Funding Constraints and Informational Efficiency^{*}

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Abstract

We analyze a tractable rational expectations equilibrium model with margin constraints. We argue that constraints affect and are affected by informational efficiency, leading to a novel amplification mechanism. A decline in wealth tightens constraints and reduces investors' incentive to acquire information, lowering price informativeness. Lower informativeness, in turn, increases the risk borne by financiers who fund trades, leading them to further tighten constraints faced by investors. This information spiral leads to (i) significant increases in risk premium and return volatility in crises, when investors' wealth declines, (ii) complementarities in information acquisition in crises, and (iii) complementarities in margin requirements.

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1 Introduction

One of the basic tenets of financial economics is that market prices aggregate investors' information. The core of the argument is that investors acquire information about future asset values and trade on it, thereby impounding that information into price. This argument presupposes that investors have incentives to acquire information and the capacity to trade on it, where each of these factors is crucially affected by investors' ability to fund their trades. Thus, an important question arises: how do funding constraints faced by investors affect price informativeness? Conversely, since lower informativeness might have an effect on the financier's risk of funding a trade, another important question is: how does price informativeness affect the tightness of funding constraints? Answering these questions requires a model in which price informativeness and funding constraints are jointly determined in equilibrium. Our paper develops such a model and examines its implications for asset pricing.

The main challenge in studying the interplay between funding constraints and informational efficiency is that most noisy rational expectation equilibrium (REE) models, which are instrumental in analyzing informational efficiency, cannot accommodate constraints in a tractable manner.¹ Our first contribution is, developing a tractable REE model with *general portfolio constraints* that can depend on prices.

We then apply our methodology to study a model in which portfolio constraints arise because of margin requirements set by financiers. This analysis underpins the second contribution of the paper, showing that investors' funding both affects and is affected by informational efficiency, which leads to a novel amplification mechanism that we call the *information spiral*. In our mechanism, a decline in wealth tightens constraints and reduces investors' incentive to acquire information, lowering price informativeness. Lower informativeness, in turn, increases the risk borne by financiers who fund trades, leading them to further tighten constraints faced by investors. As a result, risk premium, volatility, and the Sharpe ratio rise significantly when investors' wealth falls.

We consider a canonical CARA-Normal REE model in which some investors, whom we call "specialists", to highlight their ability to produce information, first acquire private signals about a

¹Two noteworthy exceptions are Yuan (2005) and Nezafat, Schroder, and Wang (2017); these authors analyze borrowing constraints and short-sale constraints, respectively.

risky asset's fundamental value, and then, trade the asset with other investors ("nonspecialists") for profit and also for hedging their endowment shocks. The novelty is that we allow for general portfolio constraints: specialists can trade up only to some maximal long and short positions of the risky asset, and these portfolio constraints can depend on price. This general price-dependent specification of portfolio constraints subsumes many types of real-world trading constraints (e.g., short-sale constraints, borrowing constraints, margin requirements). Without constraints, the model is standard: (i) the equilibrium price is linear in the fundamental value and aggregate endowment shock, and (ii) investors' initial wealth is irrelevant for asset prices.

Under portfolio constraints, the financial market equilibrium is as follows. (i) Although the price function may not be linear, it is informationally equivalent to a linear combination of the fundamental value and the aggregate endowment shock; hence, inference remains tractable. (ii) Specialists' initial wealth matters for asset prices provided that it affects constraints. With the methodology of solving equilibrium with constraints at hand, we turn to study the paper's primary concern: the reinforcing interaction between constraints and informational efficiency.

We begin with an analysis of how constraints affect informational efficiency. Without further specifying the source or form of constraints, we show that they hinder such efficiency. It is intuitive that, when constraints become tighter, specialists must take smaller positions and thus profit less on their private information. Anticipating the reduced scope for profit, they acquire less information ex ante. As specialists acquire less information, the price becomes less informative about asset fundamentals in equilibrium. And to the extent that specialists' wealth relaxes their constraints, a wealth effect emerges in our model despite investors' absolute risk aversion being constant: lower wealth impedes information acquisition, and hence, reduces informational efficiency.

Next we study the reverse channel of informational efficiency affecting constraints. Motivated by real-world margin requirements, we follow Brunnermeier and Pedersen (2009) in assuming that specialists finance their positions through collateralized borrowing from financiers who require margins that control their value-at-risk (VaR).² We show that lower informational efficiency leads to tighter margins. Here, it is intuitive that, when prices are less informative, the price tracks fundamentals less

 $^{^{2}}$ Our main results are robust to alternative risk-based margins, such as tail value-at-risk (TVaR) and expected shortfall (ES).

closely, which implies a greater risk of the trade they finance, leading them to set higher margins.

When we combine these analyses, we obtain an information-based amplification mechanism, illustrated in Figure 1, which we call the *information spiral*. Tighter funding constraints reduce the information acquired by specialists, which reduces informational efficiency; reduced informational efficiency, in turn, leads to higher margins, which tightens specialists' constraints.³

Figure 1: Amplification mechanism



This information spiral has two key implications. First, it gives rise to new sources of strategic complementarities in financial markets. The first source comes from specialists' decisions to acquire information: a reduction in information acquired by other specialists makes prices less informative, which increases the margin requirements faced by the specialist and induces him to acquire less information. The second complementarity is found in financiers' margin requirements: an increase in margins required by other financiers discourages their specialists from acquiring information. As a result, prices become less informative and the financier of interest responds by setting higher margins.

The second key implication of the information spiral is that a negative shock to specialists' wealth is amplified and causes larger changes in asset prices than in a model with fixed signal quality and/or fixed margin requirements. A drop in specialists' wealth tightens their constraints and leads to a drop in price informativeness. The effect of the wealth drop is reinforced via the information

 $^{^{3}}$ Our baseline model highlights the *information acquisition* channel of interaction between price informativeness and investors' constraints. An alternative setting in Appendix B highlights a complementary *information aggregation* channel. The interaction between investors' constraints and information efficiency is similar in both settings. In the alternative setting the noise in prices comes from exogenous noise traders who are not affected by constraints. Tightening the funding constraints of informed specialists reduces their aggregate trading intensity but not that of the noise traders. This hurts price informativeness, even for a given quality of private information. The reduction in price informativeness leads to an increase in margins, for reasons as in the baseline model.

spiral. As a result, when specialists' wealth is low, which we interpret as a crisis period, uncertainty is heightened, causing risk premium, return volatility and the Sharpe ratio to rise. These results match empirical observations made during crisis periods such as the 2007–2009 global financial crisis.

Our mechanism provides a new crisis narrative and highlights an important role of specialist investors such as hedge funds—namely in enhancing price informativeness. Consistent with empirical evidence for equities (Barrot, Kaniel, and Sraer (2016)), in our model, as a crisis deepens, specialists face tighter portfolio constraints and become less capable of holding risky assets; meanwhile, nonspecialists like commercial banks and retail investors step up to provide liquidity. Nonetheless, risk premia, volatility, and the Sharpe ratio are elevated. We claim that this is because specialists are instrumental in making price informative (see Koijen, Richmond, and Yogo (2019)), and tightened constraints hinder them from doing so. Consistent with this claim, we empirically document a negative correlation between measures of price informativeness and constraints on specialists. In short, complementary to existing intermediary-based crisis narratives in which nonspecialists are restricted from participating in the asset market, our mechanism shows how intermediaries matter even in markets where all investors can freely participate.⁴

This paper makes several methodological contributions. We present and solve a REE model with general portfolio constraints and compute the marginal value of information for a specialist facing these constraints in closed-form using stochastic calculus techniques.⁵ In our main application we consider constraints arising from margin requirements, but one can also utilize our methodology to study other types of constraints.⁶

Related Literature

This paper lies at the intersection of various strands of literature. It shares the emphasis of seminal studies that address the role played by financial markets in aggregating and disseminating information,

⁴Restricted participation is a central assumption in intermediary asset-pricing models, in which intermediaries are typically the only agents who can hold risky assets. During crisis periods, risk premia of these assets rise sharply, or prices drop substantially because otherwise the constrained intermediaries would not be able to hold the entire supply of these assets. See He and Krishnamurthy (2018) for a survey on this literature.

⁵By using stochastic calculus, we compute the marginal value of information for a specialist facing general portfolio constraints directly, without first calculating the value of information and then differentiating it with respect to specialist's choice of signal precision.

 $^{^{6}}$ In Appendix D, we study the case in which specialists have some risky assets as initial wealth, which gives rise to a form of borrowing constraints as in Yuan (2005).

which include Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980), and Diamond and Verrecchia (1981). In these models, it is generally assumed that investors can borrow or lend freely at the riskless rate—in other words, there are no funding constraints. We contribute to this literature by developing an REE model that incorporates general portfolio constraints. Some particular types of portfolio constraints have been examined before: Yuan (2005), Venter (2015), and Yuan (2006) study REE models with borrowing constraints, short-sale constraints, and both constraints, respectively. Albagli, Hellwig, and Tsyvinski (2011) derive various asset-pricing implications in a model with riskneutral investors, exogenous portfolio constraints and exogenous information. Our work differs from these papers in that we study investors' information acquisition problem and focus on the interplay between the tightness of constraints and the equilibrium informational efficiency.

Closely related to our work is Nezafat et al. (2017), who focus on how short-sale constraints affect information production and asset prices. Our paper differs in two important dimensions. First, our methodology extends their work to explore price-dependent constraints of a more general nature, allowing us to consider constraints resulting from risk-based margin requirements. Second, and more importantly, in our paper informational efficiency affects constraints, which is not present in their paper.

Our work is related to the literature on information acquisition in REE models. Grossman and Stiglitz (1980), Verrecchia (1982), Peng and Xiong (2006), and Van Nieuwerburgh and Veldkamp (2009) study investor's information acquisition problem in the case of no funding constraints. Peress (2004) and Breugem and Buss (2019) use approximation and numerical methods, respectively, to investigate the effect of investors' wealth on information acquisition in a setting with investors who exhibit constant relative risk aversion (CRRA). Our tractable model also features wealth effects, despite the investors having constant absolute risk aversion, because investors' wealth relaxes their funding constraints. Moreover, we can derive all our results analytically in the "crisis" limit when investors' wealth is small without relying on approximations. Since our paper speaks to the evidence in crises, when changes in equilibrium quantities are highly nonlinear, not relying on approximations is important.

In addition, we contribute to the literature on strategic complementarities in information acquisition, for example, Veldkamp (2006), Hellwig and Veldkamp (2009), Garcia and Strobl (2011), Ganguli and Yang (2009),Goldstein and Yang (2015), Avdis (2016),Huang (2015), and Dow, Goldstein, and Guembel (2017). In particular, the focus of complementarity between two groups of agents via information acquisition and trading is shared in Goldstein and Yang (2015). In Goldstein and Yang (2015), when one group of investors learns less about (and trades less aggressively on) one component of fundamentals, the price becomes less informative about this component. This increases uncertainty for the other group, which in turn, learns less about (and trades less aggressively on) another component. As a result, acquiring information about the two components of fundamentals is a strategic complement. We view our analysis as complementary to theirs: at the aggregate level, our paper shares the feature that more learning by one group of agents (specialists in our model) reduces the uncertainty faced by the other group (financiers in our model), whose response (more financing) in turn reinforces learning by the first group. However, there are important differences in the underlying mechanism. In our model, financiers do not produce information. The complementarity comes from financiers' funding decisions to investors. Moreover, complementarity in our paper arises even without multiple (learnable) components of fundamentals.

Except for Dow et al. (2017), the main distinguishing feature of our model is that complementarities arise in bad times, and therefore, our results have business-cycle predictions. While the mechanism in Dow et al. (2017) also generates complementarities during bad times, our paper differs from theirs in two dimensions. First, the predictions are different because bad times mean that investors, or the financial sector in general, have low wealth in our paper, while in theirs bad times mean low productivity in the real sector. Second, the amplification mechanism acts through firm managers' learning to make real investment decisions in Dow et al. (2017), whereas in ours, it is via financiers' funding decisions.

Our paper is also related to the literature on secondary financial markets as a source of information for decision makers; see Bond, Edmans, and Goldstein (2012) for a survey. We contribute to this literature by studying how financiers can use the information in prices to set their margins, and we find that lower informational efficiency leads to tighter margins.

Our work contributes to the intermediary asset-pricing literature on the effect of specialists' wealth and the associated amplification mechanisms. For example, Xiong (2001) and Kyle and Xiong (2001) study wealth constraints as amplification and spillover mechanisms, respectively. Gromb and

Vayanos (2002, 2017) develop an equilibrium model of arbitrage trading with margin constraints to explain contagion. Brunnermeier and Pedersen (2009) examine how funding liquidity and market liquidity reinforce each other. He and Krishnamurthy (2011, 2013) and Brunnermeier and Sannikov (2014) study how declines in an intermediary's capital reduce her risk-bearing capacity and lead to higher risk premia and conditional volatility; see also He and Krishnamurthy (2018) for a survey of the topic. None of these papers studies the interaction of investors' wealth (or constraints) and informational efficiency, which is the crux of our paper. Furthermore, unlike most of the models in this literature, our amplification mechanism does not stem from restricted participation of nonspecialists and hence can apply to commonly traded assets such as equities.

Finally, Dow and Han (2018) also study how constraints on information-producing specialists affect equilibrium prices. They show that as specialists become more constrained, firms with highquality assets are unwilling to sell their assets in the market because prices do not reveal their true quality. The quality of traded assets thus deteriorates, leading to market freezes and a large decline in asset prices. While they focus on the endogenous supply of risky assets of heterogeneous quality in the primary market, we show a different amplification mechanism in the secondary market when the supply and quality of traded assets are fixed.

2 An REE model with general portfolio constraints

In this section we develop a model with general portfolio constraints. In Section 3, we will apply our model to study constraints that arise from margin requirements.

2.1 Setup

There are three dates (i.e., $t \in \{0, 1, 2\}$) and two assets. The risk-free asset has exogenous (net) return normalized to zero. The payoff (fundamental value) of the risky asset is $f = v + \theta$ (which is paid at date 2), where v is the learnable (i.e., information about which can be acquired) component of fundamentals, $v \sim N(0, \tau_v^{-1})$, and θ is the unlearnable component of fundamentals, $\theta \sim N(0, \tau_{\theta}^{-1})$, and is independent of v. The aggregate supply of the asset is assumed to be constant 1 unit. The economy is populated by a unit continuum of *specialists*, indexed by $i \in [0, 1]$, with identical CARA preferences over terminal wealth with absolute risk aversion γ . There is also a unit continuum of *nonspecialists* with CARA preferences with absolute risk aversion γ_m .⁷ The fundamental distinction between the specialists and the nonspecialists is that only the specialists have the ability to acquire information about the risky asset, which happens at t = 0. Trading occurs at t = 1, and all agents consume at t = 2.

Specialists trade the risky asset for hedging and profit reasons. Specifically, at date 2, each specialist receives a random, non-tradable, and non-pledgeable endowment b_i , which has a payoff that is correlated with the unlearnable component of the risky asset's payoff, θ . We assume that the endowment is given by $b_i = e_i \theta$.⁸ The coefficient e_i measures the sensitivity of the endowment shock to the payoff of the risky asset and is known to the specialist at t = 1. Hereafter, we will refer to e_i as the endowment shock of specialist *i*. The specialist *i*'s endowment shock e_i has systematic and idiosyncratic components: $e_i = z + u_i$. Both components are normally distributed and independent of v and θ , with $z \sim N(0, \tau_z^{-1})$ and $u_i \sim N(0, \tau_u^{-1})$. Moreover, idiosyncratic shocks u_i are independent across specialists and independent of z. This formulation implies that there is uncertainty about the aggregate endowment shock z, which will create noise in the price.

At date 1, each specialist *i* receives a signal $s_i = v + \epsilon_i$, where the ϵ_i are independent across specialists with $\epsilon_i \sim N(0, \tau_{\epsilon_i}^{-1})$. The precision of his private signal τ_{ϵ_i} is optimally chosen by specialist *i* at date 0, subject to an increasing, strictly convex, twice continuously differentiable cost function $C(\tau_{\epsilon_i} - \underline{\tau_{\epsilon}})$. We assume that this cost function is identical for all specialists and that C(x) = 0 for x = 0. That is, there is no cost of acquiring information with quality below $\underline{\tau_{\epsilon}} > 0.^9$ When forming their expectations about the fundamental, specialists use all the information available to them. The information set of specialist *i* at time 1 is $\mathcal{F}_i = \{p, s_i, e_i\}$, where *p* is the equilibrium price at time 1. Nonspecialists face no endowment shocks and receive no signals about the asset payoff. Hence, the nonspecialists' information set at time 1 is $\mathcal{F}_m = \{p\}$.

⁷This specification of nonspecialist nests two commonly used model setups. When $\gamma_m = 0$, our nonspecialist is riskneutral, as in Vives (1995). When $\gamma_m = \infty$, the nonspecialist does not trade, hence, our model is equivalent to a model without a nonspecialist.

⁸The literature has also considered specification when endowment shocks are correlated with a learnable component of asset payoff (e.g., Ganguli and Yang (2009) and Manzano and Vives (2011)). The particular choice is not crucial for our results. In an earlier version of the paper, we showed that our results hold in the alternative specification as well.

⁹This is equivalent to assuming that specialists are endowed with information with quality $\underline{\tau}_{\epsilon}$ and only the incremental information is subject to a cost $C(\cdot)$. This assumption is technical and $\underline{\tau}_{\epsilon}$ can be arbitrary small: we need it to make sure that τ_{ϵ} is bounded away from zero in equilibrium, even when specialists are fully constrained.

Constraints. The specialists in our model—but not the nonspecialists—are subject to general portfolio constraints.¹⁰ Given the price p, the minimum and maximum positions that a specialist can take are a(p) and b(p), respectively, with a(p) < 0 < b(p). The functions a(p) and b(p) may depend on specialists' initial wealth W_0 and other aggregate equilibrium variables, such as volatility of returns. In short: at date 1, specialists solve the problem

$$\max_{x_i(p,s_i,e_i)} E[-\exp(-\gamma W_i) \mid p, s_i, e_i],$$
subject to $a(p) \le x_i(p, s_i, e_i) \le b(p), \text{ where } W_i = W_0 + x_i(v + \theta - p) + e_i\theta.$

$$(1)$$

The last equation above states that the terminal wealth of specialist i is the sum of his initial wealth, the profit or loss from trading the risky asset, and his endowment. Similarly, the nonspecialists at date 1 solve

$$\max_{x_m(p)} E[-\exp(-\gamma_m W_m) \mid p],$$
(2)
where
$$W_m = W_{0,m} + x_m(v + \theta - p).$$

Finally, the equilibrium price is set to clear the market as follows:

$$\int x_i(p, s_i, e_i)di + x_m(p) = 1.$$
(3)

We make the following two assumptions regarding the model parameters.

Assumption 1. $\tau_u^2 \tau_\theta^2 < 3\gamma^2 (\tau_u + \tau_z) \tau_v.$

This assumption is needed to ensure the uniqueness of financial market equilibrium.¹¹

Assumption 2. $\tau_{\theta} \min(\tau_u, \tau_z) > 2\gamma^2$.

This assumption is needed to guarantee that ex-ante utility is well defined. We discuss this in

¹⁰We assume the nonspecialists are unconstrained because our focus is on the interplay between specialists' constraints and informational efficiency. Nevertheless, our model remains tractable if the nonspecialists are also subject to constraints.

¹¹If the condition does not hold, there might be up to three equilibria in the financial market. The multiplicity can arise because traders use information about their endowments to make inferences about the noise in the price (see Ganguli and Yang (2009) and Manzano and Vives (2011)). Since this source of multiplicity is well-understood in the literature, we do not analyze it here and instead focus on our amplification mechanism.

more details in Proof of Lemma $1.^{12}$

We finish this subsection with a discussion of the economic realism of the model. We think of specialists in our model as hedge funds and nonspecialists as households and commercial banks. There are two important features of specialists: relative to nonspecialists, (i) they have an advantage in producing information, and (ii) portfolio constraints matter more for them. Consistent with (i), Koijen et al. (2019) find that hedge funds are relatively more important than other groups of investors for incorporating information into prices. In addition, in practice, hedge funds rely more on external financing, which is often subject to margin constraints (see Brunnermeier and Pedersen (2009)). Importantly, the presence of nonspecialists who can freely invest in the risky asset makes our results applicable in markets in which both sophisticated and unsophisticated investors trade, such as in the equities market.

2.2 Financial market equilibrium at t = 1

We first solve for equilibrium in the unconstrained setting (i.e., when $a(p) = -\infty$ and $b(p) = \infty$), which was studied previously in Nezafat et al. (2017).¹³ We review this setting here because it is an important benchmark in characterizing the equilibrium with constraints.

2.2.1 Unconstrained setting

Our first proposition characterizes the unconstrained equilibrium and its key features. From here on, we use superscript "u" for variables characterizing the unconstrained setting. The corresponding variables without superscripts are used for the constrained setting.

Proposition 1. (Financial market equilibrium without portfolio constraints) Suppose specialists have identical signal precision τ_{ϵ} . Then there exists a unique linear rational expectations equilibrium in which the price is informationally equivalent to a statistic $\phi^u = v - \frac{z}{\beta^u}$, where β^u is the unique root of a cubic equation (17) in Appendix. Moreover, β^u increases with τ_{ϵ} , the precision of specialists' information.

¹²See also Vayanos and Wang (2011), for a discussion of a related condition (equation (1.2) in their paper).

¹³See also Ganguli and Yang (2009) and Manzano and Vives (2011), who analyzed related settings.

The analysis of unconstrained equilibrium highlights some important features of the model that will continue to hold in the constrained setting. We observe, first of all, that in equilibrium, price is informationally equivalent to a linear combination of the (learnable) fundamental payoff v and the aggregate endowment shock z. Second, the extent of fundamental information revealed by price is captured by an endogenous signal-to-noise ratio (β^u). More precisely, the conditional variance of the learnable fundamental decreases as β^u increases, as follows:

$$Var(v|p) = Var(v|\phi^{u}) = (\tau_{v} + (\beta^{u})^{2}\tau_{z})^{-1}.$$
(4)

Hence, we refer to β^u as the *informational efficiency* of the market when specialists are unconstrained in their trading. Note that specialists' information acquisition (higher signal precision τ_{ϵ}) improves the informational efficiency β^u of the market.

2.2.2 Constrained setting

We now impose the portfolio constraints a(p) and b(p) on the specialist's problem. We posit and then verify that there exists a generalized linear equilibrium in the economy, which we define as follows.

Definition 1. A pair $\{g(p), \beta\}$ is a generalized linear equilibrium if: (i) equilibrium price is informationally equivalent to a statistic $\phi = v - \frac{z}{\beta}$ and ϕ can be expressed as $\phi = g(p)$,¹⁴ (ii) specialists' and nonspecialists' demands are optimal (i.e., they solve problems (1) and (2)), and (iii) market clears (i.e., (3) holds).

The ϕ and β defined here are the counterparts of ϕ^u and β^u in the economy without portfolio constraints. In a generalized linear equilibrium, the price function may be nonlinear, but the statistic ϕ is still linear in v and z. Therefore, ϕ is normally distributed, and the inference from price remains tractable. Since equation (4) holds in a generalized linear equilibrium, we continue using β to denote informational efficiency.

To solve for the equilibrium with constraints, one needs to pin down the informational efficiency β and the function g(p). We do so in the following proposition.

¹⁴We say that ϕ is informationally equivalent to price p if conditional distributions of $v|\phi$ and v|p are the same. Our notion of a generalized linear equilibrium follows Breon-Drish (2015).

Proposition 2. (Financial market equilibrium with portfolio constraints) Suppose that specialists face portfolio constraints and have identical signal precision τ_{ϵ} . Then there exists unique generalized linear equilibrium $\{g(p), \beta\}$ in which informational efficiency is $\beta = \beta^u$. The function g(p) is determined as follows. For every p, g(p) is the unique ϕ that solves $X(p, \phi) + x_m(p, \phi) = 1$; here, $x_m(p, \phi)$ and $X(p, \phi)$ are the aggregate demands of the nonspecialists and, respectively, specialists. If both a(p) and b(p) are continuously differentiable, then g(p) can be determined by solving the ordinary differential equation (ODE)

$$g'(p) = -\frac{\pi_1(p, g(p))a'(p) + \pi_3(p, g(p))b'(p) - \pi_2(p, g(p))c_p - c_p^m}{\pi_2(p, g(p))c_\phi + c_\phi^m}$$
(5)

subject to the boundary condition $g(0) = g_0$, where the constant g_0 is the unique solution to $X(0, g_0) + x_m(0, g_0) = 1$. The terms $\pi_1(p, \phi)$ and $\pi_3(p, \phi)$ are for the fraction of specialists for whom lower and, respectively, upper constraints binds. The term $\pi_2(p, \phi)$ is for the fraction of unconstrained specialists. The coefficients, $c_p, c_{\phi}, c_p^m, c_{\phi}^m$ characterize specialists' and non-specialists' aggregate demands in unconstrained economy, given by $X^u = x_0 + c_{\phi}\phi - c_pp$ and, respectively, $x_m^u = c_0^m + c_{\phi}^m \phi - c_p^m p$. The closed-form expressions for $c_p, c_{\phi}, c_p^m, c_{\phi}^m, x_m(p, \phi), X(p, \phi), \pi_1(p, \phi), \pi_2(p, \phi), and \pi_3(p, \phi)$ are in the Appendix.

Proposition 2 is our first main result establishing the existence of a tractable, generalized linear equilibrium in an REE model with portfolio constraints, even when price may be nonlinear. It also states that, for an *exogenously given signal precision* τ_{ϵ} , portfolio constraints are irrelevant for the informational efficiency ($\beta = \beta^u$). This result is the key to our model's tractability. Instead of solving for β in the complex model with constraints, we can solve the simpler unconstrained model. Nonetheless, it would be premature to conclude that constraints do not matter for informational efficiency: in Section 2.3, we show that constraints affect the amount of information acquired by specialists at t = 0. It is when the signal precision τ_{ϵ} becomes *endogenous* that constraints affect informational efficiency.

