ and $\mathrm{Var}(V(v) \mid P) = 1/\tau$, we obtain $\lambda(P) = \kappa/\tau$, which is constant.

Item 4. The noise calibration gives $\xi = \tau$, so $\tau^2/(\xi^2\, \tau_u) = 1/\tau_u$. The general formula (99) then yields (104).

Item 5. Using $\kappa = \tau_p/A(\kappa)$ to eliminate τ_p, equation (104) reduces to the single equation (105). ■

H.6.7 Proof of Lemma 14

Proof. The proof follows the same strategy as the proof of Lemma 13, applied to Euler condition (108).

Because the $y'(P)$ terms perfectly cancel out in the Euler–Lagrange equation, the first-order condition reduces to an algebraic equation $G(x^*, s, \mu) = 0$ in the demand level x^*. Let $H_P(\Theta(P) - x^*, v, P, \mu) \equiv H_x \Theta'(P) + H_P^{(3)}$ denote the partial derivative of $H(\Theta(P) - x^*, v, P, \mu)$ with respect to P holding the demand level x^* fixed. Then we define:

$$G(x^*, s, \mu) \equiv \int_{\mathbb{R}} \int_{\mathbb{R}} u'(W_0(a) + x^*(V(v) - P),\, a) \Big[(V(v) - P)\, H_P(\Theta(P) - x^*, v, P, \mu) \\ - x^*\, H_x(\Theta(P) - x^*, v, P, \mu) \Big]\, \mathcal{E}(v, s, \mu)\, f(v)\, f_u(u)\, dv\, du = 0,$$

where $\mathcal{E}(v, s, \mu) = \exp\!\big(t(a)(sv - \frac{\mu}{2}\, v^2)\big)$.

By the Implicit Function Theorem, $\beta(P, a) = -G_s/G_{x^*}$ evaluated at $s = 0$, $\mu = 0$, $x^* = 0$.

Step 1: Numerator (G_s). At $x^* = 0$, the term $-x^* H_x$ vanishes. Moreover, at $x^* = 0$ (and thus $x^{*\prime} = 0$), the fixed-demand derivative H_P coincides perfectly with the true density $h(P)$. The computation is therefore identical to Step 1 in the proof of Lemma 13:

$$f(P)\, G_s = u'(W_0)\, t(a)\ \mathrm{Cov}(V(v), v \mid P).$$

Step 2: Denominator (G_{x^*}). We differentiate G with respect to the scalar x^* (since G is an algebraic function of x^* and does not depend on $x^{*\prime}$). Evaluating at $x^* = 0$, $s = 0$, $\mu = 0$ (so $\mathcal{E} = 1$):

$$G_{x^*} = \underbrace{\int u''(W_0)\,(V-P)^2\, H_P\, f\, dv \ + \int u'(W_0)\,(V-P)\,\frac{\partial H_P}{\partial x^*}\, f\, dv}_{\text{from the } (V-P)\,H_P \text{ term}} \\ \underbrace{- \int u'(W_0)\, H_x\, f\, dv}_{\text{from the } -x^*\, H_x \text{ term (new in BNE)}} \quad .$$

Since $H_P = h(P)$ at $x^* = 0$, the first integral is identical to the REE derivation. For the second integral, notice that:

$$\frac{\partial H_P}{\partial x^*} = \frac{\partial}{\partial x^*}[H_x(\Theta - x^*)\Theta' + H_P^{(3)}(\Theta - x^*)] = -H_{xx}\Theta' - H_{Px}^{(3)}.$$

This perfectly matches the partial derivative of the density $h(P; x^*, x^{*\prime})$ with respect to its level argument x^* evaluated at $x^{*\prime} = 0$. Thus, evaluating this term at $x^* = 0$ yields exactly the h_{x^*} term from the REE derivation. The first two integrals combine exactly as in the proof of Lemma 13, yielding

$$f(P)[u''(W_0)\ \mathrm{Var}(V(v) \mid P) - u'(W_0)\,\lambda(P)].$$

For the new term, we claim

$$\int_{\mathbb{R}}\int_{\mathbb{R}} H_x(\Theta(P),\, v,\, P,\, 0)\, f(v)\, f_u(u)\, dv\, du \;=\; \lambda(P)\, f(P). \tag{124}$$