The intuition behind our irrelevance result is as follows. In general, price informativeness is determined by aggregate trading intensity as well as aggregate hedging intensity, where trading (hedging) intensity is the sensitivity of specialist's demand to her private signal (endowment shock). For a given signal precision, funding constraints affect trading intensity $\frac{\partial x_i}{\partial s_i}$: when constraints are tighter, a specialist is more likely to be constrained, in which case her trading intensity is zero. As a result, as constraints tighten, the aggregate trading intensity $\int \frac{\partial x_i}{\partial s_i} di$ reduces. However, constraints also affect the hedging intensity $\frac{\partial x_i}{\partial e_i}$ in a similar way, so that the ratio of aggregate trading intensity to aggregate hedging intensity $\int \frac{\partial x_i}{\partial s_i} di / \int \frac{\partial x_i}{\partial e_i} di$ remains unchanged.¹⁵ We note that our irrelevance result is a knife-edge case, as it requires that traders with speculative and hedging reasons to trade are identical ex-ante.¹⁶ However, it helps to make our model tractable and it also illustrates how prices aggregate dispersed information when traders face portfolio constraints.

In essence, our irrelevance result underscores that, even with constraints, the aggregate demand of specialists (and hence, the market-clearing price) still varies with and reflects fundamentals via changes in the *fractions* of constrained specialists. Consider an improvement in the asset fundamental v (while fixing the endowment shock z), which leads specialists to increase their demand for the risky asset. Although some specialists cannot increase their demand owing to the upper portfolio constraint, in aggregate, more (fewer) specialists become constrained by a maximal long (short) position. Aggregate demand will increase and thereby reveal the improved asset fundamentals via a higher market-clearing price.

Besides the informational efficiency β , the other important equilibrium object g'(p), which is given in equation (5), captures how much the statistic ϕ changes when the price p changes by a single unit. The numerator in (5) represents aggregate demand's price sensitivity, which derives from four sources. First is the fraction π_1 of specialists constrained by the lower constraint, whose demand has price sensitivity a'(p). Second, a similar effect applies for the fraction π_3 of specialists for whom the upper constraint b(p) binds. Third, there is a fraction π_2 of unconstrained specialists whose demand has price sensitivity $\frac{\partial X^u}{\partial p} = -c_p$. The numerator's last term is the nonspecialist's demand sensitivity to price, $\frac{\partial x_m}{\partial p} = -c_p^m$. The denominator of (5), which represents the sensitivity of aggregate demand to ϕ , can be interpreted similarly.

Figure 2 plots the price as a function of the information content ϕ (i.e., the inverse of the

¹⁵The economic forces behind the irrelevance result are similar to the ones in Dávila and Parlatore (2017), who study the impact of various forms (quadratic, linear, or fixed) of trading cost on informational efficiency. We focus on the impact of general price-dependent portfolio constraints instead.

¹⁶For the same reason, introducing noise trader in our model will undermine the irrelevance result. In Appendix B, we consider an alternative model with noise traders and study how constraints interact with information efficiency.

Figure 2: Price function

The figure plots price p as a function of statistic ϕ , summarizing the information content of price for constraints of the form $a(p) = -W_0/m$, $b(p) = W_0/m$ for different levels of wealth: $W_0 = 1$ (solid line), $W_0 = 3$ (dashed line), and $W_0 = 10$ (thin gray line). Other parameter values are set to: m = 1, $\tau_u = 0.1$, $\tau_z = 1$, $\tau_v = 1$, $\tau_{\theta} = 1$, $\tau_{\epsilon} = 2$, $\gamma = \gamma_m = 3$, and $\alpha = 0.99$.



function g(p) under price-independent constraints of the form $b(p) = -a(p) = W_0/m$. The shape of this function is intuitive. When specialists are unconstrained, the price is linear in ϕ . Correspondingly, when wealth W_0 is high (thin grey line in the figure), the function is close to linear. Constraints distort the price function, in particular, by reducing its sensitivity with respect to ϕ in the extreme region of ϕ . Consider the high- ϕ region. All traders want to take a large long position, and specialists' demand is already at the upper constraint. A further increase in ϕ will not increase specialists' demand. As a result, the price rises less than it does in the case when specialists are unconstrained.¹⁷ The same argument applies to the low- ϕ region. The intermediate- ϕ region is one in which most specialists' demands are not restricted by the constraints, and thus the price function is close to the one in unconstrained case. It follows that as constraints tighten (lower W_0), this region shrinks.¹⁸

¹⁷It is also intuitive that the price is close to being linear in ϕ in that region. The price is determined by the aggregate demand, which is a combination of: (i) the aggregate demand of specialists, which is close to constant, since most of specialists are constrained, and (ii) the aggregate demand of nonspecialists, which is linear.

¹⁸It is clear that in our setting, constraints affect the sensitivity of the function g'(p). This distinguishes our setting from those in Dávila and Parlatore (2017)) and Nezafat et al. (2017), where trading costs and short-sale constraints, respectively, are irrelevant not only for informational efficiency β but also for sensitivity g'(p).

2.3 Information acquisition at t = 0

Having solved for the financial market equilibrium at t = 1, we now study how portfolio constraints affect the incentives of specialists to acquire information at t = 0. We maintain the assumption $a(p) \leq 0 \leq b(p)$ and say that constraints are *tightened* when a(p) increases and/or b(p) decreases. We start by deriving an expression for the marginal value of information under general portfolio constraints, after which we show that a specialist's marginal value of information declines if the constraints are tightened.

At date 0, specialist i decides on the optimal amount of information to acquire by solving the following problem:

$$\max_{\tau_{\epsilon,i}} \quad E\left[-e^{-\gamma(W_i - C(\tau_{\epsilon,i}))}\right].$$

Define the marginal value of information as $\mathcal{MVI} \equiv \frac{\partial CE'_{0,i}(\tau_{\epsilon,i})}{\partial \tau_{\epsilon,i}}$, where $CE_{0,i} = -\frac{\log E_0[\exp(-\gamma W_i)]}{\gamma}$ is the date-0 certainty equivalent for investor *i*. In the next proposition, we characterize this marginal value of information under general portfolio constraints and show that it declines when a specialist's constraints tighten.

Proposition 3. (Marginal value of information) The marginal value of information for a specialist i choosing signal precision τ_{ϵ_i} , while others' signal precisions are τ_{ϵ} , is given by

$$\mathcal{MVI}(\tau_{\epsilon,i},\tau_{\epsilon}) = \underbrace{\frac{\tau_i}{2\tau_{v,i}^2\gamma}}_{\mathcal{MVI} \text{ if specialist } i \text{ was unconstrained}} + \underbrace{\frac{\tau_i}{2\tau_{v,i}^2\gamma} \left(\frac{U_0^u(\tau_{\epsilon,i},\tau_{\epsilon})}{U_0(\tau_{\epsilon,i},\tau_{\epsilon})} - 1\right)}_{Effect \text{ of constraints, } <0}, \tag{6}$$

where $\tau_{v,i} = Var(v|\mathcal{F}_i)$ is the total precision of specialist i's information about the learnable component, $U_0^u(\tau_{\epsilon,i},\tau_{\epsilon}) = E[-e^{-\gamma CE_{1,i}}\mathbb{I}_{x_i^u=x_i}]$ is the expectation of utility in the states when constraints do not bind, and $U_0(\tau_{\epsilon,i},\tau_{\epsilon}) = E[-e^{-\gamma CE_{1,i}}]$ is date-0 expected utility. The marginal value of information decreases when an individual specialist's constraints become tighter, ceteris paribus.

Proposition 3 shows how portfolio constraints affect a specialist's incentive to acquire information. If the specialist was unconstrained, he has stronger incentive to acquire information, as the term capturing the effects of constraints in equations (6) is negative. It makes sense that a specialist considers information valuable to the extent that he can profit from it. In short, constraints reduce specialists' incentives to acquire information. We will examine the equilibrium information choice in section 3.2.

3 Margin requirements and informational efficiency

So far we have studied general, price-dependent portfolio constraints. In this section, we apply our model to study the constraints arising from margin requirements. Our notion of margin requirements is standard and closely follows Brunnermeier and Pedersen (2009). To build a long position in the risky asset, a specialist can borrow from a financier at the risk-free rate, but he has to pledge a cash margin of $m^+(p) \ge 0$ per unit of asset to the financier as collateral. The specialist can similarly establish a short position by providing, as collateral, a cash margin of $m^-(p)$ per unit of asset. Thus, specialists face a funding constraint that the total margin on their positions cannot exceed their initial wealth, as follows:

$$m^{-}(p)[x_i]^{-} + m^{+}(p)[x_i]^{+} \le W_0,$$

where $[x_i]^-$ and $[x_i]^+$ are, respectively, the positive and negative parts of x_i .¹⁹ We can rewrite the margin requirements in the form of portfolio constraints as

$$a(p) = -\frac{W_0}{m^-(p)}, \quad b(p) = \frac{W_0}{m^+(p)}.$$
 (7)

Equation (7) shows that a specialist faces tighter constraints when his initial wealth is lower and/or if the financier's margin requirements are higher.

3.1 Financiers and VaR-based margins

We assume that there is a unit continuum of financiers with CARA utility, with absolute risk aversion γ_F . Financiers can participate in financial market, are uninformed, and get no endowment shocks. Thus, their problem of investing in financial market is given by (2), with γ_m substituted by γ_F . Standard aggregation results (see Mas-Colell, Whinston, and Green (1995), Chapter 4) imply that

¹⁹Since the endowment b_i is not pledgeable, it cannot be used as a collateral to satisfy the margin requirements.

financiers and nonspecialists can be aggregated into one representative agent. Thus, parameter γ_m should be interpreted as the absolute risk aversion of such representative agent, and the analysis of the model done so far is unchanged.²⁰

Motivated by the real-world margin constraints faced by hedge funds, we assume that each financier sets her margin in order to control her VaR, similar to Brunnermeier and Pedersen (2009) as follows:

$$m^{+}(p) = \inf\{m^{+}(p) \ge 0 : \Pr^{Q}(p - f > m^{+}(p) \mid p) \le 1 - \alpha\} \text{ and}$$
$$m^{-}(p) = \inf\{m^{-}(p) \ge 0 : \Pr^{Q}(f - p > m^{-}(p) \mid p) \le 1 - \alpha\},$$
(8)

where " \Pr^Q " stands for "risk-neutral probability." Equation (8) shows that the financier requires the specialists to set aside a minimum amount of cash (i.e., margin) large enough to cover, with (risk-neutral) probability α , the potential loss from trading. In Appendix C.1, we show how to calculate risk-neutral probabilities. In particular, we derive that the risk-neutral distribution of fundamental f given prices p, is Gaussian, with mean p and variance Var(f|p). (See Lemma 9.) In Appendix C.2, we provide a microfoundation for the use of VaR-based margin by financiers, when specialists can default and financiers have to incur a cost to enforce repayment.

We note that in our approach the VaR is evaluated under the risk-neutral measure, in the spirit of Ait-Sahalia and Lo (2000).²¹ In order to avoid additional effects stemming from restriction in market participation, we assume that financiers can participate in the equity market, and thus the risk-neutral measure is adopted.²²

²⁰It is easy to derive that the risk aversion of the representative agent is given by $(1/\gamma_{\text{nonspecialist}} + 1/\gamma_{\text{financier}})^{-1}$.

²¹Ait-Sahalia and Lo (2000) use risk-neutral (option-implied) probabilities to estimate the economic value of VaR. As we show in our microfoundation in Appendix C.2, financiers who participate (do not participate) in the equity market evaluate VaR under the risk-neutral (physical) measure. Note that in Brunnermeier and Pedersen (2009), the VaR is evaluated under the physical measure. In Appendix E we consider such margin specification and show that our results continue to hold, provided that nonspecialists' risk-aversion is not too large.

 $^{^{22}}$ This also allows us to contrast our results with those in intermediary asset-pricing literature, where restricted participation is a key friction.

3.2 Margins and information efficiency in equilibrium

We are now ready to define the equilibrium of the model, in which both margin requirements and specialists' information acquisition are endogenously determined.

Definition 2. An equilibrium with endogenous margins and information acquisition is defined as follows. (i) At time t = 1, financiers set their margin requirements according to (8), given a conjectured price function. (ii) At time t = 1, specialists and nonspecialists choose their optimal demand, given margin requirements and the conjectured price function. (iii) The conjectured price function is consistent with market clearing. (iv) At time t = 0, specialists optimally acquire information.

To clearly illustrate the interaction between information acquisition and margin requirements, we proceed with studying two partial equilibria—one with exogenous precision of specialists' signals and the other with exogenous margins. We first consider how an exogenous increase in specialists' information acquisition affects the endogenous VaR margins set by financiers via informational efficiency. To this end, we consider a partial equilibrium, which satisfies conditions (i), (ii), and (iii) in Definition 2, called *equilibrium with exogenous information*, and do comparative statics with respect to τ_{ϵ} in that equilibrium.

Proposition 4. (Information choice affects margin requirements) For a given level of precision of all specialists' signals τ_{ϵ} , there exists a unique equilibrium with exogenous information. In this equilibrium, margins are given by

$$m^{+} = m^{-} = \Phi^{-1}(\alpha) \sqrt{(\tau_{v} + \beta^{2} \tau_{z})^{-1} + \tau_{\theta}^{-1}}.$$
(9)

Consequently, if τ_{ϵ} decreases (and hence the informational efficiency (β) drops) the margins (m^+ and m^- both) increase. Moreover, the effect is stronger when financiers tolerate less VaR (α is higher), i.e., $\frac{\partial^2 m}{\partial \alpha \partial \beta} < 0$.

Proposition 4 states that a decrease in specialists' information acquisition, which causes a decline in informational efficiency, tightens constraints. A reduced informational efficiency implies that the price tracks fundamentals less closely. As a result, the specialists' trading profit becomes more volatile. Financiers who set margins based on the VaR of specialists' profit in turn demand higher margins. This is illustrated in steps (2) and (3) of Figure 1.

The figure plots equilibrium precision of information τ_{ϵ}^* as a function of wealth W_0 , for $m^+ = m^- = m$, where m = 1 (solid line) and m = 2 (dashed line). Other parameter values are set to $\tau_v = 1$, $\tau_z = 2$, $\tau_{\theta} = 2.75$, $\tau_u = 0.4$, $\gamma = 0.5$, $\gamma_m = 1$, $\alpha = 0.99$, and $\underline{\tau_{\epsilon}} = 0.1$. We assume that the cost function is $C(\tau_{\epsilon}) = k_0(\tau_{\epsilon} - \underline{\tau_{\epsilon}})^2$, where $k_0 = 0.1$.



Next, we study how an exogenous increase in margin requirements affect specialists' information choices. To this end, we consider a partial equilibrium that satisfies conditions (ii), (iii), and (iv) in Definition 2, called *equilibrium with exogenous margins*, and do comparative statics with respect to m^+ and m^- in that equilibrium.

Proposition 5. (Margins and wealth affect information acquisition) For given margin requirements m^+ and m^- , there exists at least one stable equilibrium with exogenous margins. In any such stable equilibrium, there exists a threshold $\hat{W}^1 > 0$ such that for all $0 < W_0 < \hat{W}^1$, specialists' equilibrium information acquisition τ_{ϵ}^* decreases when W_0 drops and/or margins m^+/m^- increase for all specialists.²³

Proposition 5 states that as specialists' face higher margin requirements, they become more constrained and the marginal value of information decreases, discouraging information acquisition. This is illustrated in steps (4) and (1) of Figure 1. Since specialists' wealth is used to satisfy margin requirements, Proposition 5 also implies that wealth plays an important role in our model—in contrast to typical CARA-Normal models.

²³Our notion of stability is as in Manzano and Vives (2011) and Cespa and Foucault (2014) and is standard in game theory (see Fudenberg and Tirole (1991), Chapter 1, 1.2.5). We call an equilibrium *stable* if the fixed point determining equilibrium precision of specialists' signals is stable. More specifically, we call an equilibrium *stable* if $|\tau_{\epsilon,i}'(\tau_{\epsilon}^*)| < 1$, where $\tau_{\epsilon,i}(\tau_{\epsilon}^*)$ is specialist *i*'s optimal choice of precision given that all other specialists' precisions are equal to τ_{ϵ}^* .

Figure 3 plots the equilibrium information choice τ_{ϵ} as a function of investors' wealth (x-axis) and for different levels of margins. Consistent with Proposition 5, as investors wealth drops, constraints tighten and investors' marginal value of information drops, which lowers the equilibrium information acquired. While a small enough W_0 is needed for the proof, numerical simulations show that the results hold more generally. Thus, we view the requirement of a small W_0 in Proposition 5 as technical and not restrictive for the economic mechanism.

We next combine the two partial equilibrium analyses and establish existence of *full equilibrium*, that is, an equilibrium that satisfies all conditions in Definition 2.

Proposition 6. (Existence of full equilibrium) Suppose that $C''(\tau_{\epsilon,i}) > \underline{C}$ for all $\tau_{\epsilon,i}$, where \underline{C} is finite and is defined in the Appendix. Then there exists at least one stable full equilibrium.

The sufficient condition for equilibrium existence is a sufficiently convex cost function to guarantee the convexity of each specialist's information choice problem. This technical restriction is needed because the value of information is not necessarily concave in the presence of constraints.²⁴ From now on we maintain this sufficient condition (stated below).

Assumption 3. The cost function is such that $C''(\tau_{\epsilon,i}) > \underline{C}$ for all $\tau_{\epsilon,i}$.

We will end this subsection with a few remarks about the results.

Remark 1. Informational efficiency affects in constraints even if financiers do not learn from prices. We emphasize that this section's results do not rely on financiers learning from prices. Indeed, one can compute the *unconditional* variance of returns under the risk-neutral measure as

$$Var^{Q}[f-p] = E^{Q}[Var^{Q}[f-p|p]] + Var^{Q}[E^{Q}[f-p|p]] = E^{Q}[Var^{Q}[f-p|p]]$$

In the equation above we used the fact that $p = E^Q[f|p]$. It follows from normality of f|p under the risk-neutral measure (Lemma 9 in Appendix C.1), that the conditional variance $Var^Q[f-p|p]$ is constant and therefore equal to the unconditional variance $Var^Q[f-p]$. This implies that the financier will set the same margins irrespective of whether (or not) she learns from prices.

Remark 2. Alternative risk-based margins. Our result that margins increase when informational efficiency falls holds also for alternative risk-based margins, such as tail value-at-risk (TVaR) and expected

 $^{^{24}}$ The potential non-concavity of value of information is due to constraints, as in Nezafat et al. (2017).

shortfall (ES). This is because all these risk measures depend on the conditional distribution of the loss p - f(f - p) for a long (short) position, given p. Under the risk-neutral measure this distribution is normal with mean zero and variance $Var^{Q}[f|p]$. Hence, the distribution is parameterized by a single parameter, $Var^{Q}[f|p]$. Since VaR, ES, and TVaR are all monotone in $Var^{Q}[f|p]$, it follows that results in this section are robust to using these alternative risk-based margins.

3.3 Information spiral

In Proposition 5, we undertook a partial equilibrium analysis and argued that given margins, tighter funding constraints (e.g., reductions in wealth) lead to lower informational inefficiency because specialists acquire less information. In Proposition 4, we argued that, for a given level of wealth, reduction in information acquisition leads to higher margins. Putting these two results together yields the amplification loop that we call the information spiral. (See Figure 1, in Section 1, for an illustration.) In this section, we discuss two main implications of the information spiral. First, the negative effect of tightening funding constraints on informational efficiency is amplified. Second, strategic complementarities arise among specialists' information choices and among financiers' margin requirements.

3.3.1 Effect of wealth on informational efficiency and margins

As illustrated in Figure 1, the information spiral amplifies a negative shock to specialist wealth into a decrease in informational efficiency (β) and an increase in margin requirements (m^+ and m^-). Thus, we have the following result.

Proposition 7. There exists a threshold $\hat{W}^2 > 0$ such that for all $0 < W_0 < \hat{W}^2$, in a stable equilibrium, a decrease in specialist wealth W_0 decreases informational efficiency β and increases VaR-based margins m^+, m^- .

Proposition 7 suggests that the effect of wealth, a determinant of constraints, on information efficiency is reinforced in two steps: specialists with less wealth have tighter portfolio constraints and hence acquire less information. As a result, prices become less informative. The reduced informational efficiency in turn induces financiers to require more margins, further tightening portfolio constraints and harming information efficiency.²⁵

3.3.2 Complementarity in information acquisition

Our mechanism underpins a novel source for strategic complementarity in specialists' decisions to acquire information. Consider a specialist i and suppose that other specialists acquire less information. The price then becomes less informative about the asset fundamentals; hence, the financier of the specialist i sets higher margin. With a tightened funding constraint, the specialist values information less and acquire less of it.

As standard in REE models (e.g., Grossman and Stiglitz (1980)), there is also a substitutability effect in information acquisition: when other specialists acquire more information, price is more informative about fundamentals, and hence, there is less incentive for a specialist i to acquire private information. The question then is: when does, if at all, the complementarity effect dominate the substitutability effect? The following proposition answers this question.

Proposition 8. Consider a specialist *i* choosing his signal precision $\tau_{\epsilon,i}$, while others' signal precisions are τ_{ϵ} . Suppose also that the parameters of the model are such that the condition (C1) in the proof holds. Then there exists a threshold $\hat{W}^3 > 0$ such that for any $0 < W_0 < \hat{W}^3$ we have that $\frac{\partial \mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon}; W_0)}{\partial \tau_{\epsilon}} > 0$. That is, if (C1) holds and specialists' wealth W_0 is small enough, there is complementarity in information acquisition.

Condition (C1) of Proposition 8 characterizes the combination of parameters of the model, for which the complementarity in information acquisition emerges when wealth is low. This combination of parameters is shown in the grey region in Figure 4, panel (a). Hence, Point A (B) in panel (a) of Figure 4 presents a combination of parameters for which this condition holds (does not hold). Panel (b) shows how the partial derivative of marginal value of information with respect to other investors' information choice varies with wealth. There is complementarity when this partial derivative is positive. One can see that for parameters corresponding to point A (solid line), there is complementarity when wealth is small. In contrast, for parameters corresponding to point B (dashed line), there is substitutability in information acquisition for all levels of wealth.

²⁵As in Proposition 5, we view the requirement of a small W_0 as technical and the result holds numerically for a wide range of values of W_0 we have tried.

Figure 4: Complementarity in information acquisition

The shaded region of panel (a) represents the combination of parameters τ_u and τ_v , for which condition (C1) holds. Panel (b) plots $\partial \mathcal{MVI}(\tau_{\epsilon,i}, \tau_{\epsilon}, W_0)/\partial \tau_{\epsilon}$ against W_0 for the paremeters corresponding to points A and B, marked on the plot in panel (a). Point A: $\tau_u = 0.4$, $\tau_v = 0.5$. Point B: $\tau_u = 0.8$, $\tau_v = 1.2$. Other parameter values are set to: $\tau_z = 2$, $\tau_{\theta} = 2.75$, $\tau_{\epsilon,i} = \tau_{\epsilon} = 0.2$, $\gamma = 0.5$, $\gamma_m = 1$, and $\alpha = 0.99$.





(a) Combination of parameters τ_u and τ_v , for which the model exhibits complementarity in information acquisition.

(b) Dependence of $\partial \mathcal{MVI}(\tau_{\epsilon,i}, \tau_{\epsilon}, W_0) / \partial \tau_{\epsilon}$ on W_0 .

Condition (C1) ensures that the complementarity effect operating through changes in investor i's likelihood of being constrained is stronger than the Grossman-Stiglitz substitutability effect. To get more intuition about condition (C1), Figure 4, panel (a) plots the region of parameters where condition (C1) holds (in grey). First, note that for any given τ_u , condition (C1) doesn't hold for high τ_v , that is, less volatile fundamentals. Intuitively, there is no complementarity when τ_v is high, since when fundamental volatility is low, the price tracks fundamentals closely even if specialists acquire little information. Thus, margins are low and are insensitive to how much information specialists acquire, weakening step 3 in information spiral in Figure 1.

Second, for any given τ_v , condition (C1) holds for τ_u not too high and not too low, that is, for moderately volatile idiosyncratic endowment shocks. If idiosyncratic endowment shocks are less volatile (high τ_u), specialists who know $z + u_i$ can almost perfectly infer the aggregate endowment shock z, thereby filtering out the noise in prices and learning the fundamentals from price. This strengthens the Grossman-Stiglitz substitutability effect. Thus, there is no complementarity for very high τ_u . When idiosyncratic endowment shocks are very volatile (low τ_u), the desired demand of a specialist *i* is very volatile, and thus, he is likely to be constrained even if other specialists acquired a lot of information and margins are low. This weakens step 1 in the information spiral in Figure 1. Thus, there is no complementarity for very low τ_u .

3.3.3 Complimentarity in margin requirements

Another strategic complementarity that arises from the information spiral is among financiers' margin requirements. Consider a financier indexed by i. An increase in margins required by other financiers discourages their specialists from acquiring information (by Proposition 5). As a result, prices become less informative, and financier i responds by setting higher margins (cf. equation (9)). This result is formally proven in the proposition below and is illustrated in Figure 5. One can see that unlike the complementarity in information choice, a complementarity in margin requirements is always present.²⁶

Proposition 9. Suppose that the conditions of Proposition 5 hold. Then, when all the financiers other than *i* increase their margins *m*, it is optimal for financier *i* to increase his margin as well.

Figure 5: Complimentarity in margin requirements



The optimal margin $m_i(m)$ for financier *i*, given that other financiers set margin equal to *m* for $W_0 = 1.5$ (solid line) and $W_0 = 0.7$ (dashed line). Other parameter values are set to: $\tau_v = 1$, $\tau_z = 2$, $\tau_\theta = 2.75$, $\tau_u = 0.4$, $\gamma = 0.5$, $\gamma_m = 1$, $\alpha = 0.99$, and $\underline{\tau_{\epsilon}} = 0.1$. We assume that the cost function is $C(\tau_{\epsilon}) = k_0(\tau_{\epsilon} - \underline{\tau_{\epsilon}})^2$, where $k_0 = 0.1$.

This complementarity suggests that changes in some financiers' margin requirements, for example, due to changes in regulations, can cause otherwise unrelated financiers to change their margin requirements, affecting wider equilibrium outcomes like aggregate asset prices and information efficiency. Our mechanism highlights the general equilibrium effects that policymakers may want to take

²⁶Here we make the same remark as for Proposition 5 that while a small enough W_0 is needed for analytical proof, numerical simulations show that the result holds generally.

into account.