Proof of (124). Define $F(x,P) = \int H(x,v,P,0)\, f(v)\, dv = \mathbb{P}(R_{-i}(P) < x)$. For a fixed realization of u, $\mathbb{P}(\tilde{P} \le P \mid u) = F(\Theta(P),\, P)$. Since $\Theta(P) = \bar{\theta} + \theta(P) + u\,\tilde{\theta}(P)$ and $F(x,P)$ does not depend on $\bar{\theta}$:

$$\frac{\partial}{\partial\bar{\theta}}\mathbb{P}(\tilde{P} \le P \mid u) = F_x(\Theta(P),\, P) = \int H_x(\Theta(P),\, v,\, P,\, 0)\, f(v)\, dv.$$

Under Assumption 6, the price response to supply, $\partial\tilde{P}/\partial\bar{\theta}$, is a deterministic function of $\tilde{P}$: the noise-trader terms cancel from the excess-supply slope (because $\tilde{\theta}(P) \propto \eta(P)$). Since $\partial\tilde{P}/\partial\bar{\theta} = -\lambda(P)$ at $\tilde{P} = P$:

$$\frac{\partial}{\partial\bar{\theta}}\mathbb{P}(\tilde{P} \le P \mid u) = f(P \mid u)\, \lambda(P).$$

Averaging over u:

$$\int_{\mathbb{R}}\int_{\mathbb{R}} H_x(\Theta(P),\, v,\, P,\, 0)\, f(v)\, f_u(u)\, dv\, du = \lambda(P) \int f(P \mid u)\, f_u(u)\, du = \lambda(P)\, f(P). \qquad \square$$

Combining all terms:

$$f(P)\, G_{x^*} = u''(W_0)\ \mathrm{Var}(V(v) \mid P)\, f(P) - 2\, u'(W_0)\, \lambda(P)\, f(P).$$

The first copy of λ arises from h_{x^*} (the inference channel, identical to REE); the second from H_x (the trader's internalization of price impact, new in BNE).

Step 3: Combining.

$$\beta(P,a) = -\frac{G_s}{G_{x^*}} = \frac{t(a)\ \mathrm{Cov}(V(v), v \mid P)}{\rho(a)\ \mathrm{Var}(V(v) \mid P) + 2\lambda(P)}.$$

■

H.6.8 Statement and proof of Lemma 16

The derivation of $\delta(P,a)$ follows the same steps as in the competitive REE (Lemma 15), applied to Wilson's Euler condition (108). As with β, the $-x^* H_x$ term in the Euler condition generates additional contributions compared with the REE.

Lemma 16. *Suppose that Assumptions 5 and 6 are satisfied and that $0 < \lambda(P) < \infty$ for every P. Let $\Sigma_{XY}(P) = \mathrm{Cov}(X, Y \mid P)$. Then:*

$$\delta(P,a) = \frac{1}{D(P,a)}\Bigg(\psi(P) + \beta(P,a)\Big[-\lambda(P)\,\Sigma'_{Vv}(P) - \rho(a)\ \mathrm{Cov}((V-P)^2,\ v \mid P) - \lambda(P)\,\mathbb{E}[v \mid P]\Big]$$
$$+ \frac{\beta(P,a)^2}{2t(a)}\Big[\pi(a)\rho(a)\ \mathrm{Skew}(V \mid P) + 2\rho(a)\,\lambda(P)\,\Sigma'_{VV}(P) + \lambda(P)\,\lambda'(P) + 2\Lambda(P)\Big]\Bigg),$$

where $D(P,a) = \rho(a)\,\Sigma_{VV}(P) + 2\lambda(P)$, $\pi(a) = -u'''(W_0)/u''(W_0)$, $\psi(P) = \frac{\partial}{\partial\mu}\mathbb{E}[V - P \mid P]$, and

$$\Lambda(P) \equiv \frac{\lambda(P)}{f(P)}\frac{d}{dP}[\lambda(P)\,f(P)] = \lambda(P)\,\lambda'(P) - \lambda(P)^2\left(s_p\frac{\tau_v}{\mathbb{E}[V'(v) \mid P]} + \frac{\mathbb{E}[V''(v) \mid P]}{\mathbb{E}[V'(v) \mid P]^2}\right),$$

where $f(P)$ is the unconditional density of the clearing price, $\tau = \tau_v + \tau_p$, and $s_p = \mathcal{P}^{-1}(P)$ is the sufficient statistic.