4 Asset-pricing implications

In this section we derive the implications of a decline in specialists' wealth on the risky asset's equilibrium risk premium and return volatility. The main result in this section is that a drop in wealth, ceteris paribus, leads to an increase in the risk premium, return volatility, and the Sharpe ratio, when specialists' wealth W_0 is small.

Definition 3. The risk premium, variance, and the Sharpe ratio of returns are defined as follows:

$$\bar{rp}(W_0, \tau_{\epsilon}) \equiv E[f-p], \ \mathcal{V}(W_0, \tau_{\epsilon}) \equiv Var[f-p], \ SR(W_0, \tau_{\epsilon}) = \frac{E[f-p]}{\sqrt{Var[f-p]}}.$$
(10)

The volatility of returns is defined as the square root of return variance.

Results

The change in risk premium in response to a change in the specialists' wealth can be decomposed as follows:

$$\frac{d\bar{r}p(W_0, \tau_{\epsilon})}{dW_0} = \underbrace{\frac{\partial \bar{r}p}{\partial W_0}}_{\text{Direct Effect}} + \underbrace{\frac{\partial \bar{r}p}{\partial \tau_{\epsilon}} \frac{\partial \tau_{\epsilon}}{\partial W_0}}_{\text{Indirect Effect}} .$$
(11)

The first term in the right-hand side of equation (11) captures the direct effect that a change in specialists' wealth has on the risk premium; the second term captures the indirect effect resulting from specialists' endogenous information acquisition decisions. Similarly, we decompose the change in volatility and the Sharpe ratio in response to a change in specialists' wealth into the direct and indirect effect. The following proposition establishes the effect of wealth W_0 on risk premium, volatility, and the Sharpe ratio.

Proposition 10. (Asset-pricing implications) There exists a threshold $\hat{W} > 0$ such that for all $W_0 \in (0, \hat{W})$, we have $\frac{d\bar{r}p}{dW_0} < 0$, $\frac{d\sqrt{y}}{dW_0} < 0$ and $\frac{dSR}{dW_0} < 0$. Moreover, (i) the direct effect of wealth on volatility is second order compared to indirect effect when wealth is low, that is, $\lim_{W_0 \to 0} \frac{\frac{\partial\sqrt{y}}{\partial W_0}}{\frac{\partial\sqrt{y}}{\partial T_c} \frac{d\tau_c}{dW_0}} = 0$; (ii) the direct effect of wealth on risk premium is negative for all levels of wealth, that is, $\frac{\partial\bar{r}p}{\partial W_0} < 0$ for all W_0 .

Figure 6: Effect of wealth on risk premium, volatility, and the Sharpe ratio

The figure plots the direct effect, $\frac{\partial Y}{\partial W_0}$ (dashed lines), indirect effect $\frac{\partial Y}{\partial \tau_{\epsilon}} \frac{d\tau_{\epsilon}}{dW_0}$ (dotted lines), and total effect $\frac{\partial Y}{\partial W_0} + \frac{\partial Y}{\partial \tau_{\epsilon}} \frac{d\tau_{\epsilon}}{dW_0}$ (solid lines) of wealth on risk premium (panel (a), $Y = \bar{r}p$), volatility (panel (b), $Y = \sqrt{\mathcal{V}}$), and the Sharpe ratio (panel (c), Y = SR). The parameter values are set to: $\tau_v = 1, \tau_z = 2, \tau_u = 0.1, \tau_{\theta} = 2.75, \gamma = 0.5, \gamma_m = 1, \tau_{\epsilon} = 0.1$, and $\alpha = 0.99$. We assume that the cost function is $C(\tau_{\epsilon}) = k_0(\tau_{\epsilon} - \tau_{\epsilon})^2$, where $k_0 = 0.1$.



Figure 6 plots the derivative of unconditional risk premium (panel (a)), volatility of returns (panel (b)), and the Sharpe ratio (panel (c)) with respect to wealth as a function of wealth. Note that the solid line in all panels is always below zero, which implies that as wealth drops, risk premium, volatility, and the Sharpe ratio rises. Moreover, note that as wealth drops, the effect is getting stronger. This implies that effect of small change in wealth on each of these objects is stronger when the wealth is already low, that is, crisis periods. Next, we split the total effect into direct and indirect effect as in equation (11) to see the effect of endogenous information choice of specialists.

Risk Premium: The results for direct and indirect effects are shown in Figure 6, panel (a) in dashed line and dotted line, respectively. Holding τ_{ϵ} fixed, as wealth drops, constraints become tighter and specialists' capacity to go long or short the asset is diminished, which is similar to the effect of lowering their risk-bearing capacity (i.e., increasing their risk aversion). Therefore, the risk premium rises. This argument implies that absent the information acquisition channel (i.e., holding τ_{ϵ} fixed), the wealth drop would cause an increase in risk premium. This reflects the direct effect in equation (11) and is shown in the dashed line. Moreover, as wealth drops, because of the information spiral, specialists in equilibrium acquire less information (lower τ_{ϵ}), which leads to an additional increase in risk premium, which is the indirect effect given in (11) and is shown in the dotted line. In Proposition 10, while small enough wealth is needed for the indirect effect to be negative, numerical simulations show that the results are more general and the requirement of small wealth is only technical.

Return Volatility: We split the total effect for volatility into direct and indirect effect as in equation (12) to see the effect of endogenous information choice of specialists.

$$\frac{d\sqrt{\mathcal{V}}(W_0, \tau_{\epsilon})}{dW_0} = \underbrace{\frac{\partial\sqrt{\mathcal{V}}}{\partial W_0}}_{\text{Direct Effect}} + \underbrace{\frac{\partial\sqrt{\mathcal{V}}}{\partial \tau_{\epsilon}}}_{\text{Indirect Effect}} \cdot \frac{\partial\tau_{\epsilon}}{\partial W_0}.$$
(12)

The results are shown in Figure 6, panel (b) in the dashed line (direct effect) and dotted line (indirect effect). The direct effect is almost zero when wealth is small. However, specialists acquire less information when they are constrained, so there is an increase in volatility corresponding to the indirect effect (which operates through the information acquisition channel). Overall, return volatility increases as wealth declines. For small wealth, Proposition 10 implies that indirect effect always dominates direct effect, highlighting the importance of our information acquisition channel.

The Sharpe ratio: Figure 6, panel (c) plots the derivative of the Sharpe ratio with respect to wealth as a function of wealth, that is, $\frac{dSR}{dW_0}$ vs W_0 (solid line). Even though risk premium and volatility both rise as wealth falls, the Sharpe ratio also increases when wealth is small enough.

5 Empirical implications

We start by providing suggestive evidence for our mechanism and then discuss its other empirical implications. The main and novel implication of our mechanism is that specialists' wealth and stock price informativeness are positively related. We proxy for specialists' wealth with the primary dealers' capital ratio, as in He, Kelly, and Manela (2017). Following Weller (2017), we construct a measure of stock price informativeness called the price jump ratio. The price jump ratio divides the return at the time of earnings announcement to the total return over the pre-announcement period. The higher this measure, the less information has entered the stock prices before the announcement, indicating lower price informativeness.²⁷ Our model therefore implies that the primary dealers' capital ratio and

 $^{^{27}}$ Weller (2017) argues that the price jump ratio measures information acquisition because it captures the fraction of acquirable information about an event being incorporated into prices before the event's public disclosure—in line with information acquisition channel that our baseline model highlights.

Figure 7: Price informativeness and financial constraints

The figure plots the quarterly price jump ratio (as constructed by Weller (2017)) and quarterly equity capital ratio of primary dealers (as constructed by He et al. (2017)) between 1995 and 2013. Both the time series are normalized to have mean 0 and standard deviation of 1. The plot also shows the time trends (dotted lines) of both series. The correlation between these two series is statistically significant -0.59.



the price jump ratio should be negatively correlated.

Data Description: We conduct our analysis using quarterly data from 1995 to 2013. We obtain equity capital ratio of primary dealers in He et al. (2017) from the authors' website. We obtain stock price data from the Center for Research in Security Prices (CRSP) to compute the price jump ratio. The sample contains publicly listed non-financial firms from 1995 to 2013. After constructing the price jump ratios at the stock level, we value-weight these ratios to construct the price jump ratio at the aggregate level.

Findings: Figure 7 illustrates the main empirical finding of the paper. It shows that the time series of the aggregate price jump ratio (blue) and the equity capital ratio of primary dealers (red). It is evident from the figure that the two ratios are negatively related. The correlation coefficient (-0.6) is also statistically significant.

Other predictions: Our model also implies that an exogenous decrease in margin requirements should lead to an increase in informational efficiency of prices (Proposition 5). For proxies for price

informativeness, see Weller (2017), Bai, Philippon, and Savov (2016), Farboodi, Matray, and Veldkamp (2018). and Dávila and Parlatore (2018). One source of exogenous shocks to margin requirements could come from policy changes. For instance, Jylha (2018) argues that the New York Stock Exchange's portfolio Margining Pilot Program of 2005–2007 was an exogenous shock that relaxed the margin constraints of index options but not of individual equity options. Our model predicts that following the pilot program, the price informativeness of index options should increase more than that of individual equity options.

Finally, our model implies that an exogenous drop in informational efficiency should result in an increase in margin requirements (Proposition 4). The shutting down of a broker (Kelly and Ljungqvist, 2012) or the merger of brokers (Hong and Kacperczyk, 2010) could be exogenous shocks to the activity of analysts, and hence, to price informativeness. Moreover, the magnitude of this mechanism is likely to vary over time. Proposition 4 implies that the impact of changes in informational efficiency on margin requirements should be stronger when the financiers are less risk-tolerant to specialists' trading loss. Financiers in reality may become effectively less risk-tolerant due to new regulations. For example, Boyarchenko, Eisenbach, Gupta, Shachar, and Van Tassel (2018) provide evidence that the post-crisis regulations make global systemically important banks less willing to finance hedge funds' arbitrage activities. Our model suggests that in the post-crisis regulations era, margin requirements should become more sensitive to informational efficiency of prices.

6 Discussion and Extensions

We have made several assumptions in the analysis for tractability and to highlight the underlying mechanism in the clearest manner. In this section, we show the robustness of our main results in alternative environments and discuss the additional implications we derive in these alternative environments.

1. Noise trading: As discussed in the introduction, constraints in general can affect price informativeness via traders' information acquisition incentives and the information aggregation function of prices. In our baseline model, we focus solely on the information acquisition channel. This is possible because of the irrelevance result, which in turn stems from the assumption that noise in prices comes from specialists' hedging needs. In Appendix B, we consider an alternative setting in which the noise in prices comes from exogenous noise traders who are not affected by constraints. We argue that in the alternative setting, constraints and price informativeness interact in a similar way via the information aggregation channel.

In the alternative setup, tightening the funding constraints of informed specialists reduce their aggregate trading intensity but not the noisy supply. This hurts price informativeness, even for a given quality of private information. The reduction in price informativeness leads to an increase in margins, for similar reasons discussed in the baseline model. Hence, via a different channel, the self-reinforcing interaction between tightness of constraints and price informativeness continues to hold. Thus, the channel presented in Appendix B would perhaps reinforce the results in the baseline model. The baseline model keeps the focus on the information acquisition channel while maintaining tractability.

- 2. Endowment of risky assets: In the baseline model, we assume that specialists have cash as initial endowment. In Online Appendix D, we instead assume that specialists are endowed with some risky assets and show that the information spiral continues to hold in this economy.
- 3. VaR under the physical measure: In the baseline model, we assume that the financiers use the risk-neutral measure to compute the VaR. The advantage of using the risk-neural measure is that the VaR-based margins will be independent of price level. In Online Appendix E, we study the case in which the financiers use the physical measure to evaluate risk.²⁸ There, margins will depend on the price level. We show that when the nonspecialists' risk aversion is not too high, all the results associated with the information spiral continue to hold.
- 4. Robustness of low wealth results: In the baseline model, we prove that Propositions 5, 7, 8, and 10 hold when wealth is low enough. This does not imply that wealth is high, those results do not hold. While a small enough wealth is needed for the analytical proofs, numerical simulations show that Propositions 5, 7, and the part of Proposition 10 concerning risk premium hold generally.²⁹

 $^{^{28}}$ We also micro-found the use of physical measures by the financiers who do not participate in the equity market in Appendix E.

²⁹For all combinations of parameters of the model we tried we could not find finite wealth thresholds for these propositions. Thus, numerically, these propositions seem to hold for all levels of wealth.

7 Conclusion

In this paper we developed a tractable REE model with general portfolio constraints, and we applied our methodology to study an REE model with margin constraints. We argued that funding constraints affect and are affected by informational efficiency, leading to a novel amplification mechanism that we call the information spiral. This spiral implies that the risk premium, return volatility, and the Sharpe ratio each rise as specialists' wealth declines. The information spiral also generates complementaries in: (i) margin requirements and, during crises, (ii) the specialists' acquisition of information. Complementary to existing intermediary-based crisis narratives in which nonspecialists are restricted from participating in the asset market, our mechanism shows how intermediaries matter even in markets such as the equity market, where all investors can freely participate.

While other papers describe mechanisms for amplification over the business cycle and highlight the importance of specialist investors for asset prices, our paper is different because it involves changes in price informativeness and the interaction with constraints. Given the important role financial markets play in aggregating and disseminating information, as argued by Bond et al. (2012), our mechanism could have significant real implications.

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Appendices

A Proofs

Lemma 1. If Assumption 2 holds, then $E[-\exp(-\gamma CE_{1,i})]$ is finite.

Proof. We decompose the expectation into three parts, as follows:

$$E[-\exp(-\gamma C E_{1,i})] = E[-\exp(-\gamma C E_{1,i}) I(x_i^u > b)] +$$
(13)

$$+ E\left[-\exp\left(-\gamma C E_{1,i}\right) I\left(a \le x_i^u \le b\right)\right] \tag{14}$$

$$+ E[-\exp(-\gamma C E_{1,i}) I(x_i^u < a)].$$
(15)

Step 1. The expectation in (13) is finite if Assumption 2 holds. For $x_i^u > b$, we can write (13) as follows:

$$E[-\exp(-\gamma C E_{1,i}) I(x_i^u > b)] = E[-\exp(-\gamma W_2) I(x_i^u > b)]$$

where

$$W_2 = W_0 + b(v + \theta - p) + e_i\theta.$$

We write the expectation explicitly

$$E[-\exp\left(-\gamma W_{2}\right)I(x_{i}>b)] = \int -\exp\left(-\gamma\left(W_{0}+b(v+\theta-p)+e_{i}\theta\right)\right)I(x_{i}^{u}>b)dF_{u_{i}}dF_{\epsilon_{i}}dF_{v}dF_{z}dF_{\theta}$$
$$> \int -\exp\left(-\gamma\left(W_{0}+b(v+\theta-p)+e_{i}\theta\right)\right)dF_{u_{i}}dF_{\epsilon_{i}}dF_{v}dF_{z}dF_{\theta}.$$
(16)

It suffices to prove that (16) is finite. We compute the expectation with respect to θ :

$$E_{\theta} \left[\exp\left(-\gamma \left(W_0 + b(v + \theta - p) + e_i\theta\right)\right) \right] = \exp\left(-\gamma \left(W_0 + b(v - p) - \frac{\gamma}{\tau_{\theta}} \left(e_i + b\right)^2\right)\right)$$
$$= \exp\left(-\gamma \left(W_0 + b(v - p)\right)\right) \exp\left(\frac{\gamma^2}{\tau_{\theta}} \left(e_i + b\right)^2\right).$$

From the above it is clear that the only "problematic" term is $\exp\left(\frac{\gamma^2}{\tau_{\theta}} (e_i + b)^2\right)$. Thus, it suffices to show that

$$E_{z,u_i}\left[\exp\left(\frac{\gamma^2}{\tau_{\theta}}\left(z+u_i+b\right)^2\right)\right] < \infty.$$

For the latter to hold it suffices to show that $E_{z,u_i}\left[\exp\left(\frac{\gamma^2}{\tau_{\theta}}\left(z^2+u_i^2\right)\right)\right] < \infty$. We write the latter expectation explicitly

$$\frac{\tau_z \tau_u}{2\pi} \int \exp\left(\left(z^2 \left(\frac{\gamma^2}{\tau_\theta} - \frac{\tau_z}{2}\right) + u_i^2 \left(\frac{\gamma^2}{\tau_\theta} - \frac{\tau_u}{2}\right)\right)\right) du_i dz$$

Clearly, the integral in the preceding displayed equation converges provided that Assumption 2 holds.

Step 2. The expectation in (14) is finite if Assumption 2 holds. For $a < x_i^u < b$, we can write (14) as

follows:

$$E[-\exp\left(-\gamma C E_{1,i}\right) I(a \le x_i^u \le b)] = E\left[-\exp\left(-\gamma \left(W_0 + \frac{\gamma}{2\tau_i} \left(x_i^u\right)^2 - \frac{\gamma}{2\tau_\theta} e_i^2\right)\right) I(a \le x_i^u \le b)\right]$$
$$> \int -\exp\left(\frac{\gamma^2}{2\tau_\theta} e_i^2\right) dF_{u_i} dF_z$$

Following the analysis in step 1, one can get that the last expectation is finite if Assumption 2 holds.

Step 3. The expectation in (15) is finite if Assumption 2 holds. The proof is analogous to step 1 and is omitted for brevity. \blacksquare

A.1 Proof of Proposition 1

Proof of Proposition 1. At time 1, the first-order condition for specialist i solving problem (1) is given by

$$x_{i} = \frac{\tau}{\gamma} \left(E\left[v | \mathcal{F}_{i} \right] - p - \gamma e_{i} \tau_{\theta}^{-1} \right), \quad \text{where} \quad \tau^{-1} = Var\left[v + \theta | \mathcal{F}_{i} \right].$$

Similarly, the first-order condition for the nonspecialist solving problem (2) is

$$x_m = \tau_m \frac{E[v|\phi] - p}{\gamma_m}$$
, where $\tau_m^{-1} = Var[v + \theta|\phi]$

Using Bayes's rule for jointly normal random variables, we can write

$$E\left[v|\mathcal{F}_{i}\right] = \frac{\tau_{\epsilon}s_{i} + \beta^{2}\left(\tau_{u} + \tau_{z}\right)\phi + \beta\tau_{u}e_{i}}{\tau_{\epsilon} + \beta^{2}\left(\tau_{u} + \tau_{z}\right) + \tau_{v}} \quad \text{and} \quad \frac{1}{\tau} = \frac{1}{\tau_{\epsilon} + \beta^{2}\left(\tau_{u} + \tau_{z}\right) + \tau_{v}} + \frac{1}{\tau_{\theta}}, \text{ and}$$
$$E\left[v|\phi\right] = \frac{\beta^{2}\tau_{z}}{\beta^{2}\tau_{z} + \tau_{v}}\phi \quad \text{and} \quad \frac{1}{\tau_{m}} = \frac{1}{\beta^{2}\tau_{z} + \tau_{v}} + \frac{1}{\tau_{\theta}}.$$

Substituting these into the market clearing condition (3), we get

$$\frac{\tau}{\gamma} \left(\frac{\tau_{\epsilon} v + \beta^2 \left(\tau_u + \tau_z \right) \phi + \beta \tau_u z}{\tau_{\epsilon} + \beta^2 \left(\tau_u + \tau_z \right) + \tau_v} \right) - \frac{\tau}{\tau_{\theta}} z + \frac{\tau_m}{\gamma_m} \frac{\beta^2 \tau_z \phi}{\beta^2 \tau_z + \tau_v} = p \left(\frac{\tau}{\gamma} + \frac{\tau_m}{\gamma_m} \right).$$

One can express equilibrium price p = p(v, z) from the above equation. Since it can only depend on v and z through $\phi = v - \frac{1}{\beta}z$, it must be true that $\frac{\partial p}{\partial v} / \frac{\partial p}{\partial z} = -\beta$. This implies that β satisfies

$$\beta^{3}\gamma\left(\tau_{u}+\tau_{z}\right)-\beta^{2}\tau_{u}\tau_{\theta}+\beta\gamma\left(\tau_{\epsilon}+\tau_{v}\right)-\tau_{\theta}\tau_{\epsilon}=0.$$
(17)

It can be seen from the above equation that the solution to it is always positive and there exists at least one solution. The solution is unique if the first derivative of the above polynomial does not change sign. The first derivative of the above equation is given by

$$3\beta^2\gamma\left(\tau_u+\tau_z\right)-2\beta\tau_u\tau_\theta+\gamma\left(\tau_\epsilon+\tau_v\right).$$

At $\beta = 0$, the slope is positive, and the slope is always positive if the above equation has no roots. This is true if and only if

$$\tau_u^2 \tau_\theta^2 < 3\gamma^2 \left(\tau_u + \tau_z\right) \left(\tau_\epsilon + \tau_v\right).$$

Using implicit differentiation of (17), β increases in τ_{ϵ} if and only if $\tau_{\theta} - \beta \gamma > 0$, which always holds for β solving equation (17).

The aggregate demand of specialists and nonspecialists can be written as

$$X^{u}(p,\phi) = c_{0} + c_{\phi}\phi - c_{p}p$$
 and $x_{m}(p,\phi) = c_{0}^{m} + c_{\phi}^{m}\phi - c_{p}^{m}p$

respectively. The individual demand of specialist i can be written as follows:

 $x_i^u = X^u + \xi_i$, where $\xi_i \sim \mathcal{N}(0, \sigma_{\xi}^2)$ are i.i.d. across specialists.

We now provide expressions for the coefficients mentioned above. Since the aggregate demand of specialists and the nonspecialist can depend on v only through ϕ , we find

$$c_{\phi} = \frac{\tau}{\gamma} \frac{\partial E[v|\mathcal{F}_i]}{\partial v} = \frac{\tau}{\gamma} \left(\frac{\tau_{\epsilon} + \beta^2 \left(\tau_u + \tau_z\right)}{\tau_{\epsilon} + \beta^2 \left(\tau_u + \tau_z\right) + \tau_v} \right), \text{ and}$$
(18)

$$c_{\phi}^{m} = \frac{\tau_{m}}{\gamma_{m}} \frac{\partial E[v|\phi]}{\partial v} = \frac{\tau_{m}}{\gamma_{m}} \frac{\beta^{2} \tau_{z}}{\beta^{2} \tau_{z} + \tau_{v}}.$$
(19)

Similarly,

$$c_p = \frac{\tau}{\gamma}, \ c_p^m = \frac{\tau_m}{\gamma_m}.$$

Finally,

$$\xi_{i} = \frac{\tau}{\gamma} \left(\frac{\tau_{\epsilon} \epsilon_{i} + \beta \tau_{u} u_{i}}{\tau_{\epsilon} + \beta^{2} (\tau_{u} + \tau_{z}) + \tau_{v}} - \gamma u_{i} \tau_{\theta}^{-1} \right), \text{ and}$$
$$\sigma_{\xi}^{2} = \left(\frac{\tau}{\gamma} \right)^{2} \left(\frac{\tau_{\epsilon} + (\beta \tau_{u} - \gamma \tau_{\theta}^{-1})^{2} \tau_{u}^{-1}}{(\tau_{\epsilon} + \beta^{2} (\tau_{u} + \tau_{z}) + \tau_{v})^{2}} \right).$$

The coefficients c_0 and c_0^m are both zero.

A.2 Proof of Proposition 2

Proof of Proposition 2. We first define a function T(x; a, b) that truncates its argument x to the interval [a, b]:

$$T(x; a, b) = \begin{cases} x, & \text{if } a \le x \le b, \\ b, & \text{if } x > b, \\ a, & \text{if } x < a. \end{cases}$$
(20)

Conjecture that there exists a generalized linear equilibrium with informational efficiency β . Investor *i*'s demand can then be written as

$$x_i = T\left(x_i^d; a(p), b(p)\right).$$

Moreover, as in the proof of Proposition 1, one can find specialist *i*'s desired demand x_i^d (or the amount he would like to trade, in the absence of constraints) as

$$x_i^d = X^d + \xi_i^d,$$

where aggregate desired demand X^d is $X^d = c_0 + c_\phi \phi - c_p p$ and the idiosyncratic part of the desired demand is

$$\xi_i^d = \frac{\tau}{\gamma} \left(\frac{\tau_\epsilon \epsilon_i + \beta \tau_u u_i}{\tau_\epsilon + \beta^2 \left(\tau_u + \tau_z \right) + \tau_v} - \gamma u_i \tau_\theta^{-1} \right).$$

By the exact law of large numbers, one can write the aggregate demand of specialists as

$$X = \int x_i di = E_{\xi_i} \left[T \left(X^d + \xi_i^d; a(p), b(p) \right) \right].$$

For a given price p, the aggregate demand X is an increasing (and thus invertible) function of the aggregate desired demand X^d . Therefore, given p, one can compute X^d , from which one can express $\eta(\phi, v, z) \equiv \frac{\tau}{\gamma} \left(\frac{\tau_e v + \beta^2 (\tau_u + \tau_z) \phi + \beta \tau_u z}{\tau_e + \beta^2 (\tau_u + \tau_z) + \tau_v} \right) - \frac{\tau}{\tau_\theta} z$. Thus, the price in the constrained economy is informationally equivalent to η . However, in the generalized linear equilibrium, the price must be informationally equivalent to ϕ . For this to hold we need $-\frac{\partial \eta(\phi, v, z)}{\partial v} / \frac{\partial \eta(\phi, v, z)}{\partial z} = \beta$, which is equivalent to equation (17), which characterizes the informational efficiency in the unconstrained economy. Thus, $\beta = \beta^u$ and $x_i^d = x_i^u$. Moreover, for the aggregate demand of specialists, we can write

$$X = X(\phi, p) = E_{\xi_i} \left[T \left(X^u(\phi, p) + \xi_i; a(p), b(p) \right) \right],$$

where $X^{u}(\phi, p)$ and ξ_i are characterized in proof of Proposition 1.

We now prove that for every p there exists unique $\phi = g(p)$ such that the market clears. Indeed, the market clearing can be written as

$$X(\phi, p) + x_m(\phi, p) = 1, \text{ where} x_m(\phi, p) = c_0^m - c_p^m p + c_\phi^m \phi.$$
(21)

For a given p, aggregate specialists' demand $X(\phi, p)$ is increasing in ϕ . Thus, there is at most one solution. At least one solution exists by the Intermediate Value Theorem. The aggregate demand at $+\infty(-\infty)$ is equal to $+\infty(-\infty)$, thus, at some intermediate point, the aggregate demand has to be equal to 1.