Compared with the REE expression (Lemma 15), the BNE δ has three modifications, all traceable to the $-x^* H_x$ term in Wilson's Euler condition: (i) the denominator is $\rho\,\Sigma_{VV} + 2\lambda$ (not $\rho\,\Sigma_{VV} + \lambda$); (ii) the β-linear bracket acquires the additional term $-\lambda\,\mathbb{E}[v \mid P]$, arising from the inference channel; (iii) the β-quadratic bracket acquires $2\Lambda(P)$, which can be written entirely in terms of local conditional moments and structural parameters.

Proof. We follow the same strategy as the proof of Lemma 15, applied to Euler condition (108). As in the proof of Lemma 14, we define the algebraic equation:

$$G(x^*, s, \mu) \equiv \int_{\mathbb{R}}\int_{\mathbb{R}} u'(W_0 + x^*(V - P)) \Big[(V - P)\, H_P - x^*\, H_x\Big]\, \mathcal{E}\, f(v)\, f_u(u)\, dv\, du = 0,$$

where $H_P = H_P(\Theta(P) - x^*, v, P, \mu)$ is the fixed-demand partial derivative of the residual demand CDF with respect to P, $H_x = H_x(\Theta(P) - x^*, v, P, \mu)$, and $\mathcal{E} = \exp\!\big(t(a)(sv - \frac{\mu}{2} v^2)\big)$.

From the proof of Lemma 14, we know $f(P)\, G_{x^*} = -u'(W_0)\, f(P)\, D(P, a)$ where $D(P, a) = \rho(a)\, \Sigma_{VV}(P) + 2\lambda(P)$.

The coefficient δ is given by $\delta = \frac{1}{2t}\, x_{ss} + x_\mu$ where $x_\mu = -G_\mu / G_{x^*}$ and $x_{ss} = -\frac{1}{G_{x^*}}[G_{ss} + 2\beta\, G_{x^* s} + \beta^2\, G_{x^* x^*}]$.

Step 1: G_μ (identical to REE). At $x^* = 0$, the $-x^* H_x$ term vanishes. The computation is identical to Step 1 of the proof of Lemma 15:

$$f(P)\, G_\mu = u'(W_0)\, f(P) \Big[\psi(P) - \tfrac{t(a)}{2}\, \mathrm{Cov}(V, v^2 \mid P)\Big].$$

Step 2: G_{ss} (identical to REE). At $x^* = 0$, only the $(V - P)\, h$ term survives:

$$f(P)\, G_{ss} = u'(W_0)\, f(P)\, t(a)^2\, \mathrm{Cov}(V, v^2 \mid P).$$

The contribution $\frac{1}{2t} \frac{G_{ss}}{-G_{x^*}} = \frac{t(a)}{2D}\, \mathrm{Cov}(V, v^2 \mid P)$ cancels the $-\frac{t(a)}{2}\, \mathrm{Cov}(V, v^2 \mid P)$ from x_μ, exactly as in REE.

Step 3: $G_{x^* s}$ (new BNE term). Differentiating G_{x^*} with respect to s and evaluating at

$x^* = 0$, $s = 0$, $\mu = 0$ ($\mathcal{E}_s = t(a)\, v$):

$$G_{x^*s} = t(a) \int \{u''(W_0)(V-P)^2\, H_P + u'(W_0)(V-P)\, \frac{\partial H_P}{\partial x^*} - u'(W_0)\, H_x\}\, v\, f(v)\, dv$$
$$= G_{xs}^{\mathrm{REE}} - t(a)\, u'(W_0) \int H_x\, v\, f(v)\, dv,$$

where G_{xs}^{REE} is the cross-derivative from the REE proof.

Now note that at the limit $H_x\,(\Theta(P), v, P, 0) = f(P \mid v, u)\, \lambda(P)$ (by the same supply-shift argument used in the proof of (124)). After averaging against $f_u(u)$ (per the convention above):

$$\int_{\mathbb{R}} \int_{\mathbb{R}} H_x(\Theta(P), v, P, 0)\, v\, f(v)\, f_u(u)\, dv\, du = \lambda(P)\, f(P)\, \mathbb{E}[v \mid P].$$

Combining with the REE expression from the proof of Lemma 15:

$$f(P)\, G_{x^*s} = u'(W_0)\, f(P)\, t(a) \Big[-\rho(a)\ \mathrm{Cov}((V-P)^2, v \mid P) - \lambda(P)\, \Sigma'_{Vv}(P) - \lambda(P)\, \mathbb{E}[v \mid P]\Big].$$

The first two terms are identical to REE; the third is new and arises from H_x.

Step 4: $G_{x^*x^*}$ (new BNE term). Write $G = \int \Phi\,(A+B)\,\mathcal{E}\, f\, dv$ with $\Phi = u'(W_0 + x^*(V-P))$, $A = (V-P)\, H_P$, $B = -x^*\, H_x$. The second derivative is $G_{x^*x^*} = \int [\Phi''\,(A+B) + 2\Phi'\,(A'+B') + \Phi\,(A''+B'')]\,\mathcal{E}\, f\, dv$, where primes denote x^*-derivatives.

At $x^* = 0$ ($\mathcal{E} = 1$): $B = 0$, $B' = -H_x$, and $B'' = 2H_{xx}(\Theta, v, P)$ (because $\frac{d^2}{d(x^*)^2}[-x^*\, H_x(\Theta - x^*)] = 2H_{xx}$ at $x^* = 0$).

The terms involving A and its derivatives reproduce G_{xx}^{REE} exactly. The new BNE terms are:

$$\Delta G_{x^*x^*} = -2u''(W_0) \int (V-P)\, H_x\, f\, dv + 2u'(W_0) \int H_{xx}\, f\, dv.$$

For the first integral, substituting $H_x\,(\Theta(P), v, P, 0) = f(P \mid v, u)\, \lambda(P)$ and averaging over u

gives $\iint (V-P)\, H_x\, f\, f_u\, dv\, du = \lambda(P)\, f(P)\, \mathbb{E}[V-P \mid P] = 0$ (efficiency).

For the second integral, we differentiate identity (124) in $\bar{\theta}$. Since $\lambda(P)$ is independent of $\bar{\theta}$ under Assumption 6 and the clearing-price density satisfies the transport equation $\frac{\partial f(P)}{\partial \bar{\theta}} = \frac{d}{dP}[\lambda(P)\, f(P)]$ (for the price shift $\partial \tilde{P}/\partial \bar{\theta} = \lambda(P)$):

$$\int_{\mathbb{R}}\int_{\mathbb{R}} H_{xx}(\Theta, v, P, 0)\, f(v)\, f_u(u)\, dv\, du = \frac{\partial}{\partial \bar{\theta}}[\lambda(P)\, f(P)] = \lambda(P)\, \frac{d}{dP}[\lambda(P)\, f(P)] = \Lambda(P)\, f(P),$$

where $\Lambda(P) \equiv \frac{\lambda(P)}{f(P)} \frac{d}{dP}[\lambda(P)\, f(P)]$.

Using the equilibrium characterization that the price is a deterministic function of the normally distributed sufficient statistic $P = \mathcal{P}(s_p)$, we can apply the change-of-variables formula $f(P) = f_S(s_p)/\mathcal{P}'(s_p)$ to express the logarithmic derivative $f'(P)/f(P)$ entirely in terms of local conditional moments. Since $f_S(s_p)$ is the density of $\mathcal{N}(0, \frac{\tau}{\tau_v \tau_p})$ and $\mathcal{P}'(s_p) = \frac{\tau_p}{\tau}\mathbb{E}[V'(v) \mid P]$, a straightforward calculation gives:

$$\frac{f'(P)}{f(P)} = \frac{1}{\mathcal{P}'(s_p)}\left[\frac{f_S'(s_p)}{f_S(s_p)} - \frac{\mathcal{P}''(s_p)}{\mathcal{P}'(s_p)}\right] = -s_p \frac{\tau_v}{\mathbb{E}[V'(v) \mid P]} - \frac{\mathbb{E}[V''(v) \mid P]}{\mathbb{E}[V'(v) \mid P]^2},$$

because $\frac{f_S'(s_p)}{f_S(s_p)} = -s_p \frac{\tau_v \tau_p}{\tau}$ and $\mathcal{P}''(s_p) = \left(\frac{\tau_p}{\tau}\right)^2 \mathbb{E}[V''(v) \mid P]$. Therefore,

$$\Lambda(P) = \lambda(P)\lambda'(P) + \lambda(P)^2 \frac{f'(P)}{f(P)} = \lambda(P)\, \lambda'(P) - \lambda(P)^2 \left(s_p \frac{\tau_v}{\mathbb{E}[V'(v) \mid P]} + \frac{\mathbb{E}[V''(v) \mid P]}{\mathbb{E}[V'(v) \mid P]^2}\right).$$