We now compute a closed-form expression for the aggregate demand of specialists $X(\phi, p)$. It can be split into three parts. For a fraction π_1 of specialists, the lower constraint a(p) will bind. They contribute $\pi_1(\phi, p)a(p)$ to the aggregate demand. Similarly, a fraction π_3 of specialists for whom the upper constraint b(p) binds. They contribute $\pi_3(\phi, p)b(p)$. Finally a fraction π_2 will be unconstrained. They contribute $\pi_2 \cdot (X^u + E[\xi_i | (\xi_i + X^u) \in [a(p), b(p)]])$. Using the standard results for the mean of truncated normal distribution, the last term can be further simplified to

$$\pi_2 E[\xi_i | (\xi_i + X^u) \in [a(p), b(p)]] = \sigma_{\xi} \left(\Phi' \left(\frac{a(p) - X^u}{\sigma_{\xi}} \right) - \Phi' \left(\frac{b(p) - X^u}{\sigma_{\xi}} \right) \right)$$

where $\Phi(\cdot)$ and $\Phi'(\cdot)$ stands for the cumulative distribution function (CDF) and probability density function (PDF) of a standard normal distribution. Combining all of the terms we get

$$X(\phi, p) = \pi_1 a(p) + \pi_3 b(p) + \pi_2 X^u + \sigma_\xi \left(\Phi'\left(\frac{a(p) - X^u}{\sigma_\xi}\right) - \Phi'\left(\frac{b(p) - X^u}{\sigma_\xi}\right) \right).$$
(22)

Now we determine the fractions π_1 , π_2 , and π_3 . The fraction of specialists constrained by the lower constraint, π_1 , is given by

$$\pi_1(p,\phi) = P(x_i < a(p)) = P(X^u(p,\phi) + \xi_i < a(p)) = \Phi\left(\frac{a(p) - X^u(p,\phi)}{\sigma_{\xi}}\right)$$
(23)

The expressions for π_2 and π_3 can be derived analogously:

$$\pi_3(\phi, p) = 1 - \Phi\left(\frac{b(p) - X^u(p, \phi)}{\sigma_\xi}\right),\tag{24}$$

$$\pi_2(\phi, p) = 1 - \pi_1 - \pi_3. \tag{25}$$

Finally, we find the expression for the function g'(p). Differentiating the market-clearing condition implicitly, we have

$$g'(p) = -\frac{\frac{\partial}{\partial p}(X(p,\phi) + x_m(p))}{\frac{\partial}{\partial \phi}(X(p,\phi) + x_m(p))}.$$
(26)

For the numerator, we have

$$\frac{\partial}{\partial p}(X(p,\phi) + x_m(p)) = \pi_1 a'(p) + \pi_3 b'(p) - \pi_2 c_p - c_p^m.$$

For the denominator, we have

$$\frac{\partial}{\partial \phi}(X(p,\phi) + x_m(p)) = c_{\phi}^m + \pi_2 c_{\phi}.$$

Substituting these expressions into (26) gives us the desired result.

A.3 Proof of Proposition 3

We start with the following lemma:

Lemma 2. The certainty equivalent $CE_{1,i}$ is given by (27).

Proof of Lemma 2. The date-1 certainty equivalent solves

$$-\exp(-\gamma CE_{1,i}) = E[-\exp(-\gamma (W_0 + x_i(v + \theta - p) + e_i\theta))|\mathcal{F}_i]$$

The certainty equivalent at time 1 can thus be written as

$$CE_{1,i} = W_0 + x_i (E[v|\mathcal{F}_i] - p) - \frac{\gamma}{2\tau_{v,i}} x_i^2 - \frac{\gamma}{2\tau_{\theta}} (x_i + e_i)^2.$$

where $\tau_{v,i}^{-1} = var(v|\mathcal{F}_i)$. Next, we note that

$$x_i^u = \frac{\tau_i}{\gamma} (E[v|\mathcal{F}_i] - p - \gamma e_i \tau_{\theta}^{-1}) \Rightarrow E[v|\mathcal{F}_i] - p = \frac{\gamma}{\tau_i} x_i^u + \frac{\gamma}{\tau_{\theta}} e_i$$

where x_i^u is her demand in the unconstrained economy. Substituting this into the certainty equivalent, we get

$$CE_{1,i} = -\frac{\gamma}{2\tau_i} \left(x_i^u - x_i\right)^2 + W_0 + \frac{\gamma}{2\tau_i} \left(x_i^u\right)^2 - \frac{\gamma}{2\tau_\theta} e_i^2.$$
(27)

Proof of Proposition 3. Step 1. Observing a signal $s_i = v + \epsilon_i$, with $\tau_{\epsilon,i} = t$ is informationally equivalent to observing a stochastic process $dX_s = vds + dB_s$, with $X_0 = 0$, between s = 0 and s = t. Here, B_s is a standard Brownian motion that is independent of all other random variables in the model.

The process X_s is Markovian, therefore the history $\{X_s\}_{s\in[0,t]}$ is informationally equivalent to $\{X_t\}$, which is, in turn, informationally equivalent to $\frac{1}{t}X_t = v + \frac{1}{t}B_t$. Since $\frac{1}{t}B_t \sim N(0, t^{-1})$, the conditional distributions of $\frac{1}{t}X_t$ and s_i are the same, which establishes the claim.

The above equivalence implies that the marginal values of information for a specialist who observes X_s , $s \in [0, t]$ and s_i are the same. The process X_s , however, is more convenient to work with, since we can utilize the stochastic calculus techniques when computing \mathcal{MVI} , as we do in the following step.

Step 2. $\mathcal{MVI} = \frac{E[\frac{1}{dt}E_t[dU_1(X_t,e_i,\phi)]]}{\gamma E[U_1(X_t,e_i,\phi)]}$, where $U_1(X_t) = E[-\exp(-\gamma W_2)|X_t,e_i,\phi]$ is t = 1 expected utility of a specialist and $E_t[\cdot]$ is a shortcut for $E[\cdot|X_t,e_i,\phi]$. Thus, computing \mathcal{MVI} reduces to calculating the drift $\frac{1}{dt}E_t[dU_1(X_t,e_i,\phi)]$ of the process $U_1(X_t)$.

It follows directly from the definition of \mathcal{MVI} that it can be computed as $\mathcal{MVI} = \frac{\frac{d}{dt}E[U_1]}{\gamma E[U_1]}$. Since $\frac{1}{dt}E_t[dU_1]$ is uniformly bounded by $\frac{\tau_i}{2\tau_{v,i}^2}U_1$ (follows from step 3 below) and $E\left[\frac{\tau_i}{2\tau_{v,i}^2}U_1\right] < \infty$ the Dominated Convergence Theorem implies that we can interchange the expectation and differentiation; that is, $\frac{d}{dt}E[U_1] = E\left[\frac{dU_1}{dt}\right]$. (By the law of iterated expectations, $E\left[\frac{dU_1}{dt}\right] = E\left[\frac{1}{dt}E_t[dU_1]\right]$.)

Step 3. The drift of the process $U_1(X_t)$ is given by

$$\frac{1}{dt}E_t[dU_1(X_t)]] = \begin{cases} \frac{\tau_i}{2\tau_{v,i}^2}U_1, & \text{if } x_i^u \in (a(p), b(p)), \\ 0, & \text{otherwise.} \end{cases}$$

We first consider the situation where agent *i* is unconstrained, that is, where $\{X_t, e_i, \phi\}$ are such that $x_i^u \in (a(p), b(p))$. We proceed by noting first that

$$\frac{1}{dt}E_t[dU_1(X_t, e_i, \phi)]] = -\gamma \frac{1}{dt}E_t[d\exp(-\gamma CE_{1,i})]],$$

where in the unconstrained region

$$CE_{1,i} = \frac{\tau_i}{2\gamma} \left(v_i - p - \gamma e_i \tau_{\theta}^{-1} \right)^2 + \text{terms that do not depend on } t \text{ and } X_t$$

We have denoted

$$v_i = E[v|\mathcal{F}_i] = \frac{X_t + \beta^2 (\tau_u + \tau_z) \phi + \beta \tau_u e_i}{\tau_{v,i}},$$

$$\tau_{v,i} = Var[v|\mathcal{F}_i] = t + \beta^2 (\tau_u + \tau_z) + \tau_v.$$

We use Ito's Lemma to compute 30

$$de^{-\gamma CE_{1,i}(X_t)} = -\gamma e^{-\gamma CE_{1,i}(X_t)} dCE_{1,i} + \frac{\gamma^2}{2} e^{-\gamma CE_{1,i}(X_t)} dCE_{1,i}^2.$$
(28)

Differentiating $CE_{1,i}$ we get

$$dCE_{1,i} = \frac{d\tau_i}{2\gamma} \left(v_i - p - \gamma e_i \tau_{\theta}^{-1} \right)^2 + \frac{\tau_i}{\gamma} \left(v_i - p - \gamma e_i \tau_{\theta}^{-1} \right) dv_i + \frac{\tau_i}{2\gamma} \left(dv_i \right)^2,$$

$$(dCE_{1,i})^2 = \left(\frac{\tau_i}{\gamma} \left(v_i - p - \gamma e_i \tau_{\theta}^{-1} \right) \right)^2 \left(dv_i \right)^2.$$
(29)

We now differentiate v_i and $\tau_i = \left(\frac{1}{\tau_{v,i}} + \frac{1}{\tau_{\theta}}\right)^{-1}$ to get

$$d\tau_{i} = \left(\frac{\tau_{i}}{\tau_{v,i}}\right)^{2} dt, \ dv_{i} = \frac{dX_{t}}{\tau_{v,i}} - \frac{dt}{\tau_{v,i}}v_{i}, \ \left(dv_{i}\right)^{2} = \left(\frac{dB_{t}}{\tau_{v,i}}\right)^{2} = \frac{dt}{\tau_{v,i}^{2}}.$$

We now compute $E_t \left[de^{-\gamma C E_{1,i}(X_t)} \right]$. Note that since $E_t[v] = v_i$ and $E_t[dB_t] = 0$, we have $E_t[dv_i] = 0$. Hence,

$$E_t[dCE_{1,i}] = \frac{d\tau_i}{2\gamma} \left(v_i - p - \gamma e_i \tau_{\theta}^{-1} \right)^2 + \frac{\tau_i}{2\gamma} \frac{dt}{\tau_{v,i}^2}.$$
(30)

Substituting (29) and (30) into (28), we get

$$E_t \left[de^{-\gamma C E_{1,i}(X_t)} \right] = -e^{-\gamma C E_1} \frac{\tau_i}{2} \frac{dt}{\tau_{v,i}^2}.$$
(31)

³⁰The function $CE_{1,i}(X_t)$ is not C^2 in X_t , which makes the standard Ito rule non-applicable. However, $CE_{1,i}(X_t)$ is convex, which allows us to apply the generalized Ito rule for convex functions (see, e.g., Karatzsas and Shreve (1991), Chapter 3.6.D). Moreover, since $CE_{1,i}$ is C^1 in X_t , the local time terms in the generalized Ito rule disappear and we can write the Ito's Lemma in the usual way.

We now consider the situation where agent *i* is constrained by lower bound, that is, where $\{X_t, e_i, \phi\}$ are such that $x_i^u < a(p)$. Then, $U_1 = E_t[-\exp(-\gamma(W_2))]$ is a martingale (since W_2 does not depend on X_t or *t*). Therefore, its drift is zero. Proceeding analogously for the case $\{X_t, e_i, \phi\}$ are such that $x_i^u > b(p)$ and combining the results, we obtain the desired result.

Step 4. The marginal value of information is given by

$$\mathcal{MVI} = \frac{\tau_i}{2\tau_{v,i}^2} \frac{E\left[e^{-\gamma CE_{1,i}}\mathbb{I}(x_i^u = x_i)\right]}{E\left[e^{-\gamma CE_{1,i}}\right]}.$$
(32)

The claim follows immediately from step 3.

Step 5. The marginal value of information decreases when individual specialist's constraints become tighter, holding everything else fixed.

Consider first the nominator in (32),

$$U_0^u = E\left[-e^{-\gamma C E_1} \mathbb{I}(x_i^u = x_i)\right] = E\left[-e^{-\left(W_0 + \frac{\gamma}{2\tau_i}(x_i^u)^2 - \frac{\gamma}{2\tau_\theta}e_i^2\right)} \mathbb{I}(x_i^u = x_i)\right].$$

As constraints tighten, only the $\mathbb{I}(x_i^u = x_i)$ changes: the price function is unaffected because constraints are changing for only one, measure-zero specialist; therefore the desired demands x_i^u are the same. The term U_0^u increases (becomes less negative) as constraints become tighter: recall that specialists get negative utility; as constraints become tighter, they get it in fewer states of the world. The denominator $U_0 = -\exp(-\gamma C E_{1,i})$ decreases (becomes more negative), as with constraints, the certainty equivalent $CE_{1,i}$ in all states weakly decreases. Thus, the ratio decreases as constraints become tighter.

A.4 Proof of Proposition 4

Proof. (Proposition 4) The existence and uniqueness of financial market equilibrium follows from Proposition 2. We now derive the formula for m^+ margin. It solves

$$\Pr^Q\left(p-f > m^+|p\right) = 1 - \alpha.$$

Lemma 9 (in online appendix) implies that p - f given p is distributed normally with mean zero and variance Var[f|p] under risk-neutral measure. Therefore, one can write

$$\Pr^{Q}\left(p-f > m^{+}|p\right) = \Pr^{Q}\left(\frac{p-f}{\sqrt{Var[f|p]}} > \frac{m^{+}}{\sqrt{Var[f|p]}}|p\right)$$
$$= 1 - \Phi\left(\frac{m^{+}}{\sqrt{Var[f|p]}}\right)$$

Analogous argument is true for m^- margin. Therefore, the margins are given by

$$m^+ = m^- = \frac{\Phi^{-1}(\alpha)}{\sqrt{Var[f|p]}}$$

where $Var[f|p]^{-1} = \frac{1}{\tau_v + \beta^2 \tau_z} + \frac{1}{\tau_{\theta}}$. As informational efficiency (β) decreases, margins increase. This implies that the constraint $a(p) = -\frac{W_0}{m^+(p)}$ decreases and the constraint $b(p) = \frac{W_0}{m^-(p)}$ increases. This implies that constraints tighten as informational efficiency decreases.

We now prove that $\frac{\partial^2 m}{\partial \alpha \partial \beta} < 0$. Differentiating (9), we get that

$$\frac{\partial m}{\partial \tau_{\epsilon}} = \frac{\partial m}{\partial \beta} \frac{\partial \beta}{\partial \tau_{\epsilon}} = -\frac{\Phi^{-1}(\alpha)}{\left(\tau_{v} + \beta^{2} \tau_{z}\right)^{2} \sqrt{\left(\tau_{v} + \beta^{2} \tau_{z}\right)^{-1} + \tau_{\theta}^{-1}}} \underbrace{\frac{\partial \beta}{\partial \tau_{\epsilon}}}_{>0, \text{ does not depend on } \alpha}.$$
(33)

Differentiating (33) with respect to α we get

$$\frac{\partial^2 m}{\partial \alpha \partial \tau_{\epsilon}} = -\frac{1}{\left(\tau_v + \beta^2 \tau_z\right)^2 \sqrt{\left(\tau_v + \beta^2 \tau_z\right)^{-1} + \tau_{\theta}^{-1}}} \underbrace{\frac{\partial \beta}{\partial \tau_{\epsilon}}}_{>0} \underbrace{\frac{\partial \Phi^{-1}(\alpha)}{\partial \alpha}}_{>0} < 0.$$

A.5 Proof of Proposition 5.

Proof of Proposition 5.

Step 1. In a stable equilibrium, τ_{ϵ}^* and equilibrium informational efficiency β decrease when W_0 drops and/or margin m increases for all specialists for all $W_0 < \hat{W}^1$. We prove that in a stable equilibrium, $\frac{d\tau_{\epsilon}^*}{dW_0} > 0$. The claims for m^+ and m^- can be proved analogously. We take the threshold \hat{W}^1 to be minimum between the thresholds in Lemma 6 and Lemma 3 so that both Lemmas hold.

Given that other specialists choose precision τ_{ϵ}^* , it is optimal for a specialist *i* to choose τ_{ϵ_i} such that

$$C'(\tau_{\epsilon_i}) = \mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon}^*, W_0) \quad \text{and} \quad (34)$$
$$C''(\tau_{\epsilon_i}) - \mathcal{MVI}_1(\tau_{\epsilon_i}, \tau_{\epsilon}^*, W_0) > 0.$$

The first (second) equation above corresponds to the first (second) order condition in specialist *i*'s optimization problem and $\mathcal{MVI}_k(\cdot, \cdot, \cdot)$ denotes the derivative of $\mathcal{MVI}(\cdot, \cdot, \cdot) > 0$ with respect to its *k*-th argument. Note that the second-order condition holds by Lemma 6 below. Differentiating (34) implicitly, we get

$$\tau_{\epsilon_i}'(\tau_{\epsilon}^*) = \frac{\mathcal{MVI}_2(\tau_{\epsilon_i}, \tau_{\epsilon}^*, W_0)}{C''(\tau_{\epsilon_i}) - \mathcal{MVI}_1(\tau_{\epsilon_i}, \tau_{\epsilon}^*, W_0)}.$$
(35)

In a symmetric equilibrium $\tau_{\epsilon_i} = \tau_{\epsilon}^*$, therefore

$$C'(\tau_{\epsilon}^{*}) = \mathcal{MVI}(\tau_{\epsilon}^{*}, \tau_{\epsilon}^{*}, W_{0}) \quad \text{and} \\ C''(\tau_{\epsilon}^{*}) - \mathcal{MVI}_{1}(\tau_{\epsilon}^{*}, \tau_{\epsilon}^{*}, W_{0}) > 0.$$

Moreover, since in a stable equilibrium $|\tau'_{\epsilon_i}(\tau^*_{\epsilon})| < 1$, from (35) we also have

$$C''(\tau_{\epsilon}^*) - \mathcal{MVI}_1(\tau_{\epsilon}^*, \tau_{\epsilon}^*, W_0) - \mathcal{MVI}_2(\tau_{\epsilon}^*, \tau_{\epsilon}^*, W_0) > 0.$$
(36)

To calculate $\frac{d\tau_{\epsilon}^*}{dW_0}$, we differentiate $C'(\tau_{\epsilon}^*(W_0)) = \mathcal{MVI}(\tau_{\epsilon}^*(W_0), \tau_{\epsilon}^*(W_0); W_0)$ with respect to W_0 to get

$$\frac{d\tau_{\epsilon}^*}{dW_0} = \frac{\mathcal{MVI}_3(\tau_{\epsilon}^*, \tau_{\epsilon}^*, W_0)}{C''(\tau_{\epsilon}^*) - \mathcal{MVI}_1(\tau_{\epsilon}^*, \tau_{\epsilon}^*, W_0) - \mathcal{MVI}_2(\tau_{\epsilon}^*, \tau_{\epsilon}^*, W_0)}.$$

It follows from Lemma 3 below that $\mathcal{MVI}_3(\tau_{\epsilon}^*, \tau_{\epsilon}^*, W_0) < 0$. Combining it with (36), we get $\frac{d\tau_{\epsilon}^*}{dW_0} > 0$. To see that $\frac{d\beta}{dW_0} > 0$, note that β satisfies equation (17) and β increases as specialists acquire more information.

Step 2. There exists at least one stable equilibrium.

It suffices to prove that $f(\tau_{\epsilon}) \equiv C'(\tau_{\epsilon}) - \mathcal{MVI}(\tau_{\epsilon}, \tau_{\epsilon}, W_0)$ crosses zero from below at least once for $\tau_{\epsilon} \in [\underline{\tau_{\epsilon}}, \infty)$. Note that $f(\underline{\tau_{\epsilon}}) < 0$. For sufficiently high τ_{ϵ} it becomes positive: this is because $C'(\tau_{\epsilon})$ is increasing in τ_{ϵ} , whereas $\mathcal{MVI}(\tau_{\epsilon}, \tau_{\epsilon}, W_0)$ is bounded above by $\frac{\tau_i}{2\tau_{v,i}^2\gamma}$, which is decreasing in τ_{ϵ} . By the Intermediate Value Theorem, $f(\tau_{\epsilon})$, has to cross zero from below at least once.

Lemma 3. There exists \hat{W}^1 such that for all $W_0 < \hat{W}^1$, we have $\frac{\partial}{\partial m^+} \mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon}; m^+, m^-, W_0) < 0$, $\frac{\partial}{\partial m^-} \mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon}; m^+, m^-, W_0) < 0$ and $\frac{\partial}{\partial W_0} \mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon}; m^+, m^-, W_0) > 0$.

Proof of Lemma 3.

We prove the statement for W_0 . The proofs of statements for m^+ and m^- are analogous and are omitted for brevity.

Step 1. We prove that

$$\lim_{W_0 \to 0} \frac{\partial}{\partial W_0} \mathcal{MVI}\left(\tau_{\epsilon_i}, \tau_{\epsilon}; m^+, m^-, W_0\right) > 0.$$
(37)

We write

$$\lim_{W_0 \to 0} \frac{\partial}{\partial W_0} \mathcal{MVI}\left(\tau_{\epsilon_i}, \tau_{\epsilon}; m^+, m^-, W_0\right) = \frac{\tau_i}{2\tau_{v,i}^2 \gamma} \lim_{W_0 \to 0} \left(\frac{\frac{\partial}{\partial W_0} U_0^u}{U_0} - \frac{U_0^u \frac{\partial}{\partial W_0} U_0}{U_0^2}\right)$$

Clearly, $\lim_{W_0\to 0} \frac{\partial}{\partial W_0} U_0 < \infty$ and $\lim_{W_0\to 0} U_0^u = 0$. Lemma 4 below proves that $\lim_{W_0\to 0} \frac{\partial}{\partial W_0} U_0^u = 0$. The statement of this step then follows.

Step 2. There exists \hat{W}^1 such that for all $0 < W_0 < \hat{W}^1$, $\frac{\partial}{\partial W_0} \mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon}; m^+, m^-, W_0) > 0$.

Denote the limit in (37) by λ . By epsilon-delta definition of a limit (see, e.g., Kolmogorov and Fomin (1975)) it follows that for any $\epsilon > 0$ there exists $\delta > 0$ such that for all W_0 : $|W_0| < \delta$, we have $\left|\frac{\partial}{\partial W_0}\mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon}; m^+, m^-, W_0) - \lambda\right| < \epsilon$. Taking $\epsilon = \psi$ we get that for all W_0 such that $0 < W_0 < \delta$, $\frac{\partial}{\partial W_0}\mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon}; m^+, m^-, W_0) > 0$. Thus, the desired statement holds if we take $\hat{W}^1 = \delta$.

Lemma 4. $\lim_{W_0 \to 0} \frac{\frac{\partial}{\partial W_0} U_0^u}{U_0} > 0.$

Proof of Lemma 4.

We first compute $\frac{\partial(-U_0^u)}{\partial W_0}$ as follows:

$$\frac{\partial \left(-U_{0}^{u}\right)}{\partial W_{0}} = \frac{\partial}{\partial W_{0}} E\left[e^{-\gamma C E_{1,i}} \mathbb{I}\left(x_{i}^{u} \in \left[-W_{0}/m^{-}; W_{0}/m^{+}\right]\right)\right] =$$
(38)

$$= E\left[e^{-\gamma C E_{1,i}} \left(\frac{1}{m^+} \delta\left(x_i^u - W_0/m^+\right) + \frac{1}{m^-} \delta\left(x_i^u + W_0/m^+\right)\right)\right]$$
(39)

$$+ E\left[\frac{\partial}{\partial W_0} \left(e^{-\gamma C E_{1,i}}\right) \mathbb{I}\left(x_i^u \in \left[-W_0/m^-; W_0/m^+\right]\right)\right],\tag{40}$$

where $\delta(\cdot)$ denotes Dirac's delta function. For the expression in (39) we have

$$\lim_{W_0 \to 0} E\left[e^{-\gamma C E_{1,i}}\left(\delta\left(x_i^u - W_0/m^+\right) + \delta\left(x_i^u + W_0/m^-\right)\right)\right] = \left(\frac{1}{m^+} + \frac{1}{m^-}\right) E\left[e^{-\gamma C E_{1,i}}\delta\left(x_i^u\right)\right] > 0.$$

We now consider the term in (40) and prove that its limit is zero, as $W_0 \to 0$. Note that in the unconstrained region,

$$CE_{1,i} = W_0 + \frac{\gamma}{2\tau_i} \left(x_i^u\right)^2 - \frac{\gamma}{2\tau_\theta} e_i^2 \implies \frac{\partial CE_{1,i}}{\partial W_0} = 1 + \frac{\gamma}{\tau_i} x_i^u \frac{\partial x_i^u}{\partial W_0}.$$

It then follows that

$$\left|\frac{\partial CE_{1,i}}{\partial W_0}\right| < 1 + \frac{\gamma}{\tau_i} A \left|\frac{\partial x_i^u}{\partial W_0}\right|,\tag{41}$$

where we have denoted

$$A = \max\left(\frac{W_0}{m^-}, \frac{W_0}{m^+}\right). \tag{42}$$

We show in Lemma 5 below that

$$\left|\frac{\partial x_i^u}{\partial W_0}\right| < \frac{c_p}{c_p^m \min(m^+, m^-)}.$$
(43)

We can write now

$$\begin{split} \left| E\left[\frac{\partial}{\partial W_0} \left(e^{-\gamma C E_{1,i}}\right) \mathbb{I}\left(x_i^u \in \left[-\frac{W_0}{m^-}; \frac{W_0}{m^+}\right]\right) \right] \right| &< E\left[\gamma e^{-\gamma C E_{1,i}} \left|\frac{\partial C E_{1,i}}{\partial W_0}\right| \mathbb{I}\left(x_i^u \in \left[-\frac{W_0}{m^-}; \frac{W_0}{m^+}\right]\right) \right] \\ &< \gamma E\left[e^{-\gamma C E_{1,i}} \left(1 + \frac{\gamma}{\tau_i} A \frac{c_p}{c_p^m m^+}\right) \mathbb{I}\left(x_i^u \in \left[\frac{W_0}{m^-}; \frac{W_0}{m^+}\right]\right) \right] \\ &= \gamma \left(1 + \frac{\gamma}{\tau_i} A \frac{c_p}{c_p^m m^+}\right) E\left[e^{-\gamma C E_{1,i}} \mathbb{I}\left(x_i^u \in \left[-\frac{W_0}{m^-}; \frac{W_0}{m^+}\right]\right)\right]. \end{split}$$

Since $\lim_{W_0 \to 0} E\left[e^{-\gamma C E_{1,i}} \mathbb{I}\left(x_i^u \in \left[-\frac{W_0}{m^-}; \frac{W_0}{m^+}\right]\right)\right] = 0$, the statement follows. **Lemma 5.** $\left|\frac{\partial x_i^u}{\partial W_0}\right| < \frac{c_p}{c_p^m \min(m^+, m^-)}.$

Proof of Lemma 5. The unconstrained demand is given by

$$x_i^u = c_\phi \phi - c_p l(\phi; W_0) + \xi_i$$

and depends on W_0 only through $l(\phi, W_0) \equiv g^{-1}(\phi)$ (i.e., the inverse of g(p) for a given W_0). Therefore

$$\frac{\partial x_i^u}{\partial W_0} = -c_p \frac{\partial l(\phi; W_0)}{\partial W_0}$$

The inverse of g(p), $l(\phi)$ solves

We thus have that

$$X_{W_0}^{agg}(l(\phi; W_0), \phi; W_0) + X_p^{agg}(l(\phi; W_0), \phi; W_0) \frac{\partial l(\phi; W_0)}{\partial W_0} = 0.$$

 $X^{agg}(l(\phi), \phi; W_0) = 1.$

Computing the derivatives of aggregate demand and expressing $\frac{\partial l(\phi; W_0)}{\partial W_0}$ yields

$$\frac{\partial l(\phi; W_0)}{\partial W_0} = \frac{(\pi_3/m^+ - \pi_1/m^-)}{c_p^m + \pi_2 c_p}.$$

It is clear from above that

$$\left|\frac{\partial l(\phi;W_0)}{\partial W_0}\right| < \frac{1}{c_p^m\min(m^+,m^-)}$$

The statement follows. \blacksquare

Lemma 6. There exists a threshold \hat{W}^1 such that for all $W_0 \in (0, \hat{W}^1)$ we have $\frac{\partial \mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon}; W_0)}{\partial \tau_{\epsilon, i}} - C''(\tau_{\epsilon, i}) < 0.$

Proof of Lemma 6. We know that

$$\mathcal{MVI}\left(\tau_{\epsilon.i}, \tau_{\epsilon}; W_{0}\right) = \frac{\tau_{i}}{2\tau_{v,i}^{2}\gamma} \frac{U_{0}^{u}}{U_{0}}.$$

Hence,

$$\frac{\partial \mathcal{MVI}\left(\tau_{\epsilon_{i}},\tau_{\epsilon};W_{0}\right)}{\partial \tau_{\epsilon,i}} = \frac{U_{0}^{u}}{U_{0}}\frac{\partial}{\partial \tau_{\epsilon,i}}\left(\frac{\tau_{i}}{2\tau_{v,i}^{2}\gamma}\right) - \frac{\tau_{i}}{2\tau_{v,i}^{2}\gamma}\frac{U_{0}^{u}}{U_{0}^{2}}\frac{\partial U_{0}}{\partial \tau_{\epsilon,i}} + \frac{\tau_{i}}{2\tau_{v,i}^{2}\gamma}\frac{1}{U_{0}}\frac{\partial U_{0}^{u}}{\partial \tau_{\epsilon,i}}$$

We prove below that

$$\lim_{W_0 \to 0} \frac{\partial \mathcal{MVI}\left(\tau_{\epsilon_i}, \tau_{\epsilon}; W_0\right)}{\partial \tau_{\epsilon, i}} = 0.$$

First, it is clear that

$$\lim_{W_0 \to 0} \underbrace{\frac{U_0^u}{U_0}}_{\to 0} \underbrace{\frac{\partial}{\partial \tau_{\epsilon,i}} \left(\frac{\tau_i}{2\tau_{v,i}^2 \gamma}\right)}_{<\infty} = 0.$$

We then proceed in three steps. In step 1 we show that $\lim_{W_0 \to 0} \frac{\partial U_0}{\partial \tau_{\epsilon,i}} = 0$. In step 2 we show that $\lim_{W_0 \to 0} \frac{\partial U_0^u}{\partial \tau_{\epsilon,i}} = 0$. In step 3 we combine these results to prove the statement of the proposition.

Step 1. $\lim_{W_0\to 0} \frac{\partial U_0}{\partial \tau_{\epsilon,i}} = 0$. Recall that

$$CE_{1,i} = W_0 + \frac{\gamma}{2\tau_i} (x_i^u)^2 - \frac{\gamma}{2\tau_\theta} e_i^2 - \frac{\gamma}{2\tau_i} (x_i^u - x_i)^2$$

Hence,

$$\frac{\partial CE_{1,i}}{\partial \tau_{\epsilon,i}} = 1 + \frac{\gamma}{\tau_i} x_i^u \frac{\partial x_i^u}{\partial \tau_{\epsilon,i}} - \frac{\gamma}{\tau_i} \left(x_i^u - x_i \right) \left(\frac{\partial x_i^u}{\partial \tau_{\epsilon,i}} - \frac{\partial x_i}{\partial \tau_{\epsilon,i}} \right) + \frac{\gamma}{2} \left(\left(x_i^u \right)^2 - \left(x_i^u - x_i \right)^2 \right) \frac{\partial}{\partial \tau_{\epsilon,i}} \left(\frac{1}{\tau_i} \right)$$

$$= 1 + \frac{\gamma}{\tau_i} x_i \frac{\partial x_i^u}{\partial \tau_{\epsilon,i}} + \frac{\gamma}{\tau_i} \left(x_i^u - x_i \right) \frac{\partial x_i}{\partial \tau_{\epsilon,i}} + \frac{\gamma}{2} \left(\left(x_i^u \right)^2 - \left(x_i^u - x_i \right)^2 \right) \frac{\partial}{\partial \tau_{\epsilon,i}} \left(\frac{1}{\tau_i} \right).$$
(44)

Note that

$$\lim_{W_0 \to 0} \underbrace{x_i}_{\to 0} \underbrace{\frac{\partial x_i^u}{\partial \tau_{\epsilon,i}}}_{<\infty} = 0$$

Consider $x_i^u > x_i$. For the third term in (44), $\frac{\gamma}{\tau_i} (x_i^u - x_i) \frac{dx_i}{d\tau_{e,i}} = 0$ (recall that $x_i = W_0/m$ in that case). A similar argument shows that this term goes to 0 for $x_i^u < x_i$. For $x_i^u = x_i$, this term is zero. Thus,

$$\lim_{W_0 \to 0} \frac{\gamma}{\tau_i} \left(x_i^u - x_i \right) \frac{dx_i}{d\tau_{\epsilon,i}} = 0$$

Since

$$\lim_{W_0 \to 0} \left(\left(x_i^u \right)^2 - \left(x_i^u - \underbrace{x_i}_{\to 0} \right)^2 \right) = 0,$$

we have

$$\lim_{W_0 \to 0} \frac{\gamma}{2} \underbrace{\left((x_i^u)^2 - (x_i^u - x_i)^2 \right)}_{\to 0} \underbrace{\frac{d}{d\tau_{\epsilon,i}} \left(\frac{1}{\tau_i} \right)}_{<\infty} = 0.$$

Thus,

$$\lim_{W_0 \to 0} \frac{\partial U_0}{\partial \tau_{\epsilon,i}} = 0.$$

Step 2. $\lim_{W_0 \to 0} \frac{\partial U_0^u}{\partial \tau_{\epsilon,i}} = 0.$

We write the desired demand x_i^u as follows:

$$x_i^u = E[x_i^u | v, z, u_i] + \sqrt{Var[x_i^u | v, z, u_i]} \epsilon_i^n,$$

where $\epsilon_i^n = \tau_{\epsilon,i}^{-1/2} \epsilon_i$ and $\epsilon_i^n | \{v, z, u_i\}$ is distributed normally with zero mean and unit variance. Denote

$$r \equiv \frac{E[x_i^u|v, z, u_i]}{\sqrt{Var[x_i^u|v, z, u_i]}}, \text{ and } q \equiv \frac{1}{m\left(\tau_\epsilon\right)\sqrt{Var[x_i^u|v, z, u_i]}}.$$
(45)

Following the steps of Proposition 1, from the expression for unconstrained demand one can derive

$$\sqrt{Var[x_i^u|v, z, u_i]} = \frac{\tau_i \sqrt{\tau_{\epsilon, i}}}{\gamma \tau_{v, i}}$$

and similarly, for r,

$$r = \frac{E[x_i|v, z, u_i]}{\sqrt{Var[x_i|v, z, u_i]}}$$

$$\tag{46}$$

$$=\frac{\tau_i}{\gamma\sqrt{Var[x_i|v,z,u_i]}}\left(\frac{\tau_{\epsilon,i}v+\beta^2\left(\tau_u+\tau_z\right)\phi+\beta\tau_u e_i}{\tau_{v,i}}-p-\gamma e_i\tau_{\theta}^{-1}\right).$$
(47)

We write

$$\lim_{W_0 \to 0} \frac{d}{d\tau_{\epsilon,i}} U_0^u(\tau_{\epsilon_i}, \tau_{\epsilon}) =$$
(48)

$$= -\lim_{W_0 \to 0} \int \gamma e^{-\gamma C E_{1,i}} \frac{\partial C E_{1,i}}{\partial \tau_{\epsilon,i}} \mathbb{I}\left(\epsilon_i^n \in \left[-qW_0 - r; qW_0 - r\right]\right) dF_v dF_z dF_{u_i} dF_{\epsilon_i^n} +$$
(49)

$$+\lim_{W_0\to 0} W_0 \int \frac{\partial q}{\partial \tau_{\epsilon,i}} \left(e^{-\gamma C E_{1,i}} \left(\delta\left(\epsilon_i^n + q W_0 + r\right) + \delta\left(\epsilon_i^n - q W_0 + r\right) \right) \right) dF_v dF_z dF_{u_i} dF_{\epsilon_i^n} + \tag{50}$$

$$-\lim_{W_0\to 0} \int \frac{\partial r}{\partial \tau_{\epsilon,i}} \left(e^{-\gamma C E_{1,i}} \left(\delta \left(\epsilon_i^n - q W_0 + r \right) - \delta \left(\epsilon_i^n + q W_0 + r \right) \right) \right) dF_v dF_z dF_{u_i} dF_{\epsilon_i^n}.$$
(51)

The limit in (49) is

$$\lim_{W_0 \to 0} \int \gamma e^{-\gamma C E_{1,i}} \frac{\partial C E_{1,i}}{\partial \tau_{\epsilon,i}} \mathbb{I}\left(\epsilon_i^n \in \left[-qW_0 - r; qW_0 - r\right]\right) dF_v dF_z dF_{u_i} dF_{\epsilon_i^n} = 0.$$

The limit in (50) is simply

$$\lim_{W_0 \to 0} W_0 \int \frac{\partial q}{\partial \tau_{\epsilon,i}} \left(e^{-\gamma C E_{1,i}} \left(\delta \left(\epsilon_i^n + q W_0 + r \right) + \delta \left(\epsilon_i^n - q W_0 + r \right) \right) \right) dF_v dF_z dF_{u_i} dF_{\epsilon_i^n} = \lim_{W_0 \to 0} 2W_0 \int e^{-\gamma C E_{1,i}} \delta \left(\epsilon_i^n + r \right) dF_v dF_z dF_{u_i} dF_{\epsilon_i^n} = 0.$$

For (51), we write

$$\lim_{W_0 \to 0} \int \frac{\partial r}{\partial \tau_{\epsilon,i}} \left(e^{-\gamma C E_{1,i}} \left(\delta \left(\epsilon_i^n - q W_0 + r \right) - \delta \left(\epsilon_i^n + q W_0 + r \right) \right) \right) dF_v dF_z dF_{u_i} dF_{\epsilon_i^n} = 0.$$

Step 3. There exists threshold \hat{W} such that for all $W_0 \in (0, \hat{W})$, we have $\frac{\partial \mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon}; W_0)}{\partial \tau_{\epsilon,i}} - C''(\tau_{\epsilon,i}) < 0$. By the previous steps of the proposition,

$$\lim_{W_0 \to 0} \frac{\partial \mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon}; W_0)}{\partial \tau_{\epsilon, i}} = 0.$$

Given the strict convexity of the cost function,

$$\lim_{W_0 \to 0} \left(\frac{\partial \mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon}; W_0)}{\partial \tau_{\epsilon, i}} - C''(\tau_{\epsilon, i}) \right) < 0.$$

Denote this limit $\lambda < 0$ and write epsilon-delta definition of a limit (see, e.g., Kolmogorov and Fomin (1975)): for any $\epsilon > 0$, there exists $\delta > 0$ such that for all $0 < W_0 < \delta$, we have $\left| \frac{\partial \mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon_i}; W_0)}{\partial \tau_{\epsilon,i}} - C''(\tau_{\epsilon,i}) - \lambda \right| < \epsilon$. Take $\epsilon = \lambda$. For such ϵ there exists $\delta > 0$ such that for all W_0 such that $0 < W_0 < \delta$, we have $\frac{\partial \mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon_i}; W_0)}{\partial \tau_{\epsilon,i}} - C''(\tau_{\epsilon,i}) < 0$. Therefore, the desired statement holds if we take $\hat{W}^1 = \delta$.

A.6 Proof of Proposition 6

Proof of Proposition 6.

Denote the marginal value of information and time-0 certainty equivalent when a specialist *i* chooses his signal precision $\tau_{\epsilon,i}$, other specialists choose precision τ_{ϵ} , and a financier has set the margins $m^+ = m^- = m$ for a specialist of interest as $\mathcal{MVI}(\tau_{\epsilon,i}, \tau_{\epsilon}, m)$ and $CE_{i,0}(\tau_{\epsilon,i}, \tau_{\epsilon}, m)$ respectively.

In equilibrium,

$$m = m(\tau_{\epsilon}) = \Phi^{-1}(\alpha)\sqrt{(\tau_{\nu} + \beta^2(\tau_{\epsilon})\tau_z)^{-1} + \tau_{\theta}^{-1}}.$$

By Proposition 2, the financial market equilibrium (at t = 1) exists and is unique. Similarly to the proof of Proposition 5, at least one stable full equilibrium exists provided that (i) $\mathcal{MVI}(\tau_{\epsilon}, \tau_{\epsilon}, m(\tau_{\epsilon}))$ crosses $C(\tau_{\epsilon})$ from above and (ii) the point of intersection is indeed a maximum. We prove these statements in the three steps below.

Step 1. There exists at least one $\tau_{\epsilon} = \tau_{\epsilon}^*$ such that $\mathcal{MVI}(\tau_{\epsilon}, \tau_{\epsilon}, m(\tau_{\epsilon}))$ crosses $C(\tau_{\epsilon})$ from above at $\tau_{\epsilon} = \tau_{\epsilon}^*$.

It suffices to prove that $f(\tau_{\epsilon}) \equiv C'(\tau_{\epsilon}) - \mathcal{MVI}(\tau_{\epsilon}, \tau_{\epsilon}, m(\tau_{\epsilon}))$ crosses zero from below at least once for $\tau_{\epsilon} \in (\underline{\tau_{\epsilon}}, \infty)$. Note that $f(\underline{\tau_{\epsilon}}) < 0$. For sufficiently high τ_{ϵ} , it becomes positive: this is because $C'(\tau_{\epsilon})$ is increasing in τ_{ϵ} , whereas $\mathcal{MVI}(\tau_{\epsilon}, \tau_{\epsilon}, W_0)$ is bounded above by $\frac{\tau_i}{2\tau_{v,i}^2\gamma}$, which is decreasing in τ_{ϵ} . By the Intermediate Value Theorem, $f(\tau_{\epsilon})$ has to cross zero from below at least once.

Step 2. Denote
$$\hat{\tau}_{\epsilon}$$
 the t that solves $\frac{\tau_i}{2\tau_{v,i}^2 \gamma}\Big|_{\tau_{\epsilon,i}=t,\tau_{\epsilon}=\tau_{\epsilon}^*} = C'(t)$. There are no maxima of
 $CE_{i,0}(\tau_{\epsilon,i},\tau_{\epsilon}^*,m(\tau_{\epsilon}^*)) - C(\tau_{\epsilon,i}),$
(52)

for $\tau_{\epsilon,i} > \hat{\tau_{\epsilon}}$.

Indeed, $\mathcal{MVI}(\tau_{\epsilon,i}, \tau_{\epsilon}^*, m(\tau_{\epsilon}^*)) < \frac{\tau_i}{2\tau_{v,i}^2 \gamma} \Big|_{\tau_{\epsilon} = \tau_{\epsilon}^*} < C'(\tau_{\epsilon,i}), \text{ for } \tau_{\epsilon,i} > \hat{\tau_{\epsilon}}.$ This implies that $CE_{i,0}(\tau_{\epsilon,i}, \tau_{\epsilon}^*, m(\tau_{\epsilon}^*)) - C(\tau_{\epsilon,i})$ decreases for $\tau_{\epsilon,i} > \hat{\tau_{\epsilon}}$ and thus has no maxima in that region.

Denote

$$\underline{C} = \max_{\tau_{\epsilon,i} \in [\underline{\tau_{\epsilon}}, \hat{\tau_{\epsilon}}]} \frac{\partial \mathcal{MVI}(\tau_{\epsilon,i}, \tau_{\epsilon}^{*}, m(\tau_{\epsilon}^{*}))}{\partial \tau_{\epsilon,i}}.$$

Such maximum exists by the Weierstrass extreme value theorem.

Step 3. If $C''(\tau_{\epsilon,i}) > \underline{C}$ for $\tau_{\epsilon,i} \in [\underline{\tau_{\epsilon}}, \hat{\tau_{\epsilon}}]$ then the function (52) attains a maximum at $\tau_{\epsilon,i} = \tau_{\epsilon}^*$.

Condition $C''(\tau_{\epsilon,i}) > \underline{C}$ for $\tau_{\epsilon,i} \in [\underline{\tau_{\epsilon}}, \hat{\tau_{\epsilon}}]$ implies that the investor *i*'s information choice problem is convex on $[\underline{\tau_{\epsilon}}, \hat{\tau_{\epsilon}}]$. Then τ_{ϵ}^* is a maximum of (52) on $[\underline{\tau_{\epsilon}}, \hat{\tau_{\epsilon}}]$. By step 2, there are no maxima on $(\underline{\tau_{\epsilon}}, \infty)$. Thus, τ_{ϵ}^* is a maximum of (52) on $[\underline{\tau_{\epsilon}}, \infty)$.

A.7 Proof of Proposition 7

Proof of Proposition 7. The proof follows closely the proof of Proposition 5.

Step 1. In a stable equilibrium, when W_0 drops, τ_{ϵ}^* and β decrease, whereas margin m increases for all specialists for all $W_0 < \hat{W}^2$. We prove that in a stable equilibrium, $\frac{d\tau_{\epsilon}^*}{dW_0} > 0$. The claims for β and $m = m^+ = m^-$ follow because these are monotone functions of τ_{ϵ}^* .

Given that other specialists choose precision τ_{ϵ}^* , it is optimal for a specialist *i* to choose τ_{ϵ_i} such that:

$$C'(\tau_{\epsilon_i}) = \mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon}^*, m(\tau_{\epsilon}^*), W_0),$$
(53)

$$C''(\tau_{\epsilon_i}) - \mathcal{MVI}_1(\tau_{\epsilon_i}, \tau_{\epsilon}^*, m(\tau_{\epsilon}^*), W_0) > 0.$$

The first (second) equation above corresponds to the first (second) order condition in specialist *i*'s optimization problem and $\mathcal{MVI}_k(\cdot, \cdot, \cdot, \cdot)$ denotes the derivative of $\mathcal{MVI}(\cdot, \cdot, \cdot, \cdot) > 0$ with respect to its *k*-th argument. Note that the second-order condition holds since Assumption 3 holds. Differentiating (53) implicitly, we get

$$\tau_{\epsilon_i}'(\tau_{\epsilon}^*) = \frac{\mathcal{MVI}_2(\tau_{\epsilon_i}, \tau_{\epsilon}^*, m(\tau_{\epsilon}^*), W_0) + \mathcal{MVI}_3(\tau_{\epsilon_i}, \tau_{\epsilon}^*, m(\tau_{\epsilon}^*)W_0)m'(\tau_{\epsilon}^*)}{C''(\tau_{\epsilon_i}) - \mathcal{MVI}_1(\tau_{\epsilon_i}, \tau_{\epsilon}^*, W_0)}.$$
(54)

In a symmetric equilibrium $\tau_{\epsilon_i} = \tau_{\epsilon}^*$, therefore

$$C'(\tau_{\epsilon}^*) = \mathcal{MVI}(\tau_{\epsilon}^*, \tau_{\epsilon}^*, m(\tau_{\epsilon}^*), W_0),$$

$$C''(\tau_{\epsilon}^*) - \mathcal{MVI}_1(\tau_{\epsilon}^*, \tau_{\epsilon}^*, m(\tau_{\epsilon}^*), W_0) > 0.$$

Moreover, since in a stable equilibrium $|\tau_{\epsilon_i}'(\tau_{\epsilon}^*)| < 1$, from (54) we also have

$$C''(\tau_{\epsilon}^{*}) - \mathcal{MVI}_{1}(\tau_{\epsilon}^{*}, \tau_{\epsilon}^{*}, m(\tau_{\epsilon}^{*}), W_{0}) - \mathcal{MVI}_{2}(\tau_{\epsilon}^{*}, \tau_{\epsilon}^{*}, m(\tau_{\epsilon}^{*}), W_{0}) - \mathcal{MVI}_{3}(\tau_{\epsilon}^{*}, \tau_{\epsilon}^{*}, m(\tau_{\epsilon}^{*}), W_{0})m'(\tau_{\epsilon}^{*}) > 0.$$
(55)

To calculate $\frac{d\tau_{\epsilon}^*}{dW_0}$, we differentiate $C'(\tau_{\epsilon}^*(W_0)) = \mathcal{MVI}(\tau_{\epsilon}^*(W_0), \tau_{\epsilon}^*(W_0), m(\tau_{\epsilon}^*(W_0)); W_0)$ with respect to W_0 to get

$$\frac{d\tau_{\epsilon}^{*}}{dW_{0}} = \frac{\mathcal{MVI}_{4}(\tau_{\epsilon}^{*},\tau_{\epsilon}^{*},m(\tau_{\epsilon}^{*}),W_{0})}{C^{\prime\prime}(\tau_{\epsilon}^{*}) - \mathcal{MVI}_{1}(\tau_{\epsilon}^{*},\tau_{\epsilon}^{*},m(\tau_{\epsilon}^{*}),W_{0}) - \mathcal{MVI}_{2}(\tau_{\epsilon}^{*},\tau_{\epsilon}^{*},m(\tau_{\epsilon}^{*}),W_{0}) - \mathcal{MVI}_{3}(\tau_{\epsilon}^{*},\tau_{\epsilon}^{*},m(\tau_{\epsilon}^{*}),W_{0})m^{\prime}(\tau_{\epsilon}^{*})}$$

It follows from Lemma 3 that $\mathcal{MVI}_4(\tau_{\epsilon}^*, \tau_{\epsilon}^*, m(\tau_{\epsilon}^*), W_0) < 0$. Combining it with (55) we get $\frac{d\tau_{\epsilon}^*}{dW_0} > 0$.

Step 2. There exists at least one stable equilibrium.

It suffices to prove that $f(\tau_{\epsilon}) \equiv C'(\tau_{\epsilon}) - \mathcal{MVI}(\tau_{\epsilon}, \tau_{\epsilon}, m(\tau_{\epsilon}^{*}), W_{0})$ crosses zero from below at least once for $\tau_{\epsilon} \in [\underline{\tau_{\epsilon}}, \infty)$. Note that $f(\underline{\tau_{\epsilon}}) < 0$. For sufficiently high τ_{ϵ} , it becomes positive; this is because $C'(\tau_{\epsilon})$ is increasing in τ_{ϵ} , whereas $\mathcal{MVI}(\tau_{\epsilon}, \tau_{\epsilon}, m(\tau_{\epsilon}), W_{0})$ is bounded above by $\frac{\tau_{i}}{2\tau_{v,i}^{2}\gamma}$, which is decreasing in τ_{ϵ} . By the Intermediate Value Theorem, $f(\tau_{\epsilon})$ has to cross zero from below at least once.

A.8 Proof of Proposition 8

Proof of Proposition 8. We know that

$$\mathcal{MVI}\left(\tau_{\epsilon_{i}}, \tau_{\epsilon}; W_{0}\right) = \frac{\tau_{i}}{2\tau_{v,i}^{2}\gamma} \frac{U_{0}^{u}}{U_{0}},$$

where $U_0^u(\tau_{\epsilon_i}, \tau_{\epsilon}) = E[-e^{-\gamma C E_{1,i}} \mathbb{I}_{x_i^u = x_i}]$ and $U_0(\tau_{\epsilon_i}, \tau_{\epsilon}) = E[-e^{-\gamma C E_{1,i}}]$. Since

$$\frac{\partial \log \mathcal{MVI}}{\partial \tau_{\epsilon}} = \frac{1}{\mathcal{MVI}} \frac{\partial \mathcal{MVI}}{\partial \tau_{\epsilon}}$$

and $\mathcal{MVI} > 0$, we may instead prove that $\frac{\partial \log \mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon}; W_0)}{\partial \tau_{\epsilon}} > 0$. We write

$$\frac{\partial \log \mathcal{MVI}}{\partial \tau_{\epsilon}} = \frac{\partial}{\partial \tau_{\epsilon}} \log \left(\frac{\tau_i}{2\gamma \tau_{v,i}^2} \right) + \frac{\partial \log(-U_0^u)}{\partial \tau_{\epsilon}} - \frac{\partial \log(-U_0)}{\partial \tau_{\epsilon}}.$$

We then proceed in three steps. In step 1, we show $\lim_{W_0 \to 0} \frac{\partial \log(-U_0)}{\partial \tau_{\epsilon}} = 0$. In step 2, we derive the closed-form expression for $\lim_{W_0 \to 0} \frac{\partial \log(-U_0^u)}{\partial \tau_{\epsilon}}$. In step 3, we combine these results to prove the proposition.