Combining:

$$f(P)\, \Delta G_{x^* x^*} = 2\, u'(W_0)\, f(P)\, \Lambda(P).$$

So the full BNE expression is

$$f(P)\, G_{x^* x^*} = u'(W_0)\, f(P)\, \Big[\pi(a)\rho(a)\ \mathrm{Skew}(V \mid P) + 2\rho(a)\, \lambda(P)\, \Sigma_{VV}'(P) \\ + \lambda(P)\, \lambda'(P) + 2\Lambda(P)\Big].$$

Step 5: Combining terms. Assembling $\delta = x_\mu + \frac{1}{2t} x_{ss}$ with $x_{ss} = -\frac{1}{G_{x^*}}[G_{ss} + 2\beta\, G_{x^* s} + \beta^2\, G_{x^* x^*}]$, and using $-1/G_{x^*} = 1/(u'(W_0)\, f(P)\, D)$, the $\mathrm{Cov}(V, v^2 \mid P)$ terms cancel (as in REE), and the surviving terms yield the expression in the lemma. ■

H.6.9 Proof of Theorem 8

Proof. The proof proceeds identically to the proof of Theorem 7, with $\rho(a) + \kappa$ replaced by $\rho(a) + 2\kappa$ throughout. We sketch the key steps.

Step 1. By Lemma 14 and Stein's lemma, the trading intensity factorizes as

$$\beta(P, a) = \eta(P) \cdot \frac{t(a)}{\rho(a) + 2\kappa(P)}, \qquad \eta(P) = \frac{\mathbb{E}[V'(v) \mid P]}{\tau\ \mathrm{Var}(V(v) \mid P)}.$$

The coefficient $\delta(P, a)$ is given by Lemma 16.

Step 2. Market clearing defines the sufficient statistic s_p exactly as in Theorem 7, with weights $\omega(a) = \beta(a, P) / \int_0^1 \beta(b, P)\, db$.

Step 3. Substituting the factorized form of β into the precision formula (123), the $\eta(P)$ factors cancel in the signal-noise term. Under Assumption 6, $\tilde{\theta}(P)/\eta(P) = \tau/\xi$ is P-independent, so the $\eta(P)$ factors cancel in the supply-noise term as well:

$$\tau_p = \frac{A(\kappa)^2}{B(\kappa) + \tau^2/(\xi^2 \tau_u)},$$

where $A(\kappa) = \int_0^1 \frac{t(a)}{\rho(a)+2\kappa}\, da$ and $B(\kappa) = \int_0^1 \frac{t(a)}{(\rho(a)+2\kappa)^2}\, da$. Thus τ_p is P-independent.

Step 4. Standard differentiation of the market-clearing condition yields $\lambda(P) = \frac{\tau_p}{\tau}\, \mathbb{E}[V'(v) \mid P]/(\eta(P)\, A(\kappa))$. Substituting $\eta(P)$ and using $\lambda = \kappa\ \mathrm{Var}(V|P)$ gives $\kappa = \tau_p / A(\kappa)$, which is P-independent.

Step 5. The efficiency condition $\mathbb{E}[V(v) \mid P] = P$ determines the price function (109).

Step 6. Combining $\kappa = \tau_p / A(\kappa)$ with the expression for τ_p, and writing $\tau = \tau_v + \kappa\, A(\kappa)$, yields the single fixed-point equation (114). ■

H.6.10 Proof of Corollary 7

Proof. The proof is identical to that of Corollary 5, with $\rho(a) + \kappa$ replaced by $\rho(a) + 2\kappa$ throughout. ■

H.6.11 Proof of Proposition 18

Proof. We give the argument for REE; the BNE case is identical with $\rho(a) + \kappa$ replaced by $\rho(a) + 2\kappa$ throughout.

Part 1. In the fixed-point equation (100), the denominator includes the term $(\tau_v + \kappa A(\kappa))^2/(\xi^2 \tau_u)$. Since $\tau_u^{-1} = \sigma_u^2 + \sigma_{\bar{u}}^2 \to \infty$ as $\sigma_{\bar{u}}^2 \to \infty$, this term diverges for any $\kappa > 0$, while the numerator $A(\kappa)$ remains bounded. Hence $\kappa \to 0$. It follows that $A(\kappa) \to A(0)$, $B(\kappa) \to B(0)$, $\tau_p = \kappa A(\kappa) \to 0$, $\tau \to \tau_v$, and $\xi \to \xi_0$.