Step 1. $\lim_{W_0 \to 0} \frac{\partial \log(-U_0)}{\partial \tau_{\epsilon}} = 0.$

Recall that

$$CE_{1,i} = W_0 + \frac{\gamma}{2\tau_i} (x_i^u)^2 - \frac{\gamma}{2\tau_\theta} e_i^2 - \frac{\gamma}{2\tau_i} (x_i^u - x_i)^2.$$

Hence,

$$\frac{\partial CE_{1,i}}{\partial \tau_{\epsilon}} = 1 + \frac{\gamma}{\tau_{i}} x_{i}^{u} \frac{\partial x_{i}^{u}}{\partial \tau_{\epsilon}} - \frac{\gamma}{\tau_{i}} \left(x_{i}^{u} - x_{i} \right) \left(\frac{\partial x_{i}^{u}}{\partial \tau_{\epsilon}} - \frac{\partial x_{i}}{\partial \tau_{\epsilon}} \right) + \frac{\gamma}{2} \left(\left(x_{i}^{u} \right)^{2} - \left(x_{i}^{u} - x_{i} \right)^{2} \right) \frac{\partial}{\partial \tau_{\epsilon}} \left(\frac{1}{\tau_{i}} \right)$$

$$= 1 + \frac{\gamma}{\tau_{i}} x_{i} \frac{\partial x_{i}^{u}}{\partial \tau_{\epsilon}} + \frac{\gamma}{\tau_{i}} \left(x_{i}^{u} - x_{i} \right) \frac{\partial x_{i}}{\partial \tau_{\epsilon}} + \frac{\gamma}{2} \left(\left(x_{i}^{u} \right)^{2} - \left(x_{i}^{u} - x_{i} \right)^{2} \right) \frac{\partial}{\partial \tau_{\epsilon}} \left(\frac{1}{\tau_{i}} \right).$$
(56)

Note that

$$\lim_{W_0 \to 0} \underbrace{x_i}_{\to 0} \underbrace{\frac{\partial x_i^u}{\partial \tau_{\epsilon}}}_{<\infty} = 0$$

Consider $x_i^u > x_i$. For the third term in (56), $\frac{\gamma}{\tau_i} (x_i^u - x_i) \frac{dx_i}{d\tau_{\epsilon}}$, we have (recall that $x_i = W_0/m(\tau_{\epsilon})$ in that case)

$$\begin{aligned} \left| \frac{\gamma}{\tau_i} \left(x_i^u - x_i \right) \frac{dx_i}{d\tau_{\epsilon}} \right| &< \frac{\gamma}{\tau_i} \left| x_i^u \right| \left| \frac{dx_i}{d\tau_{\epsilon}} \right| \\ &= \frac{\gamma}{\tau_i} \underbrace{|x_i^u|}_{<\infty} \frac{W_0}{m^2} m'(\tau_{\epsilon}) \\ &\to 0 \text{ as } W_0 \to 0. \end{aligned}$$

A similar argument shows that this term goes to 0 for $x_i^u < x_i$. For $x_i^u = x_i$, this term is zero. Thus,

$$\lim_{W_0 \to 0} \frac{\gamma}{\tau_i} \left(x_i^u - x_i \right) \frac{dx_i}{d\tau_\epsilon} = 0$$

Since

$$\lim_{W_0 \to 0} \left((x_i^u)^2 - \left(x_i^u - \underbrace{x_i}_{\to 0} \right)^2 \right) = 0,$$

we have

$$\lim_{W_0 \to 0} \frac{\gamma}{2} \underbrace{\left((x_i^u)^2 - (x_i^u - x_i)^2 \right)}_{\to 0} \underbrace{\frac{d}{d\tau_\epsilon} \left(\frac{1}{\tau_i} \right)}_{<\infty} = 0.$$

Thus,

$$\lim_{W_0 \to 0} \frac{d \log U_0}{d \tau_{\epsilon}} = 0.$$

Step 2. $\lim_{W_0 \to 0} \frac{\partial \log(-U_0^u)}{\partial \tau_{\epsilon}} = \frac{\partial \log(q\psi)}{\partial \tau_{\epsilon}}$, where q is given by (45) and ψ is given by (66).

We now consider

$$\frac{\partial \log \left(-U_0^u\right)}{\partial \tau_{\epsilon}} = \frac{1}{-U_0^u} \frac{\partial \left(-U_0^u\right)}{\partial \tau_{\epsilon}}$$

We write the desired demand x_i^u as follows:

$$x_i^u = E[x_i^u | v, z, u_i] + \sqrt{Var[x_i^u | v, z, u_i]} \epsilon_i^n,$$

where $\epsilon_i^n = \tau_{\epsilon,i}^{-1/2} \epsilon_i$ and $\epsilon_i^n | \{v, z, u_i\}$ is distributed normally with zero mean and unit variance. Denote

$$r \equiv \frac{E[x_i^u | v, z, u_i]}{\sqrt{Var[x_i^u | v, z, u_i]}}, \text{ and } q \equiv \frac{1}{m(\tau_\epsilon)\sqrt{Var[x_i^u | v, z, u_i]}}.$$
(57)

Following the steps of Proposition 1, from the expression for unconstrained demand one can derive

$$\sqrt{Var[x_i^u|v, z, u_i]} = \frac{\tau_i \sqrt{\tau_{\epsilon, i}}}{\gamma \tau_{v, i}},$$

and similarly, for r,

$$r = \frac{E[x_i|v, z, u_i]}{\sqrt{Var[x_i|v, z, u_i]}}$$
(58)

$$=\frac{\tau_i}{\gamma\sqrt{Var[x_i|v,z,u_i]}}\left(\frac{\tau_{\epsilon,i}v+\beta^2\left(\tau_u+\tau_z\right)\phi+\beta\tau_u e_i}{\tau_{v,i}}-p-\gamma e_i\tau_{\theta}^{-1}\right).$$
(59)

Multiply U_0^u by $\frac{1}{2qW_0}$ and write

$$\begin{split} \lim_{W_0 \to 0} -\frac{U_0^u}{2qW_0} &= \lim_{W_0 \to 0} \int e^{-\gamma C E_{1,i}} \frac{\mathbb{I}\left(\epsilon_i^n \in \left[-qW_0 - r; qW_0 - r\right]\right)}{2qW_0} dF_v dF_z dF_{u_i} dF_{\epsilon_i^n} \\ &= \int e^{-\gamma C E_{1,i}} \delta\left(\epsilon_i^n + r\right) dF_v dF_z dF_{u_i} dF_{\epsilon_i^n} \\ &= \frac{1}{\sqrt{2\pi}} \int \exp\left(\frac{\gamma^2}{2\tau_\theta} e_i^2 - \frac{r^2}{2}\right) dF_v dF_z dF_{u_i} \\ &\equiv \psi. \end{split}$$

In the above, $\delta(\cdot)$ denotes Dirac's delta function. Note that ψ can be computed in closed form, which we do in Lemma 7 below.

Similarly, multiply $\frac{d(-U_0^u)}{d\tau_{\epsilon}}$ by $\frac{1}{2qW_0}$ and write

$$\lim_{W_0 \to 0} \frac{-1}{2qW_0} \frac{d}{d\tau_{\epsilon}} U_0^u(\tau_{\epsilon_i}, \tau_{\epsilon}) =$$
(60)

$$= -\lim_{W_0 \to 0} \int \gamma e^{-\gamma C E_{1,i}} \frac{\partial C E_{1,i}}{\partial \tau_{\epsilon}} \frac{\mathbb{I}\left(\epsilon_i^n \in \left[-qW_0 - r; qW_0 - r\right]\right)}{2qW_0} dF_v dF_z dF_{u_i} dF_{\epsilon_i^n} +$$
(61)

$$+\lim_{W_0\to 0} \int \frac{1}{2q} \frac{\partial q}{\partial \tau_{\epsilon}} \left(e^{-\gamma C E_{1,i}} \left(\delta\left(\epsilon_i^n + q W_0 + r\right) + \delta\left(\epsilon_i^n - q W_0 + r\right) \right) \right) dF_v dF_z dF_{u_i} dF_{\epsilon_i^n} +$$
(62)

$$-\lim_{W_0\to 0} \int \frac{1}{2qW_0} \frac{\partial r}{\partial \tau_{\epsilon}} \left(e^{-\gamma C E_{1,i}} \left(\delta \left(\epsilon_i^n - qW_0 + r \right) - \delta \left(\epsilon_i^n + qW_0 + r \right) \right) \right) dF_v dF_z dF_{u_i} dF_{\epsilon_i^n}.$$
(63)

The limit in (61) is

$$\lim_{W_0 \to 0} \int \gamma e^{-\gamma C E_{1,i}} \frac{\partial C E_{1,i}}{\partial \tau_{\epsilon}} \frac{\mathbb{I}\left(\epsilon_i^n \in \left[-qW_0 - r; qW_0 - r\right]\right)}{2qW_0} dF_v dF_z dF_{u_i} dF_{\epsilon_i^n} = \int \gamma e^{-\gamma C E_{1,i}} \frac{dC E_{1,i}}{d\tau_{\epsilon}} \delta\left(\epsilon_i^n + r\right) dF_v dF_z dF_{u_i} dF_{\epsilon_i^n}.$$

Note that in the unconstrained region

$$\begin{split} \frac{\partial CE_{1,i}}{d\tau_{\epsilon}} &= \frac{d}{\partial\tau_{\epsilon}} \left(\frac{\gamma}{2\tau_{i}} \left(x_{i}^{u} \right)^{2} \right) \\ &= \frac{\gamma \left(x_{i}^{u} \right)^{2}}{2} \underbrace{\frac{d}{d\tau_{\epsilon}} \left(\frac{1}{\tau_{i}} \right)}_{<\infty} + \frac{\gamma}{\tau_{i}} x_{i}^{u} \underbrace{\frac{d}{d\tau_{\epsilon}} \left(x_{i}^{u} \right)}_{<\infty}. \end{split}$$

Once we substitute $\epsilon_i^n = -r$, the desired demand x_i^u becomes zero, as does $\frac{dCE_{1,i}}{d\tau_{\epsilon}}$. Thus,

$$\int \gamma e^{-\gamma C E_{1,i}} \frac{dC E_{1,i}}{d\tau_{\epsilon}} \delta\left(\epsilon_i^n + r\right) dF_v dF_z dF_{u_i} dF_{\epsilon_i^n} = 0.$$

The limit in (62) is simply

$$\begin{split} \lim_{W_0 \to 0} \int \frac{1}{2q} \frac{\partial q}{\partial \tau_{\epsilon}} \left(e^{-\gamma C E_{1,i}} \left(\delta \left(\epsilon_i^n + q W_0 + r \right) + \delta \left(\epsilon_i^n - q W_0 + r \right) \right) \right) dF_v dF_z dF_{u_i} dF_{\epsilon_i^n} = \\ \frac{\partial \log q}{\partial \tau_{\epsilon}} \int e^{-\gamma C E_{1,i}} \delta \left(\epsilon_i^n + r \right) dF_v dF_z dF_{u_i} dF_{\epsilon_i^n} = \frac{\partial \log q}{\partial \tau_{\epsilon}} \psi. \end{split}$$

For (63), we write

$$\begin{split} &\lim_{W_0\to 0} \int \frac{1}{2qW_0} \frac{\partial r}{\partial \tau_{\epsilon}} \left(e^{-\gamma C E_{1,i}} \left(\delta \left(\epsilon_i^n - qW_0 + r \right) - \delta \left(\epsilon_i^n + qW_0 + r \right) \right) \right) dF_v dF_z dF_{u_i} dF_{\epsilon_i^n} \\ &= \lim_{W_0\to 0} \int \frac{1}{2qW_0} \frac{\partial r}{\partial \tau_{\epsilon}} \left(e^{-\gamma C E_{1,i}} f_{\epsilon_i^n} \Big|_{\epsilon_i^n = -qW_0 - r}^{\epsilon_i^n = qW_0 - r} \right) dF_v dF_z dF_{u_i} \\ &= \int \frac{\partial r}{\partial \tau_{\epsilon}} \left. \frac{\partial}{\partial \epsilon_i^n} \left(e^{-\gamma C E_{1,i}} f_{\epsilon_i^n} \right) \right|_{\epsilon_i^n = -r} dF_v dF_z dF_{u_i}. \end{split}$$

Direct calculation yields

$$\left. \frac{\partial}{\partial \epsilon_i^n} \left(e^{-\gamma C E_{1,i}} f_{\epsilon_i^n} \right) \right|_{\epsilon_i^n = -r} = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{\gamma^2}{2\tau_\theta} e_i^2 - \frac{r^2}{2} \right) r.$$

The limit in (63) can then be written as

$$\begin{split} \lim_{W_0 \to 0} \int \frac{1}{2qW_0} \frac{\partial r}{\partial \tau_{\epsilon}} \left(e^{-\gamma C E_{1,i}} \left(\delta \left(\epsilon_i^n - qW_0 + r \right) - \delta \left(\epsilon_i^n + qW_0 + r \right) \right) \right) dF_v dF_z dF_{u_i} dF_{\epsilon_i^n} \\ &= \frac{1}{\sqrt{2\pi}} \int r \frac{\partial r}{\partial \tau_{\epsilon}} \exp\left(\frac{\gamma^2}{2\tau_{\theta}} e_i^2 - \frac{r^2}{2} \right) dF_v dF_z dF_{u_i} \\ &= \frac{-1}{\sqrt{2\pi}} \frac{\partial}{\partial \tau_{\epsilon}} \int \exp\left(\frac{\gamma^2}{2\tau_{\theta}} e_i^2 - \frac{r^2}{2} \right) dF_v dF_z dF_{u_i} \\ &= -\frac{\partial \psi}{\partial \tau_{\epsilon}}. \end{split}$$

Combining all the intermediary results of this step, we get

$$\lim_{W_0 \to 0} \frac{\partial \log \left(-U_0^u \right)}{\partial \tau_\epsilon} = \frac{\partial \log q}{\partial \tau_\epsilon} + \frac{\partial \log \psi}{\partial \tau_\epsilon}.$$

Step 3. Suppose that the parameters of the model other than W_0 are such that

$$\frac{\partial \log(q\psi)}{\partial \tau_{\epsilon}} + \frac{\partial}{\partial \tau_{\epsilon}} \log\left(\frac{\tau_i}{2\gamma \tau_{v,i}^2}\right) > 0, \tag{C1}$$

where r is defined in (57) and ψ is given by (66) Then there exists threshold \hat{W}^3 such that for all $W_0 \in (0, \hat{W}^3)$, we have $\frac{\partial \mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon}; W_0)}{\partial \tau_{\epsilon}} > 0$.

By the previous steps of the proposition, if the condition (C1) holds, then the limit

$$\lim_{W_0 \to 0} \frac{\partial \log \mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon}; W_0)}{\partial \tau_{\epsilon}} > 0$$

Denote this limit $\lambda > 0$ and write the epsilon-delta definition of a limit (see, e.g., Kolmogorov and Fomin (1975)): for any $\epsilon > 0$, there exists $\delta > 0$ such that for all $0 < W_0 < \delta$, we have $\left| \frac{\partial \log \mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon}; W_0)}{\partial \tau_{\epsilon}} - \lambda \right| < \epsilon$. Take $\epsilon = \lambda$. For such ϵ , there exists $\delta > 0$ such that for all W_0 such that $0 < W_0 < \delta$, we have $\frac{\partial \log \mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon}; W_0)}{\partial \tau_{\epsilon}} > 0$. For such values of W_0 , we also have $\frac{\partial \mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon}; W_0)}{\partial \tau_{\epsilon}} > 0$ (since $\frac{\partial \log \mathcal{MVI}(\tau_{\epsilon_i}, \tau_{\epsilon}; W_0)}{\partial \tau_{\epsilon}} = \frac{1}{\mathcal{MVI}} \frac{\partial \mathcal{MVI}}{\partial \tau_{\epsilon}}$ and $\mathcal{MVI} > 0$). Therefore, the desired statement holds if we take $\hat{W}^3 = \delta$.

Lemma 7. A closed-form expression for ψ is given by (66).

Proof. We write ψ as follows:

$$\psi = \frac{1}{\sqrt{2\pi}} E\left[\exp\left(\frac{\gamma^2}{2\tau_{\theta}}e_i^2 - \frac{r^2}{2}\right)\right].$$
(64)

Note that r is given by (59). Moreover, in the limit as $W_0 \to 0$, we have $p = E[v|\phi] - \frac{\gamma_m}{\tau_m}$. Thus, one can write

$$r = \rho_v v + \rho_z z + \rho_u u_i + \rho_0,$$

where

$$\begin{split} \rho_v &= \frac{\tau_i}{\gamma \sqrt{Var[x_i|v,z,u_i]}} \left(\frac{\tau_{\epsilon,i} + \beta^2 \left(\tau_u + \tau_z\right)}{\tau_{v,i}} - \frac{\beta^2 \tau_z}{\beta^2 \tau_z + \tau_v} \right), \\ \rho_z &= \frac{\tau_i}{\gamma \sqrt{Var[x_i|v,z,u_i]}} \left(\frac{\beta \tau_z}{\beta^2 \tau_z + \tau_v} + \frac{\beta \tau_u - \beta \left(\tau_u + \tau_z\right)}{\tau_{v,i}} - \gamma \tau_{\theta}^{-1} \right), \\ \rho_u &= \frac{\tau_i}{\gamma \sqrt{Var[x_i|v,z,u_i]}} \left(\frac{\beta \tau_u}{\tau_{v,i}} - \gamma \tau_{\theta}^{-1} \right), \text{ and} \\ \rho_0 &= \frac{\tau_i}{\gamma \sqrt{Var[x_i|v,z,u_i]}} \frac{\gamma_m}{\tau_m}. \end{split}$$

Then, e_i and r are jointly normally distributed as follows:

$$\begin{pmatrix} e_i \\ r \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ \rho_0 \end{pmatrix}, \begin{pmatrix} v_e & c_{er} \\ c_{er} & v_r \end{pmatrix}\right),$$
(65)

where we have introduced the notation v_e , v_r , and c_{er} for, respectively, variance of e_i , variance of r, and covariance between e_i and r.

It is easy to express them as follows:

$$v_e = \tau_z^{-1} + \tau_u^{-1},$$

$$v_r = \rho_v^2 \tau_v^{-1} + \rho_z^2 \tau_z^{-1} + \rho_u^2 \tau_u^{-1}, \text{ and}$$
$$c_{er} = \rho_z \tau_z^{-1} + \rho_u \tau_u^{-1}.$$

Substituting the density (65) and computing the expectation in (64), we get

$$\psi = \frac{\sqrt{\tau_{\theta}}}{\sqrt{2\pi}} \frac{\exp\left(\frac{\rho_0^2(\gamma^2 v_e - \tau_{\theta})}{2\gamma^2 c_{er}^2 + 2(v_r + 1)(\tau_{\theta} - \gamma^2 v_e)}\right)}{\sqrt{\gamma^2 c_{er}^2 + (v_r + 1)(\tau_{\theta} - \gamma^2 v_e)}}.$$
(66)

A.9 Proof of Proposition 9

The fact that β falls when financiers $j \neq i, j \in [0, 1]$ increase margins follows from Proposition 5. Given that β falls, by (9), it is optimal for financier *i* to increase his margin.

A.10 Proof of Proposition 10

We split the proof into three parts, devoted to risk premium, volatility, and the Sharpe ratio, respectively.

A.10.1 Risk premium

The direct effect

The proof follows a sequence of steps. Consider the ODE for the function g(p)

$$g'(p) = h(p, g(p)) \equiv \frac{\pi_2(p, g(p))c_p + c_p^m}{\pi_2(p, g(p))c_\phi + c_\phi^m} > 0,$$
(67)

where $\pi_2(p,\phi) = \Phi\left(\frac{W_0/m - X^u(p,\phi)}{\sigma_{\xi}}\right) - \Phi\left(\frac{-W_0/m - X^u(p,\phi)}{\sigma_{\xi}}\right).$

Step 1. The function $h(p, \phi)$ is an even function, that is, $h(p, \phi) = h(-p, -\phi)$.

We prove that $\pi_2(p,\phi) = \pi_2(-p,-\phi)$ from which the claim follows. Note that $X_u(p,\phi) = c_\phi \phi - c_p p = -X_u(-p,-\phi)$. Then

$$\begin{aligned} \pi_2(-p,-\phi) &= \Phi\left(\frac{W_0/m - X^u(-p,-\phi)}{\sigma_{\xi}}\right) - \Phi\left(\frac{-W_0/m - X^u(-p,-\phi)}{\sigma_{\xi}}\right) \\ &= \Phi\left(\frac{-(-W_0/m - X^u(p,\phi))}{\sigma_{\xi}}\right) - \Phi\left(\frac{-(W_0/m - X^u(p,\phi))}{\sigma_{\xi}}\right) \\ &= 1 - \Phi\left(\frac{-W_0/m - X^u(p,\phi)}{\sigma_{\xi}}\right) - \left(1 - \Phi\left(\frac{W_0/m - X^u(p,\phi)}{\sigma_{\xi}}\right)\right) \\ &= \Phi\left(\frac{W_0/m - X^u(p,\phi)}{\sigma_{\xi}}\right) - \Phi\left(\frac{-W_0/m - X^u(p,\phi)}{\sigma_{\xi}}\right).\end{aligned}$$

Step 2. The solution to the ODE (67) with boundary condition g(0) = 0 is an odd function. That is, $g(p) + g(-p) = 0 \forall p$.

We prove the statement by contradiction. Denote by \hat{p} the largest p such that for all $p \leq \hat{p}$, we have g(p)+g(-p) =

0. Consider $g(\hat{p} + dp) = g(\hat{p}) + h(\hat{p}, g(\hat{p})) dp$. On the other hand,

$$g(-\hat{p} - dp) = g(-\hat{p}) - h(-\hat{p}, g(-\hat{p})) dp$$

= $-g(\hat{p}) - h(-\hat{p}, -g(\hat{p})) dp$
= $-(g(\hat{p}) + h(\hat{p}, g(\hat{p})) dp)$
= $-g(\hat{p} + dp),$

which means that \hat{p} can be increased to $\hat{p} + dp$, which contradicts the fact that \hat{p} is the largest.

Step 3. The solution to the ODE (67) with boundary condition g(0) > 0 is such that g(p) + g(-p) > 0 $\forall p$. The solution to the ODE (67) with boundary condition g(0) < 0 is such that g(p) + g(-p) < 0 $\forall p$.

We prove the first statement; the second is proved analogously. Denote the solution to the ODE (67) with a bounday condition g(0) = 0 by $g_0(p)$. We have shown in the lemma above that $g_0(p)$ is odd.

Any solution to the ODE (67) with a boundary condition g(0) > 0 is a function g(p) that is always above the function $g_0(p)$. Hence, $g(p) + g(-p) > g_0(p) + g_0(-p) = 0$.

Step 4. In equilibrium, g(0) > 0.

The aggregate demand of specialists is given by

$$\begin{split} X(p,\phi) &= -\pi_1(p,\phi) \frac{W_0}{m} + \pi_3(p,\phi) \frac{W_0}{m} + \pi_2(p,\phi) X^u(p,\phi) + \\ &\sigma_{\xi} \left(\Phi' \left(\frac{-W_0/m - X^u(p,\phi)}{\sigma_{\xi}} \right) - \Phi' \left(\frac{W_0/m - X^u(p,\phi)}{\sigma_{\xi}} \right) \right) \end{split}$$

It can be shown that X(0,0) = 0. Denote the aggregate demand $X_{agg}(p,\phi) = X(p,\phi) + c_{\phi}\phi - c_{p}p$. The value g(0) is ϕ^{*} such that $X_{agg}(0,\phi^{*}) = 1$. Since $X_{agg}(0,0) = 0$ and $\frac{\partial}{\partial \phi} X_{agg}(p,\phi) = c_{\phi}^{m} + \pi_{2}c_{\phi} > 0$, we have g(0) > 0.

Step 5. Suppose that f(x) is a positive even function and a function l(x) is such that l(x) + l(-x) < 0. Then $\int_{-\infty}^{\infty} f(x)l(x)dx < 0$.

Given symmetry, the integral can be written as

$$\int_0^\infty f(x)\left(l(x)+l(-x)\right)dx>0.$$

Step 6. $X^u(p, g(p)) + X^u(-p, g(-p)) < 0.$

 $X^{u}(-p,g(-p)) = -(c_{p}p + c_{\phi}g(-p)) < -(c_{p}p - c_{\phi}g(p)) = -X^{u}(p,g(p)).$

Step 7. For any a > 0, the function $k(x) = \Phi(x+a) - \Phi(x-a)$ decreases (increases) in x for x > 0 < 0). Moreover, suppose that x + y < 0 and x < 0, then k(x) < k(y).

By symmetry $\Phi(x+a) - \Phi(x-a)$ attains unique maximum at x = 0; hence, it decreases to the right of it and increases to the left of it. The second claim follows from the symmetry of k(x).

Step 8. $\pi_2(p, g(p)) > \pi_2(-p, g(-p))$ for p such that $X^u(p, g(p)) > 0$. Vice versa, $\pi_2(p, g(p)) < \pi_2(-p, g(-p))$ for p such that $X^u(p, g(p)) < 0$.

Consider the case $X^u(p, g(p)) > 0$. In that case, $-X^u(-p, g(-p)) > X^u(p, g(p)) > 0$ (Step 6)

$$\pi_2(-p, g(-p)) = \Phi\left(\frac{W_0/m - X^u(-p, g(-p))}{\sigma_{\xi}}\right) - \Phi\left(\frac{-W_0/m - X^u(-p, g(-p))}{\sigma_{\xi}}\right).$$

Applying step 7, we get

$$\pi_2(-p,g(-p)) < \Phi\left(\frac{W_0/m + X^u(p,g(p))}{\sigma_{\xi}}\right) - \Phi\left(\frac{-W_0/m - X^u(p,g(p))}{\sigma_{\xi}}\right)$$
$$= \pi_2(p,g(p))$$

Consider the case $X^u(p, g(p)) < 0$. In that case, $X^u(p, g(p)) + X^u(-p, g(-p)) < 0$ (step 6). Applying step 7 (second claim), we get

$$\pi_2(-p, g(-p)) < \Phi\left(\frac{W_0/m + X^u(p, g(p))}{\sigma_{\xi}}\right) - \Phi\left(\frac{-W_0/m - X^u(p, g(p))}{\sigma_{\xi}}\right) = \pi_2(p, g(p))$$

Step 9. $\pi_3(p, g(p)) - \pi_1(p, g(p)) + \pi_3(-p, g(-p)) - \pi_1(-p, g(-p)) < 0.$

One can write

$$\begin{aligned} \pi_{3}(-p,g(-p)) - \pi_{1}(-p,g(-p)) &= 1 - \Phi\left(\frac{W_{0}/m - X^{u}(-p,g(-p))}{\sigma_{\xi}}\right) - \Phi\left(\frac{-W_{0}/m - X^{u}(-p,g(-p))}{\sigma_{\xi}}\right) \\ &= \Phi\left(\frac{-W_{0}/m + X^{u}(-p,g(-p))}{\sigma_{\xi}}\right) - \Phi\left(\frac{-W_{0}/m - X^{u}(-p,g(-p))}{\sigma_{\xi}}\right) \\ &< \Phi\left(\frac{-W_{0}/m - X^{u}(p,g(p))}{\sigma_{\xi}}\right) - \Phi\left(\frac{-W_{0}/m + X^{u}(p,g(p))}{\sigma_{\xi}}\right) \\ &= -\left(\pi_{3}(p,g(p)) - \pi_{1}(p,g(p))\right).\end{aligned}$$

Step 10.
$$\frac{\partial g(p;W)}{\partial W} + \frac{\partial g(-p;W)}{\partial W} < 0 \text{ and } \frac{\partial l(\phi;W)}{\partial W} + \frac{\partial l(-\phi;W)}{\partial W} > 0.$$

Write

$$\frac{\partial g(p;W)}{\partial W} = -\frac{\left(\pi_3 - \pi_1\right)/m}{c_{\phi}^m + \pi_2 c_{\phi}} \equiv \frac{n(p)}{d(p)}.$$

where

$$n(p) = -(\pi_3(p, g(p)) - \pi_1(p, g(p)))$$

and

$$d(p) = c_{\phi}^m + \pi_2(p, g(p))c_{\phi}.$$

We have

$$\frac{n(p)}{d(p)} + \frac{n(-p)}{d(-p)} = \overbrace{\frac{n(-p) + n(p)}{d(-p)}}^{>0} - n(p) \left(\frac{d(p) - d(-p)}{d(-p)d(p)}\right)$$

Note also that n(p) is $< 0 \ (> 0)$ if and only if $X(p, g(p)) > 0 \ (< 0)$, which, by step 9, implies that sign(n(p)) = $-\operatorname{sign}(d(p) - d(-p))$. Therefore n(p)(d(p) - d(-p)) < 0. The second statement can be proven analogously.

Step 11.
$$\frac{\partial}{\partial W_0}\overline{rp} < 0.$$

$$\frac{\partial}{\partial W_0}\overline{rp}=-\frac{\partial}{\partial W_0}E[p]=-\int_0^\infty\frac{\partial}{\partial W_0}(l(\phi)+l(-\phi))f_\phi(\phi)d\phi<0.$$

The indirect effect

Step 1. The indirect effect is $\frac{\partial \overline{rp}}{\partial \tau_{\epsilon}} \frac{d\tau_{\epsilon}}{dW_0}$ is negative for W_0 small enough, that is, $\lim_{W_0 \to 0} \frac{\partial \overline{rp}}{\partial \tau_{\epsilon}} \frac{d\tau_{\epsilon}}{dW_0} < 0$.

The risk premium can be written as

$$\overline{rp} = \frac{Var[f|p]}{\gamma_m} (1 - E[X]).$$

For $\frac{\partial \overline{rp}}{\partial \tau_{\epsilon}}$, we write

$$\frac{\partial \overline{rp}}{\partial \tau_{\epsilon}} = \underbrace{\frac{(1 - E[X])}{\gamma_m}}_{\rightarrow 1/\gamma_m} \underbrace{\frac{\partial Var[f|p]}{\partial \tau_{\epsilon}}}_{<0} - \frac{Var[f|p]}{\gamma_m} \frac{\partial E[X]}{\partial \tau_{\epsilon}}.$$

It follows from Lemma 8 below that $\lim_{W_0\to 0} \frac{\partial E[X]}{\partial \tau_{\epsilon}} = 0$, thus

$$\lim_{W_0 \to 0} \frac{\partial \overline{rp}}{\partial \tau_{\epsilon}} = \frac{\partial \overline{rp}}{\partial \tau_{\epsilon}} = \frac{1}{\gamma_m} \underbrace{\frac{\partial Var[f|p]}{\partial \tau_{\epsilon}}}_{<0}$$

The indirect effect is $\frac{\partial \overline{rp}}{\partial \tau_{\epsilon}} \frac{d\tau_{\epsilon}}{dW_0}$. Given that for small enough W_0 , $\frac{d\tau_{\epsilon}}{dW_0} > 0$, the claim of this step follows.

Step 2. There exists threshold \hat{W} such that for all $W_0 \in (0, \hat{W})$ we have that the indirect effect $\frac{\partial \overline{r_p}}{\partial \tau_{\epsilon}} \frac{d\tau_{\epsilon}}{dW_0}$ is negative.

Denote the limit in the previous step $\lambda < 0$ and write the epsilon-delta definition of a limit (see, e.g., Kolmogorov and Fomin (1975)): for any $\epsilon > 0$, there exists $\delta > 0$ such that for all $0 < W_0 < \delta$, we have $\left|\frac{\partial \overline{\tau}\overline{p}}{\partial \tau_{\epsilon}} \frac{d\tau_{\epsilon}}{dW_0} - \lambda\right| < \epsilon$. Take $\epsilon = -\lambda$. For such ϵ , there exists $\delta > 0$ such that for all W_0 such that $0 < W_0 < \delta$, we have $\left|\frac{\partial \overline{\tau}\overline{p}}{\partial \tau_{\epsilon}} \frac{d\tau_{\epsilon}}{dW_0} - \lambda\right| < \epsilon$. Therefore, the desired statement holds if we take $\hat{W} = \delta$.

Lemma 8. $\lim_{W_0 \to 0} \frac{dX}{d\tau_{\epsilon}} = 0.$

Proof. The aggregate demand of specialists changes with τ_{ϵ} because: (1) constraints $A(\tau_{\epsilon}) = W/m(\tau_{\epsilon})$ change (2) price function $p(v, z; A(\tau_{\epsilon}), \tau_{\epsilon})$ changes (both directly and through changes in constraints) and (3) τ_{ϵ} affects X directly. Correspondingly, we can write

$$X = X(v, z, p(v, z; A(\tau_{\epsilon}), \tau_{\epsilon}); A(\tau_{\epsilon}), \tau_{\epsilon})$$

and

$$\frac{dX}{d\tau_{\epsilon}} = \left(\frac{\partial X}{\partial A} + \frac{\partial X}{\partial p}\frac{\partial p}{\partial A}\right)A'(\tau_{\epsilon}) + \frac{\partial X}{\partial p}\frac{\partial p}{\partial \tau_{\epsilon}} + \frac{\partial X(v,z)}{\partial \tau_{\epsilon}}.$$

Consider

$$\frac{\partial X(v,z)}{\partial \tau_{\epsilon}} = E\left[\left(c'_{v}(\tau_{\epsilon})v + c'_{z}(\tau_{\epsilon})z + c'_{p}(\tau_{\epsilon})p + c'_{u}(\tau_{\epsilon})u_{i} + c'_{\epsilon}(\tau_{\epsilon})\epsilon_{i}\right)\mathbb{I}\left(x_{i}^{u} \in [-A;A]\right)|v,z\right].$$

From the above, it is clear that $\lim_{W_0\to 0} \frac{\partial X(v,z)}{\partial \tau_{\epsilon}} = 0$. We now compute $\frac{\partial p}{\partial \tau_{\epsilon}}$. Differentiating market clearing condition implicitly, we get $\rightarrow 0$

$$\frac{\partial p}{\partial \tau_{\epsilon}} = \frac{\frac{\partial X^{agg}}{\partial \tau_{\epsilon}}}{c_{p}^{m} + \pi_{2}c_{p}} = \frac{\overbrace{\frac{\partial X}{\partial \tau_{\epsilon}}}^{\infty} + \frac{\partial x_{m}}{\partial \tau_{\epsilon}}}{c_{p}^{m} + \pi_{2}c_{p}}.$$

It is clear that

$$\lim_{W_0 \to 0} \frac{\partial p}{\partial \tau_{\epsilon}} = \frac{c_{\phi}^m(\tau_{\epsilon})'\left(v - \frac{1}{\beta(\tau_{\epsilon})}z\right) + \frac{c_{\phi}^m(\tau_{\epsilon})}{\beta(\tau_{\epsilon})^2}\beta'(\tau_{\epsilon})z - c_p^m(\tau_{\epsilon})'p}{c_p^m} < \infty.$$

On the other hand,

$$\lim_{W_0 \to 0} \frac{\partial X}{\partial p} = \lim_{W_0 \to 0} \pi_2 c_p = 0,$$

hence,

$$\lim_{W_0 \to 0} \frac{\partial X}{\partial p} \frac{\partial p}{\partial \tau_{\epsilon}} = 0.$$

We now consider $\frac{\partial X}{\partial A}$:

$$\frac{\partial X}{\partial A} = \pi_3 - \pi_1 < \infty$$

On the other hand,

$$\lim_{W_0 \to 0} \frac{\partial X}{\partial p} = \lim_{W_0 \to 0} \pi_2 c_p = 0.$$

We finally compute $\frac{\partial p}{\partial A}$. Differentiating the market-clearing condition implicitly, we get

0.17

$$\frac{\partial p}{\partial A} = \frac{\pi_3 - \pi_1}{c_p^m + \pi_2 c_p} \Rightarrow \lim_{W \to 0} \frac{\partial p}{\partial A} < \infty.$$

Therefore,

$$\lim_{W_0 \to 0} \left(\frac{\partial X}{\partial A} + \frac{\partial X}{\partial p} \frac{\partial p}{\partial A} \right) A'(\tau_{\epsilon}) = \lim_{W_0 \to 0} \left(-\underbrace{\left(\frac{\partial X}{\partial A} + \frac{\partial X}{\partial p} \frac{\partial p}{\partial A} \right) \frac{m(\tau_{\epsilon})'}{m(\tau_{\epsilon})^2}}_{<\infty} W_0 \right) = 0.$$

A.10.2 Volatility

The proof follows a sequence of steps.

Step 1.
$$\lim_{W_0\to 0} \frac{d\sqrt{\mathcal{V}}}{dW_0} = \lim_{W_0\to 0} \frac{\partial\sqrt{\mathcal{V}}}{\partial\tau_{\epsilon}} \frac{d\tau_{\epsilon}}{dW_0} < 0.$$

Using the law of total variance, we write

$$Var(f - p) = Var(f - p|p) + Var(E[f - p|p]).$$

For the first term, we already know that

$$\frac{d}{dW_0}Var(f-p|p) = \frac{d}{dW_0}Var(f|p) < 0,$$

for W_0 small enough. For the second term, we write

$$E[f - p|p] = \gamma_m Var[f|p](1 - X).$$

Therefore,

$$Var\left(E\left[f-p|p\right]\right) = \left(\gamma_m Var[f|p]\right)^2 Var\left(X\right).$$

Taking the derivative,

$$\frac{d}{dW_0} Var\left(E\left[f-p|p\right]\right) = \underbrace{\frac{d}{dW_0} \left(\gamma_m Var[f|p]\right)^2}_{<0} Var\left(1-X\right) + \left(\gamma_m Var[f|p]\right)^2 \left(\frac{d}{dW_0} Var(X)\right).$$

Since $\lim_{W_0 \to 0} Var(1-X) = 0$, we have

$$\lim_{W_0 \to 0} \frac{d}{dW_0} \left(\gamma_m Var[f|p] \right)^2 Var\left(1 - X\right) = 0.$$

For $\frac{d}{dW_0}Var(X)$ note that $\lim_{W_0\to 0} Var(X) = 0$, Therefore,

$$\lim_{W_0 \to 0} \frac{d}{dW_0} Var(X) = \lim_{W_0 \to 0} \frac{Var(X) - \lim_{W_0 \to 0} Var(X)}{W_0} = \lim_{W_0 \to 0} \frac{Var(X)}{W_0}$$

Note that

$$0 \le Var(X) \le E[X^2] \le \frac{W_0^2}{m^2}$$

Therefore,

$$0 \le \lim_{W_0 \to 0} \frac{Var(X)}{W_0} \le \lim_{W_0 \to 0} \frac{W_0}{m^2} = 0.$$

Combining all the above, we have

$$\lim_{W_0 \to 0} \frac{d}{dW_0} Var\left(E\left[f - p|p\right]\right) = 0.$$

It then follows that the direct effect is zero. Summarizing

$$\lim_{W_0 \to 0} \frac{d\mathcal{V}}{dW_0} = \frac{d}{dW_0} Var(f|p) < 0$$

Similarly,

$$\lim_{W_0 \to 0} \frac{d\sqrt{\mathcal{V}}}{dW_0} = \frac{1}{2\sqrt{Var(f|p)}} \frac{d}{dW_0} Var(f|p) < 0.$$

Step 2. There exists threshold \hat{W} such that for all $W_0 \in (0, \hat{W})$, we have $\frac{d\sqrt{\mathcal{V}}}{dW_0} < 0$.

Denote the limit in the previous step $\lambda < 0$ and write the epsilon-delta definition of a limit (see, e.g., Kolmogorov and Fomin (1975)): for any $\epsilon > 0$ there exists $\delta > 0$ such that for all $0 < W_0 < \delta$, we have $\left|\frac{d\sqrt{\nu}}{dW_0} - \lambda\right| < \epsilon$. Take $\epsilon = -\lambda$. For such ϵ there exists $\delta > 0$ such that for all W_0 such that $0 < W_0 < \delta$, we have $\frac{d\nu}{dW_0} < 0$. Therefore, the desired statement holds if we take $\hat{W} = \delta$.

A.10.3 the Sharpe ratio

Step 1. We compute $\lim_{W_0 \to 0} \frac{dSR}{dW_0}$.

By definition $SR = \frac{\overline{rp}}{\sqrt{\nu}}$. Therefore,

$$\frac{dSR}{dW_0} = \frac{\frac{d\overline{r}\overline{p}}{dW_0}\sqrt{\mathcal{V}} + \frac{1}{2\sqrt{\mathcal{V}}}\frac{d\mathcal{V}}{dW_0}\overline{r}\overline{p}}{\mathcal{V}}.$$

Note that

$$\lim_{W_0 \to 0} \overline{rp} = \lim_{W_0 \to 0} \frac{Var[f|p]}{\gamma_m} (1 - E[X]) = \frac{Var[f|p]}{\gamma_m}$$

and $\lim_{W_0\to 0} \mathcal{V} = Var[f|p]$. For the derivatives, we have shown that

$$\frac{d\overline{rp}}{dW_0} < \frac{\partial\overline{rp}}{\partial\tau_\epsilon} \frac{d\tau_\epsilon}{dW_0}$$

The inequality is true since there is also a direct effect, which is negative. Moreover, we have shown before that

$$\lim_{W_0 \to 0} \frac{\partial \overline{rp}}{\partial \tau_{\epsilon}} = \frac{1}{\gamma_m} \frac{dVar[f|p]}{d\tau_{\epsilon}}$$

We have also shown before that

$$\lim_{W_0 \to 0} \frac{d\mathcal{V}}{dW_0} = \lim_{W_0 \to 0} \frac{d}{dW_0} Var(f|p) = \lim_{W_0 \to 0} \frac{dVar[f|p]}{d\tau_{\epsilon}} \frac{d\tau_{\epsilon}}{dW_0}$$

Combining all of the above we get

$$\lim_{W_0 \to 0} \frac{dSR}{dW_0} < \underbrace{\frac{d\tau_{\epsilon}}{dW_0}}_{>0} \underbrace{\frac{dVar[f|p]}{d\tau_{\epsilon}}}_{<0} \underbrace{\frac{\frac{1}{\gamma_m}\sqrt{Var[f|p]} + \frac{1}{2\sqrt{Var[f|p]}}\frac{Var[f|p]}{\gamma_m}}_{>0}}_{>0} < 0.$$

Step 2. There exists threshold \hat{W} such that for all $W_0 \in (0, \hat{W})$, we have $\frac{dSR}{dW_0} < 0$.

Denote the limit in the previous step $\lambda < 0$ and write epsilon-delta definition of a limit (see, e.g., Kolmogorov and Fomin (1975)): for any $\epsilon > 0$ there exists $\delta > 0$ such that for all $0 < W_0 < \delta$, we have $\left|\frac{dSR}{dW_0} - \lambda\right| < \epsilon$. Take $\epsilon = -\lambda$. For such ϵ there exists $\delta > 0$ such that for all W_0 such that $0 < W_0 < \delta$, we have $\frac{dSR}{dW_0} < 0$. Therefore, the desired statement holds if we take $\hat{W} = \delta$.

B For Online Publication: A Setup with Noise Traders

In our baseline setting, we assume that noise in prices comes from aggregate hedging needs. It contributes to the tractability of our baseline setting due to the irrelevance result: given exogenous private information, informational efficiency of prices is independent of the constraints that specialists face. Thus, the informational efficiency can be found by solving for equilibrium in the unconstrained economy. What matters for the irrelevance result is that constraints affect not only the informed demand, but also the demand from hedging needs of the specialists.

In this appendix, we eliminate the hedging needs of specialists and assume that the noise in prices comes from classic noise traders. We assume that the noise is exogenous, in particular, it is not affected by constraints. The question we answer here is "will the information spiral continue to hold in this setting?". We argue that the answer is positive. Moreover, the setting in this section highlights a novel channel for the interaction between constraints and information efficiency. We call this channel the information aggregation channel: as constraints tighten, noise is unaffected by constraints, whereas informed trading is more constrained; thus, the price informativeness decreases, even with exogenous information.

In the baseline model, because of the irrelevance result, there was no information aggregation channel. In order to focus on the new mechanism, in this section we consider a setting with exogenous information (shutting down the information acquisition channel).³¹ We show that an exogenous shock that tightens constraints of specialists leads to lower informational efficiency. In response, financiers set higher margins, further tightening specialists' constraints. This information spiral mechanism is similar to the one studied in the baseline model, with the difference that it acts through the information aggregation function of price. We illustrate this new channel in Figure 8.

Figure 8: Amplification mechanism



Setup

Consider a two-date model with $t = \{1, 2\}$. Suppose the payoff of the risky asset at t = 2 is $f = v + \theta$, where v is the learnable component and θ is the unlearnable component. We assume that fundamental v is drawn from an improper uniform distribution, whereas $\theta \sim \mathcal{N}(0, \tau_{\theta}^{-1})$. There are three classes of agents in the economy: specialists, noise traders, and nonspecialists. There is a unit mass of specialists with constant absolute risk aversion γ who have wealth W_0 and observe the same signal $s = v + \epsilon, \epsilon \sim \mathcal{N}(0, \tau_{\epsilon}^{-1})$. Among these specialists, fraction $\lambda \in (0, 1)$ is subject to margin constraints while the remaining fraction $(1 - \lambda)$ is unconstrained. As in

 $^{^{31}}$ It is also worth noting that information acquisition is much less tractable in the setting that we consider in this section.

the baseline model, we first study fixed margins m for both long and short positions and later study VaR-based margins. The noise traders submit exogenous liquidity demands $u \sim \mathcal{N}(0, \tau_u^{-1})$, and, finally, the uninformed nonspecialist is unconstrained and risk-neutral, that is, $\gamma_m = 0$.

Financial Market Equilibrium

Denote $\tau = \frac{\tau_{\epsilon} \tau_{\theta}}{\tau_{\epsilon} + \tau_{\theta}}$. The optimal demand for an unconstrained specialist is given by

$$x_{i,u} = \frac{\tau}{\gamma} \left(s - p \right).$$

For a constrained specialist, his demand is

$$x_{i,c} = \begin{cases} x_{i,u}, & \text{if } -\frac{W_0}{m} < \frac{\tau}{\gamma} (s-p) < \frac{W_0}{m}, \\ -\frac{W_0}{m}, & \text{if } \frac{\tau}{\gamma} (s-p) < -\frac{W_0}{m}, \\ +\frac{W_0}{m}, & \text{if } \frac{\tau}{\gamma} (s-p) > \frac{W_0}{m}. \end{cases}$$
and

Hence, the aggregate demand of specialists is

$$X \equiv \lambda x_{i,c} + (1-\lambda) x_{i,u} + u = \begin{cases} \frac{\tau}{\gamma} \left(s-p\right) + u & , \text{ if } s-p \in \left[-\frac{W_0 \gamma}{m\tau}, \frac{W_0 \gamma}{m\tau}\right];\\ -\frac{W_0}{m} \lambda + \frac{\tau}{\gamma} \left(s-p\right) \left(1-\lambda\right) + u & , \text{ if } s-p < -\frac{W_0 \gamma}{m\tau};\\ \frac{W_0}{m} \lambda + \frac{\tau}{\gamma} \left(s-p\right) \left(1-\lambda\right) + u & , \text{ if } s-p > \frac{W_0 \gamma}{m\tau}. \end{cases}$$

The nonspecialist's inferred information from price s_p is an affine transformation of the intercept of the above aggregate demand as follows:

$$s_p = \begin{cases} s + \frac{\gamma}{\tau}u & \text{, if } s - p \in \left[-\frac{W_0\gamma}{m\tau}, \frac{W_0\gamma}{m\tau}\right], \\ s + \frac{\gamma}{(1-\lambda)\tau}u & \text{, if } s - p < -\frac{W_0\gamma}{m\tau}, \\ s + \frac{\gamma}{(1-\lambda)\tau}u & \text{, if } s - p > \frac{W_0\gamma}{m\tau}. \end{cases}$$

Given that nonspecialist is risk-neutral, she sets the semi-strong efficient price $p = \mathbb{E}[v|s_p]$.

Proposition 11. There exists a piecewise linear REE with price function, given by

$$p = \begin{cases} s + \frac{\gamma}{\tau}u, & \text{if } u \in \left[-\frac{W_0}{m}, +\frac{W_0}{m}\right];\\ s + \frac{\gamma}{\tau(1-\lambda)}\left(u - \lambda\frac{W_0}{m}\right), & \text{if } u > \frac{W_0}{m}; & \text{and}\\ s + \frac{\gamma}{\tau(1-\lambda)}\left(u + \lambda\frac{W_0}{m}\right), & \text{if } u < -\frac{W_0}{m}. \end{cases}$$

In the piecewise linear REE, the price function takes different forms in different states of the world. Consider $u \in \left[-\frac{W_0}{m}, +\frac{W_0}{m}\right]$. In this case, prices are the same as in the economy without constraints. However, if $u < -\frac{W_0}{m}$, lower constraint binds for informed specialists and their demand is information insensitive (a constant $-\frac{W_0}{m}$). In this case, the aggregate demand that the market maker observes is given by

$$X = \underbrace{-\frac{W_0}{m}\lambda}_{\text{constrained demand}} + \underbrace{\frac{\tau}{\gamma}\left(s-p\right)\left(1-\lambda\right)}_{\text{unconstrained demand}} + \underbrace{\frac{u}{\gamma}\left(s-p\right)\left(1-\lambda\right)}_{\text{noise}} + \underbrace{u}_{\text{noise}}.$$

This implies that the aggregate demand (and hence, the market-clearing price) is more sensitive to noise trading in this region. Hence, informational efficiency is lower in this region (compared to the economy without constraints). Similar logic follows in the case of $u > \frac{W_0}{m}$. As a result, the nonspecialist's posterior is not normal The left panel shows the distribution in a model without constraints. The right panel shows the distribution in a model with constraints. Other parameters are $\tau_u = 1, \lambda = 0.9, \gamma = 3$, and $\alpha = 0.99$.



and is characterized by the conditional probability density function:

$$f_{s|p} = \begin{cases} \frac{\tau}{\gamma} \sqrt{\tau_u} \phi\left(\frac{\tau}{\gamma} \sqrt{\tau_u} \left(s-p\right)\right), & \text{if } s-p \in \left[-\frac{W_0 \gamma}{m\tau}, \frac{W_0 \gamma}{m\tau}\right];\\ (1-\lambda) \frac{\tau}{\gamma} \sqrt{\tau_u} \phi\left((1-\lambda) \frac{\tau}{\gamma} \sqrt{\tau_u} \left(s-p\right) - \lambda \sqrt{\tau_u} \frac{W_0}{m}\right), & \text{if } s-p < -\frac{W_0 \gamma}{m\tau}; & \text{and}\\ (1-\lambda) \frac{\tau}{\gamma} \sqrt{\tau_u} \phi\left((1-\lambda) \frac{\tau}{\gamma} \sqrt{\tau_u} \left(s-p\right) + \lambda \sqrt{\tau_u} \frac{W_0}{m}\right), & \text{if } s-p > \frac{W_0 \gamma}{m\tau} \end{cases}$$

where $\phi(.)$ denotes the pdf of a standard normal distribution.

Figure 9 illustrates the distribution of s conditional on the price with and without the constraints. The left panel shows the distribution without constraints and it is Gaussian. The right panel shows the distribution with constraints and it is not Gaussian. The various lines show the distribution when the agents face different constraints. For the states of the world in which constraints do not bind (i.e., center region of the distribution), the posterior variance is the same as in the unconstrained case. For the states in which constraints bind for some agents (tails of the distribution), there is less informed trade in the market and hence the posterior variance is higher and leads to fatter tails. We therefore arrive at the following proposition about the interaction between constraints and informational efficiency.

Proposition 12. If constraints become tighter for all specialists, that is, if $\frac{W_0}{m}$ decreases, (1) price informativeness (defined as the inverse of the conditional variance of the payoff) decreases (2) conditional distribution of losses on short and long positions becomes heavier-tailed, that is, the probability of a loss greater than x on a long position (given by Pr(p - f > x|p)) and probability of a loss greater than x on a short position (given by Pr(f - p > x|p)) increase for any x > 0.

The proposition above demonstrates that the first part of our informational spiral holds in the setting with noise traders. As constraints tighten, informational efficiency falls. As a result, conditional distribution of losses becomes heavier-tailed. We show in the section below that these heavier tails feed back into higher margins, closing the loop in our spiral.