Part 2. As noted in the Market quality subsection, the actual precision of the price signal is obtained by replacing τ_u^{-1} with σ_u^2 in the derivation of Theorem 7 (Step 3 of the proof). This yields

$$\tau_p^*(\kappa) = \frac{A(\kappa)^2}{B(\kappa) + \dfrac{\tau^2 \sigma_u^2}{\xi^2}}.$$

Substituting the limits from Part 1 gives (119). ■

H.6.12 Proof of Corollary 8

Proof. The limiting precision (119) has the form $\tau_p^* = \phi(A(0), B(0))$ where $\phi(A, B) = A^2/(B + c)$ with $c = c_0 \sigma_u^2 \geq 0$. Its partial derivatives satisfy $\phi_A = 2A/(B + c) > 0$ and $\phi_B = -A^2/(B + c)^2 < 0$—the same signs as for the price-taking precision A^2/B. Since $A(0)$ and $B(0)$ depend on $t(a)$ and $\rho(a)$ in the same way as in the price-taking equilibrium ($\kappa = 0$), the Gateaux derivatives of $\mathcal{I}^*$ with respect to $W_0(a)$ and $t(a)$ inherit the signs established in Propositions 2–3 and Corollary 1.

Since $\mathcal{I}$ depends continuously on $\sigma_{\bar{u}}^2$ (through κ), the strict inequalities in these comparative statics are preserved for all $\sigma_{\bar{u}}^2$ above some finite threshold. ■

I Risk premium in the CRRA–log-normal example

We derive the aggregate risk premium $\psi(P)$ for the CRRA–log-normal specialization of Section 6 and examine its dependence on the HHI of the wealth distribution.

I.1 Derivation of $\psi(P)$

With $V(v) = e^v$, $\rho(a) = \eta/W_0(a)$, $t(a) = t$, the conditional moments appearing in the demand coefficient $\delta(a, P)$ from (16) specialize as follows. The posterior $v \mid P \sim N(\mu_P, \tau^{-1})$ with $\mu_P = \ln P - 1/(2\tau)$ and $\tau = \tau_v + \tau_p$. Standard log-normal moment formulas give

$$\mathrm{Var}[V(v) \mid P] = P^2(e^{1/\tau} - 1), \tag{125}$$

$$\frac{\mathrm{Sk}[V(v) \mid P]}{\mathrm{Var}[V(v) \mid P]} = P\,(e^{1/\tau} - 1)(e^{1/\tau} + 2),$$

$$\frac{\mathbb{E}[v(V(v) - P)^2 \mid P]}{\mathrm{Var}[V(v) \mid P]} = \ln P + \frac{3}{2\tau}. \tag{126}$$

From (19), $\beta(a, P) = W_0(a)\, c(P)/\eta$ where $c(P) = t/(\tau\, P\,(e^{1/\tau} - 1))$. Prudence is $\pi(a) = (\eta{+}1)/W_0(a)$. Integrating $\delta(a, P)$ over a, each of the three terms in (16) picks up $\int_0^1 W_0(a)\, da = \bar{W}$, because $\beta^2\pi \propto W_0(a)$, $\beta \propto W_0(a)$, and $1/\rho \propto W_0(a)$. The market clearing condition $\int_0^1 \delta(a, P)\, da + s_p \int_0^1 \beta(a, P)\, da = \Theta(P)$ then yields, after dividing by $\bar{W}/\eta$,

$$\frac{\psi(P)}{\mathrm{Var}[V \mid P]} = -s_p\, c(P) - \frac{(\eta + 1)\, c(P)^2}{2t\,\eta}\, \frac{\mathrm{Sk}[V \mid P]}{\mathrm{Var}[V \mid P]} + c(P)\, \frac{\mathbb{E}[v(V - P)^2 \mid P]}{\mathrm{Var}[V \mid P]} + \frac{\eta\, \Theta(P)}{\bar{W}}.$$

Substituting (125)–(126) and $s_p = \tau(\ln P - 1/(2\tau))/\tau_p$, one obtains

$$\psi(P) = P\left[\frac{t}{\tau}\left(\ln P + \frac{3}{2\tau}\right) - \mathrm{HHI}\left(\ln P - \frac{1}{2\tau}\right)\right] - \frac{(\eta+1)\,t\,P\,(e^{1/\tau}+2)}{2\eta\,\tau^2} + \frac{\eta\,\Theta(P)}{\bar{W}}\,P^2(e^{1/\tau}-1), \tag{127}$$

where we have used $\tau_p = t/\mathrm{HHI}$ from (20).

I.2 Dependence on HHI

The aggregate risk premium (127) depends on HHI through two channels:

1. *Direct channel*: the term $-\mathrm{HHI}\,P(\ln P - 1/(2\tau))$, which appears because the sufficient statistic s_p depends on $\tau_p = t/\mathrm{HHI}$ (so $t/\tau_p = \mathrm{HHI}$);
2. *Indirect channel*: the posterior precision $\tau = \tau_v + t/\mathrm{HHI}$ enters every conditional moment through $e^{1/\tau}$.

Differentiating (127) with respect to HHI, the direct channel increases $|\psi|$ when $\ln P > 1/(2\tau)$ (i.e., when the asset price exceeds its unconditional median) and decreases it otherwise. The indirect channel operates through $\partial\tau/\partial\mathrm{HHI} = -t/\mathrm{HHI}^2 < 0$: a higher HHI lowers the posterior precision, which raises the conditional variance $P^2(e^{1/\tau} - 1)$ and the conditional third central moment, generating competing effects on ψ. In contrast to $\mathcal{I}$ and $\mathcal{L}$—whose comparative statics with respect to wealth inequality are unambiguous (Proposition 2)—the comparative statics of ψ with respect to HHI are richer and can be nonmonotone.

J Average vs. aggregate quantities in large economies

This section illustrates, in the simplest possible setting, how individual trades and risk premia can vanish as the number of agents grows while their economy-wide aggregates remain finite and nonzero. The setting is a standard competitive economy with n identical, uninformed, risk-averse agents—no private information and no features specific to CHILE. The phenomenon is a generic consequence of splitting a fixed amount of aggregate risk among an increasing number of agents.

Consider an economy with n identical agents, and let $\mu = 1/n$ denote the mass of each agent. Each agent has initial wealth W_0 and preferences $u(\cdot)$ over terminal wealth, where u is increasing, strictly concave, and twice continuously differentiable. There is a risky asset with payoff $V(v)$ and supply $\Theta(P)$, and a risk-free asset with gross return normalized to 1. Agents have no private information: each agent observes only the price P and chooses demand x to maximize $\mathbb{E}[u(W_0 + x(V(v) - P))]$.

By symmetry, all agents hold the same position $x^*(\mu)$ in equilibrium. Market clearing requires that aggregate demand equals supply:

$$n \cdot x^*(\mu) = \Theta(P), \qquad \text{i.e.,} \qquad x^*(\mu) = \mu\, \Theta(P).$$

Substituting the market-clearing condition into the first-order condition, the equilibrium price $P(\mu)$ is determined by

$$G(\mu, P) \;\equiv\; \mathbb{E}\Big[u'(W_0 + \mu\, \Theta(P)\,(V(v) - P))\,(V(v) - P)\Big] = 0. \tag{128}$$

Proposition 19 (Individual trade)**.** *Let $\rho = -u''(W_0)/u'(W_0)$ denote absolute risk aversion at W_0. As $\mu \to 0$ (equivalently, $n \to \infty$):*

(i) *The equilibrium price satisfies*

$$P(\mu) = \mathbb{E}[V(v)] - \mu\,\rho\,\Theta(\mathbb{E}[V(v)])\,\mathrm{Var}(V(v)) + O(\mu^2). \tag{129}$$

(ii) *Individual demand vanishes:* $x^*(\mu) = \mu\,\Theta(\mathbb{E}[V(v)]) + O(\mu^2) \;\to\; 0.$

(iii) *Aggregate demand converges to a finite, generically nonzero limit:* $n\,x^*(\mu) = \Theta(\mathbb{E}[V(v)]) + O(\mu) \;\to\; \Theta(\mathbb{E}[V(v)]).$

The key to understanding why vanishing individual demands are compatible with a finite aggregate is the distinction between *averages* and *aggregates*. The average demand across agents is $\frac{1}{n}\sum_{i=1}^{n}|x_i^*| = |x^*(\mu)| \to 0$, which indeed vanishes as the proposition states. The aggregate demand is $\sum_{i=1}^{n}|x_i^*| = n\,|x^*(\mu)|$—the product of a diverging number of agents and a vanishing per-agent demand—and this product converges to a finite limit.

Proposition 20 (Individual risk premium)**.** *Under the conditions of Proposition 19, as* $\mu \to 0$*:*

(i) *The per-agent risk premium vanishes:*

$$\mathbb{E}[V(v)] - P(\mu) = \mu\,\rho\,\Theta(\mathbb{E}[V(v)])\,\mathrm{Var}(V(v)) + O(\mu^2) \;\to\; 0.$$

(ii) *The aggregate risk premium converges to a finite, generically nonzero limit:*

$$\frac{1}{\mu}\Big(\mathbb{E}[V(v)] - P(\mu)\Big) = \rho\,\Theta(\mathbb{E}[V(v)])\,\mathrm{Var}(V(v)) + O(\mu).$$

The economics are transparent: a finite amount of aggregate risk, $\Theta(\mathbb{E}[V(v)])\,\mathrm{Var}(V(v))$, is shared among n agents. Each agent bears a $1/n$ share and earns a proportionally small premium. The per-agent premium $\mathbb{E}[V(v)] - P(\mu) = O(\mu)$ vanishes, yet the economy-wide total $n\,(\mathbb{E}[V(v)] - P(\mu))$ remains finite, exactly as with aggregate demand.

J.1 Proofs

Proof of Proposition 19. We expand $P(\mu)$ around $\mu = 0$ using the implicit function theorem applied to $G(\mu, P) = 0$ defined in (128).

Zeroth order. At $\mu = 0$, the per-agent holding is $x^*(0) = 0$, so terminal wealth equals W_0 with certainty. Equation (128) becomes $u'(W_0)\,\mathbb{E}[V(v) - P_0] = 0$, which gives $P_0 \equiv P(0) = \mathbb{E}[V(v)]$.

First order. Differentiating $G(\mu, P(\mu)) = 0$ with respect to μ and evaluating at $\mu = 0$:

$$\left.\frac{\partial G}{\partial \mu}\right|_{\mu=0} + \left.\frac{\partial G}{\partial P}\right|_{\mu=0} \cdot P'(0) = 0.$$

For the partial with respect to P: at $\mu = 0$ the only surviving term is $\mathbb{E}[u'(W_0)\cdot(-1)] = -u'(W_0)$, since the chain-rule term through u'' carries a factor of μ that vanishes. For the partial with respect to μ:

$$\left.\frac{\partial G}{\partial \mu}\right|_{\mu=0} = \mathbb{E}\Big[u''(W_0)\,\Theta(P_0)\,(V(v) - P_0)^2\Big] = u''(W_0)\,\Theta(\mathbb{E}[V(v)])\,\mathrm{Var}(V(v)).$$

Solving for $P_1 \equiv P'(0)$:

$$P_1 = -\left.\frac{\partial G/\partial \mu}{\partial G/\partial P}\right|_{\mu=0} = \frac{u''(W_0)}{u'(W_0)}\,\Theta(\mathbb{E}[V(v)])\,\mathrm{Var}(V(v)) = -\rho\,\Theta(\mathbb{E}[V(v)])\,\mathrm{Var}(V(v)),$$

which establishes part (i). Parts (ii) and (iii) follow from the market-clearing condition $x^*(\mu) = \mu\,\Theta(P(\mu))$:

$$x^*(\mu) = \mu\,\Theta(\mathbb{E}[V(v)] + \mu\,P_1 + O(\mu^2)) = \mu\,\Theta(\mathbb{E}[V(v)]) + O(\mu^2),$$

so $x^*(\mu) \to 0$ and $n\,x^*(\mu) = \Theta(\mathbb{E}[V(v)]) + O(\mu)$. ■

Proof of Proposition 20. Immediate from the price expansion (129): part (i) is $\mathbb{E}[V(v)] - P(\mu) = -\mu\,P_1 + O(\mu^2) = \mu\,\rho\,\Theta(\mathbb{E}[V(v)])\,\mathrm{Var}(V(v)) + O(\mu^2)$, and part (ii) follows by dividing by μ. ■