VaR-based margins

Up to now, we assumed that margins are exogenously fixed. Next, as in the baseline model, we study how the price (and its informational efficiency) affects margins when they are set to control financiers' value-at-risk. VaR-based margins are described in (8) and we have the following result.

Proposition 13. Suppose margins are VaR-based. Then $m^+ = m^- = m$ and margins solve

$$1 - \alpha = E_{\eta} \left[Pr\left(p - v > m + \eta | p, \eta \right) \right],$$

where $\eta \equiv \theta - \epsilon$ and $\eta \sim N(0, \tau_{\eta}^{-1})$. Both τ_{η} and $E_{\eta} [Pr(p-v > m + \eta | p, \eta)]$ are explicitly solved in the proof. Moreover, as the conditional distribution of losses becomes heavier-tailed, margins become higher.

Proposition 13 shows that heavier tails of conditional loss distribution imply higher margins set by financiers. Combining the results of Propositions 13 and 12, we get that the following version of information spiral holds. As constraints tighten, informational efficiency drops and the distribution of losses becomes heavier-tailed. This implies higher margins, feeding back into tighter constraints. This is represented in the Figure 8. One consequence of the information spiral highlighted in this section is the following complementarity.

Corollary 1. Suppose that margins are increased for all specialists except specialist *i*. Then informational efficiency drops and the distribution of losses becomes heavier-tailed. As a result, specialist *i* will face higher margins as well.

Figure 10 illustrates the above proposition. The various lines in panel (b) represent the function f(m): the margin that a specialist of interest faces, given that the margins faced by other specialists is m. The fact that the function f(m) is upward-sloping signifies the complementarity outlined in the proposition above. Different lines correspond to the functions f(m) for different levels of wealth. Note that for some wealth levels there could be multiple equilibria (since f(m) crosses the 45-degree line in multiple points). We conclude Appendix B by arguing that the analysis provided here highlights that the economic mechanism of the informational spiral is present even in a setting with standard noise traders.

Proofs for Appendix **B**

Proof. (Propostion 11) Let $A = \frac{W_0}{m}$. Conjecture that a piecewise linear REE can be written with a price function of the form

$$p = \begin{cases} s + \frac{\gamma}{\tau} (u - u_0), & \text{if } u \in [u_0 - A, u_0 + A]; \\ s + \frac{\gamma}{\tau(1 - \lambda)} (u - u_0 - \lambda A), & \text{if } u > u_0 + A; \\ s + \frac{\gamma}{\tau(1 - \lambda)} (u - u_0 + \lambda A), & \text{if } u < u_0 - A. \end{cases}$$

The constant u_0 is pinned down by the equilibrium condition p = E[v|p], which implies

$$E[p] = E[v]$$

The above condition gives a unique solution, $u_0 = 0$. It can be then verified that the condition p = E[v|p] holds with $u_0 = 0$.

Proof. (Proposition 12) Part 1 (Tightening of constraints leads to increase in conditional variance). Since the price is efficient, one can write

$$Var(v|p) = E\left[(v - E[v|p])^2|p\right] = E\left[(v - p)^2|p\right].$$

We can further expand the above as follows:

$$E\left[(v-p)^{2}|p\right] = E\left[(v-p)^{2} 1 \left[u \in [-A,A]\right]|p\right] + E\left[(v-p)^{2} 1 \left[u > A\right]|p\right] + E\left[(v-p)^{2} 1 \left[u < -A\right]|p\right].$$

Figure 10: VaR-based margins set by a financier

The figure shows the VaR-based margins set by a financier. The left panel plots the margins as a function of specialists' wealth W_0 . The right panel shows the margins as a function of margins set by other financiers for different levels of wealth as indicated in the legend. Other parameters are: $\tau_u = 1; \lambda = 0.9; \gamma = 3; \alpha = 0.99$.



(a) as a function of specialists' initial wealth W_0



(b) as a function of other financiers' margins

Now we expand the terms above as follows:

$$E\left[\left(v-p\right)^{2} 1\left[u \in \left[-A,A\right]\right]|p\right] = E\left[\left(\epsilon + \frac{\gamma}{\tau}u\right)^{2} 1\left[u \in \left[-A,A\right]\right]|p = v + \epsilon + \frac{\gamma}{\tau}u\right]$$
$$= E\left[\left(\epsilon + \frac{\gamma}{\tau}u\right)^{2} 1\left[u \in \left[-A,A\right]\right]\right]$$

The last equality is true because $\tau_v = 0$, and hence, price is infinitely noisy signal of $\epsilon + \frac{\gamma}{\tau}u$. Proceeding similarly we get

$$E\left[\left(v-p\right)^{2} 1\left[u > A\right] | p\right] = E\left[\left(\epsilon + \frac{\gamma}{\tau(1-\lambda)}\left(u-\lambda A\right)\right)^{2} 1\left[u > A\right]\right]$$
$$E\left[\left(v-p\right)^{2} 1\left[u < -A\right] | p\right] = E\left[\left(\epsilon + \frac{\gamma}{\tau(1-\lambda)}\left(u+\lambda A\right)\right)^{2} 1\left[u < -A\right]\right]$$

To derive the sign of $\frac{\partial}{\partial A} E\left[(v-p)^2 | p\right]$ we note that due to symmetry we can evaluate this sign conditional on $\epsilon = 0$. This is true since the effect of a decrease in the upper constraint A on $E\left[(v-p)^2 | p, \epsilon\right]$ will be the opposite of the effect of an increase in the lower constraint -A on $E\left[(v-p)^2 | p, -\epsilon\right]$, and they will therefore cancel out once we integrate with respect to ϵ . Now it is easy to see that

$$E\left[\left(v-p\right)^{2} 1\left[u \in \left[-A,A\right]\right]|p,\epsilon=0\right] = E\left[\left(\frac{\gamma}{\tau}u\right)^{2} 1\left[u \in \left[-A,A\right]\right]\right]$$
$$+ E\left[\left(\frac{\gamma}{\tau(1-\lambda)}\left(u-\lambda A\right)\right)^{2} 1\left[u>A\right]\right]$$
$$+ E\left[\left(\frac{\gamma}{\tau(1-\lambda)}\left(u+\lambda A\right)\right)^{2} 1\left[u<-A\right]\right]$$

increases as A decreases.

Part 2 (Distribution of losses). Let $\eta = \theta - \epsilon$. Then $\eta \sim \mathcal{N}\left(0, \tau_{\theta}^{-1} + \tau_{\epsilon}^{-1}\right)$. We first derive the distribution of losses conditional on p and η . We split it into three parts, as follows:

$$\begin{aligned} \Pr\left(p - s > x + \eta | p, \eta\right) = & \Pr\left(p - s > x + \eta | u > A\right) \cdot \Pr\left(u > A\right) \\ & + \Pr\left(p - s > x + \eta | -A < u < A\right) \cdot \Pr\left(-A < u < A\right) \\ & + \Pr\left(p - s > x + \eta | u < -A\right) \cdot \Pr\left(u < -A\right). \end{aligned}$$

For the first part, we can write

$$\begin{aligned} \Pr\left(p-s > x+\eta | u > A\right) \cdot \Pr\left(u > A\right) = &\Pr\left(p-s > x+\eta, u > A\right) \\ = &\Pr\left(u > \max\left\{\frac{(1-\lambda)(x+\eta)}{\gamma\sigma_{\theta}^2 + \lambda A}, A\right\}\right). \end{aligned}$$

Proceeding analogously with the other two term, one can obtain

$$Pr\left(p-s > x+\eta | p,\eta\right) = \begin{cases} 1 - \Phi\left(\frac{1}{\sigma_u}\left(\frac{(1-\lambda)\tau(x+\eta)}{\gamma} + \lambda a\right)\right), & \text{if } x+\eta > \frac{\gamma}{\tau}a;\\ 1 - \Phi\left(\frac{1}{\sigma_u}\frac{(x+\eta)\tau}{\gamma}\right), & \text{if } -\frac{\gamma}{\tau}a \le x+\eta \le \frac{\gamma}{\tau}a\\ 1 - \Phi\left(\frac{1}{\sigma_u}\left(\frac{(1-\lambda)\tau(x+\eta)}{\gamma} - \lambda a\right)\right) & otherwise \end{cases}$$

Consider $\eta = y > 0$. It can be seen from above that

$$-\frac{\partial}{\partial A}Pr\left(p-s>x+\eta|p,\eta=y\right)>\frac{\partial}{\partial A}Pr\left(p-s>x+\eta|p,\eta=-y\right)>0.$$

Given the symmetry of the distribution of η this implies

$$\frac{\partial}{\partial A} Pr(p-s>x+\eta|p) = E_\eta \left[\frac{\partial}{\partial A} Pr\left(p-s>x+\eta|p,\eta=y\right)\right] < 0.$$

Thus if A decreases, conditional distribution of losses becomes heavier-tailed.

Proof. (Proposition 13) Conjecture that $m^+ = m^- = m$. For long positions, the financier sets margins such that $Pr(p-f > m|p) = 1 - \alpha$. Note that

$$\Pr(p - f > m|p) = \Pr(p - s + \epsilon - \theta > m|p)$$
$$= E_n \left[\Pr(p - s > m + \eta|p, \eta)\right]$$

The expression for $\Pr(p - s > m + \eta | p, \eta)$ was derived in the proof of the Proposition 12. The margins are given by $1 - \alpha$ quantile of the conditional distribution of losses derived in Proposition 12. Therefore, as this distribution become heavier-tailed, the margins increase.

C For Online Publication: Risk-neutral measure and microfoundation for VaR-based margins

Specialists in our model borrow from financiers, who impose a risk-based margin per unit of risky asset invested (long or short). While fully endogenizing the risk-based margin as an optimal contract is beyond the scope of our paper, in this appendix we attempt to describe the problem and frictions faced by the financier to rationalize the use of risk-based margin. We start by defining the risk-neutral measure, which is used to compute the VaR in our baseline specification.

C.1 Risk-neutral measure

Consider a nonspecialist who solves

$$\max_{x} E[-\exp(-\gamma x(f-p))|p].$$

The first-order condition implies

$$p = E\left[\frac{\exp(-\gamma x^*(f-p))}{E[\exp(-\gamma x^*(f-p))|p]}f|p\right],\tag{68}$$

where x^* denotes the nonspecialist's optimal holding. Define a random variable $Z = \frac{\exp(-\gamma x^*(f-p))}{E[\exp(-\gamma x^*(f-p))|p]}$ which is a Radon-Nikodym derivative that defines the risk-neutral measure. Substituting the optimal demand of a nonspecialist $x^* = \frac{E[f|p]-p}{\gamma Var[f|p]}$, the Radon-Nikodym derivative can be written as

$$Z = \frac{\exp\left(-\frac{E[f|p]-p}{Var[f|p]}(f-p)\right)}{E\left[\exp\left(-\frac{E[f|p]-p}{Var[f|p]}(f-p)\right)|p\right]}.$$
(69)

Definition 4. The risk-neutral meausre is defined by the Radon-Nikodym derivative Z, given by (69). That is, for any event A measurable with respect to information in prices, the risk-neutral conditional probability of that event $Pr^Q(A|p) = E[Z \cdot I(A)|p]$. The unconditional probability is defined as $Pr^Q(A) = E[Z \cdot I(A)]$.

The first-order condition (68) can be rewritten as $p = E^Q[f|p]$, which justifies the name of the new measure. Our definition implies that the risk-neutral distribution of f|p is characterized by the following probability density function

$$g_{f|p}(f|p) = \phi\left(\frac{f - E[f|p]}{\sqrt{Var(f|p)}}\right) \cdot Z$$

where $\phi(\cdot)$ is a density of a standard normal random variable. Direct calculation leads to the following result.

Lemma 9. Under the risk-neutral measure, $f|p \sim \mathcal{N}(p, \sqrt{Var(f|p)})$.

Proof. (Lemma 9). We proceed by direct calculation

$$g_{f|p}(f|p) = \phi\left(\frac{f - E[f|p]}{\sqrt{Var(f|p)}}\right) \cdot Z$$

Substituting Z above and collecting the terms that depend on f, we get

$$g_{f|p}(f|p) = \psi(p) \cdot \exp\left(-\frac{1}{2Var(f|p)}\left(f^2 - 2fE[f|p]\right) - \frac{E[f|p] - p}{Var[f|p]}f\right)$$

$$= \psi(p) \cdot \exp\left(-\frac{1}{2Var(f|p)}\left(f^2 - 2f \cdot p\right)\right).$$

$$(70)$$

In the above, $\psi(p)$ combines all the terms that do not depend on f. Since the density has to integrate to 1, we can express

$$\psi(p) = \left(\int \exp\left(-\frac{1}{2Var(f|p)}\left(f^2 - 2f \cdot p\right)\right)df\right)^{-1}.$$
(71)

Since the normal density with mean p and variance Var(f|p) can be represented by (70), and (71) the lemma follows.

C.2 Financiers' problem and VaR-margins.

We first consider the case in which specialists take a long position in the asset, and therefore, microfound the expression for m^+ .

When specialists take a long position of the asset

We assume that the financier can borrow at a rate $1 - \epsilon$ and lend to specialists at a rate of 1, that is, there are gains from trade between financiers and specialists.³² We also assume that the specialists' date-2 wealth is not pledgeable and the financier has to pay a proportional cost to enforce the specialist to repay with date-2 wealth. Therefore, for every unit of asset that the specialist has purchased, he can transfer the asset and some cash m^+ to the financier's account as a collateral. Effectively, the financier is lending an amount $(p - m^+)$ per unit to the specialist while holding the asset as collateral.

At t = 2, the specialist has to repay $(p - m^+)$ to get the asset dividend f back. If the dividend is more than the promised repayment, that is, $f > p - m^+$, costly enforcement is not needed because the financier can just take the repayment from the dividend of the asset, which is at his custody. If instead the dividend from the asset is less than the promised repayment, we assume the financier has to pay an enforcement cost $k = \frac{\epsilon}{1-\alpha}$ per dollar lent to force the specialists to pay with his date-2 wealth.³³ Thus, the financier earns a return "spread" ϵ on lending to specialists but has to pay an enforcement cost $\frac{\epsilon}{1-\alpha}$ per dollar lent in the states where f .

Assume financiers have CARA utility over terminal wealth with risk aversion parameter γ_F . The financier's time-2 wealth consists of two parts: the first one comes from his investment in the assets, W_A and the second comes from lending to specialists W_L . For the second part, we write

$$W_L = x_i \left(p - m^+ \right) \left(\epsilon - \frac{\epsilon}{1 - \alpha} \times 1 \left(f
$$= \epsilon x_i \left(p - m^+ \right) \left(1 - \frac{1}{1 - \alpha} \times 1 \left(f$$$$

We will later assume that ϵ is small and calculate our expressions in the limit as $\epsilon \to 0$. Given that the financier is uninformed and unconstrained, we can write

$$W_A = \frac{E\left[f|p\right] - p\left(1 - \epsilon\right)}{\gamma_F Var\left[f|p\right]} \left(f - p\right).$$

Note that financier's information set is $\mathcal{I}_F = \{p, x_i\}$. However, it is easy to see that information content in x_i is subsumed in prices and hence the financier only conditions on prices. The financier's utility can then be written

 $^{^{32}}$ One way to rationalize this is to assume that specialists valuation of the risk free asset is different from financiers, e.g., due to relative tax disadvantage as in Duffie, Gârleanu, and Pedersen (2005).

³³Here α is just a normalization constant, but later on we will derive that it will be equal to the VaR confidence level α .

as

$$E[U_{F}(W_{A}+W_{L})|p] = U_{F}(W_{A}) + E\left[U_{F}'(W_{A})W_{L}|p\right] + o(\epsilon)$$
(72)

$$= U_F(W_A) + E\left[U'_F(W_A) \epsilon x_i \left(p - m^+\right) \left(1 - \frac{1}{1 - \alpha} \times 1 \left(f (73)$$

$$= U_F(W_A) + \frac{\epsilon x_i \left(p - m^+\right)}{1 - \alpha} E\left[U'_F(W_A) | p\right].$$
(74)

$$\left(1 - \alpha - E\left[\frac{U'_F(W_A)}{E\left[U'_F(W_A)\mid p\right]}1\left(f
(75)$$

We assume that there is perfect competition between financiers, so that each of them should be indifferent between lending to specialists and getting (75) or not lending and getting the outside option of $U_F(W_A)$. Equalising the above expression to $U_F(W_A)$, and taking the limit as $\epsilon \to 0$, we get

$$E\left[\frac{U'_{F}(W_{A})}{E\left[U'_{F}(W_{A})|p\right]}1\left(f$$

Noting that $\lim_{\epsilon \to 0} \frac{U'_F(W_A)}{E[U'_F(W_A)]} = Z$, where Z is the Radon-Nikodym derivative associated with the rsik-neutral measure, it then follows that m^+ should be such that

$$Pr^{Q}\left[f$$

which coincides with expression for VaR-based margins.

When specialists take a short position of the asset

Financiers are also endowed with a large but finite amount of assets. We call them security lenders here because they act as such. Their outside option is to lend the security to some unmodeled agents at a return $1 - \epsilon$. Specialists can borrow the asset from the security lenders by pledging $(p + m^-)$ cash collateral per unit at t = 1. As the specialists sell the asset immediately for p, they have to put up m^- from their own wealth. At t = 2, the specialists buy back the asset at f, return it, and retrieve $(p + m^-)$ from the security lenders. We assume that if the value of the asset is higher than the cash collateral, that is, $f > p + m^-$, the security lenders have to expend k per dollar of asset lent in order to force the specialists to buy back and return the asset. Using the same steps as in the above subsection, the margin m^- should satisfy the following break-even condition:

$$\epsilon x_{i} \left(p + m^{-} \right) E \left[U'_{F} \left(W_{A} \right) | p \right] \left(1 - \frac{1}{1 - \alpha} \times E \left[\frac{U'_{F} \left(W_{A} \right)}{E \left[U'_{F} \left(W_{A} \right) | p \right]} 1 \left(f > p + m^{-} \right) | p \right] \right) = 0$$

which after simplification reduces to

$$Pr^{Q}\left[f-p>m^{-}|p\right]=1-\alpha,$$

which coincides with expression for VaR-margins.

D For Online Publication: Specialists with initial endowments of risky asset

In the baseline model, we assume that specialists have cash as initial endowment. In this section we assume that specialists are endowed with y_0 units of risky asset. As we show below, the functional form of the constraints a(p) and b(p) will change in that case. In particular, the upper constraint b(p) will resemble the borrowing constraint in Yuan (2005). The main result of this section is that the informational spiral will still be present in this economy with different form of constraints.

Setup

The model is identical to the model in Section 2.1, except that the specialists are initially endowed with $y_0 > 0$ units of risky asset. Moreover, we assume that they sell these assets to relax their constraints.³⁴ Specialists' wealth at date 2 is thus given by

$$W_i = y_0 p + x_i (v + \theta - p) + e_i \theta.$$

In this section we assume that the nonspecialist is risk-neutral. We now derive the functional form of constraints a(p) and b(p). As before, we assume that to build a long position in the risky asset, a specialist can borrow from a financier at the risk-free rate, but he has to pledge a cash margin of $m^+ \ge 0$ per unit of asset to the financier as collateral. The specialist can similarly establish a short position by providing, as collateral, a cash margin of m^- per unit of asset. The maximum positions they can take are constrained by the amount of cash C they have

$$m^{-}[x_{i}]^{-} + m^{+}[x_{i}]^{+} \le C.$$

The difference compared to baseline setting is that the amount of cash now depends on p:

$$C = y_0 p.$$

We can derive portfolio constraints as follows:

$$b(p) = \left[\frac{y_0 p}{m^+}\right]^+, \ a(p) = -\left[\frac{y_0 p}{m^-}\right]^+,$$
(76)

where in the above we accounted for the fact that by definition the maximum long position b(p) has to be positive, whereas the maximum short position a(p) has to be negative. We note the similarity between the upper constraint b(p) and the linear borrowing constraint in Yuan (2005).

Equilibrium and the informational spiral

The proposition below characterizes the equilibrium in the financial market.

Proposition 14. Suppose that specialists have identical signal precisions τ_{ϵ} and face constraints as described above, then there exists a unique linear equilibrium in which informational efficiency $\beta = \beta^u$ and the function $g(p) = \frac{\beta^2 \tau_z + \tau_v}{\beta^2 \tau_z} \cdot p.$

Proof. Follows from Proposition 2 by taking the limit $\gamma_m \to 0$.

³⁴Strictly speaking, selling the assets relaxes constraints, and hence, is always optimal only if price is greater than both margins. This is always true in practice because otherwise the agents could take larger positions without the financiers. To have this reasonable property in the model, one can choose a high enough mean payoff of the asset to ensure that the price is almost always greater than margins. This is because changing the mean payoff does not affect the margins but changes the level of price.

We now establish the two parts of the informational spiral are present in the alternative setup considered in this appendix. First, we establish that as constraints tighten, specialists' incentives to acquire information decrease. See the Proposition below.

Proposition 15. If y_0 drops, specialists' constraints become tighter and their marginal value of information decreases.

Proof. The fact that constraints tighten as y_0 drops follows directly from (76). The rest of the proof follows from Proposition 18.

As in Section 3.2 we assume that each financier sets her margin in order to control her VaR. As we show in the proposition below, the second part of the informational spiral continues to hold: as informational efficiency drops, margins increase and hence constraints tighten.

Proposition 16. If portfolio constraints are of the form of margin requirements, and if margins are VaRbased, then there exists a unique generalized linear equilibrium in which the function g(p) is as characterized by Proposition 14 and the equilibrium margins are given by $m^+ = m^- = \Phi^{-1}(\alpha)\sqrt{Var[f-p|p]} = \Phi^{-1}(\alpha)\sqrt{(\tau_v + \beta^2\tau_z)^{-1} + \tau_{\theta}^{-1}}$. Consequently, for a given specialist's wealth y_0 , if informational efficiency (β) decreases, then the margins $(m^+ \text{ and } m^-)$ both increase. This implies that the lower constraint (i.e., a(p)) increases and the upper constraint (i.e., b(p)) decreases. In other words, as informational efficiency declines, constraints become tighter.

Proof. Follows from Propositions 4 and 14.

Combining the results of Propositions 15 and 16, we get the a version of the informational spiral with risky asset endowment y_0 as wealth.
E For Online Publication: Equilibrium characterization with VaR^{P} margins

Assume that financiers set the margins under physical measure. With risk-averse nonspecialists, the price can be written as $p = \mathbb{E}[v|p] - rp(p)$. As in Brunnermeier and Pedersen (2009), we assume that the financiers use information from prices to set margin in order to control their VaR, as follows:

$$m^{+}(p) = \inf \left\{ m^{+}(p) \ge 0 : Pr(p - f > m^{+}(p)|p) \le 1 - \alpha \right\}$$
 and
$$m^{-}(p) = \inf \left\{ m^{-}(p) \ge 0 : Pr(f - p > m^{-}(p)|p) \le 1 - \alpha \right\},$$

where $m^+(p)$ and $m^-(p)$ are the margins on long and short positions (per unit of asset), respectively. We now derive the expressions for these margins. To compute $m^+(p)$, we first determine the function $m_n^+(p)$ that satisfies

$$1 - \alpha = Pr\left(E[f|p] - rp(p) - f > m_n^+(p)|p\right) = Pr\left(\sqrt{\tau_m}(E[f|p] - f) > \sqrt{\tau_m}(m_n^+(p) + rp(p))|p\right) = 1 - \Phi\left(\sqrt{\tau_m}(m_n^+(p) + rp(p))\right).$$

Thus, we can write

$$m^{+}(p) = [m_{n}^{+}(p)]^{+} = \left[\frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_{m}}} - rp(p)\right]^{+}.$$
(77)

Similarly, one can define $m_n^-(p)$ which satisfies $Pr(f - p > m_n^-(p)|p) = 1 - \alpha$ and can write

$$m^{-}(p) = [m_{n}^{-}(p)]^{+} = \left[\frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_{m}}} + rp(p)\right]^{+}.$$
(78)

The endogenous VaR margins $m^+(\cdot)$ and $m^-(\cdot)$ are determined by three variables. Both margins on long and short positions increase in the exogenous level of confidence α and decrease in the endogenous informational efficiency of price β (through $\tau_m^{-1} = (\tau_v + \beta^2 \tau_z)^{-1} + \tau_{\theta}^{-1}$). In addition, the margin on long (short) position decreases (increases) in the endogenous risk premium rp(p). We would like to emphasize the fact that informational efficiency of price always affect the tightness of margin constraint.

Financial market equilibrium with a risk-averse nonspecialist

Formally, our financial market equilibrium with endogenous margin constraints is defined as follows: (1) financiers and specialists determine demands and margins anticipating a particular price function (2) in equilibrium, demands and margins are consistent with anticipated price function. We hold the precision of specialists' signals fixed.

Proposition 17. (Equilibrium with endogenous margin requirements) When the portfolio constraints are of the form of margin as in equation (7) and margins are endogenously determined by VaR, there exists a unique generalized linear equilibrium. Moreover, in this unique equilibrium the function g(p), i.e. the sufficient statistic ϕ , is increasing in price.

Proof. (Proposition 17) One can prove that for every p there exists unique $\phi = g(p)$ such that the market clears (similar to Proposition 2). We now prove that g(p) is invertible. We plug the expression for our endogenous margins into ODE (5), assuming that both m_n^+ and m_n^- are positive. We get

$$\frac{\partial g(p)}{\partial p} = \frac{c_p^m + \pi_2 c_p - \left(\frac{\pi_1 W_0}{m^-(p)^2} + \frac{\pi_3 W_0}{m^+(p)^2}\right) \frac{\partial r p(p)}{\partial p}}{\pi_2 c_\phi + c_\phi^m}.$$
(79)

Moreover, using the fact that $rp(p) = \frac{\gamma_m}{\tau_m} \left(c_0^m + c_\phi^m g(p) - c_p^m p \right)$, we get

$$\frac{\partial rp\left(p\right)}{\partial p} = \frac{\gamma_m}{\tau_m} \left(c_{\phi}^m \frac{\partial g\left(p\right)}{\partial p} - c_p^m \right).$$
(80)

Substituting (80) into (79), we get

$$\frac{\partial g\left(p\right)}{\partial p} = \frac{c_p^m + \pi_2 c_p + \frac{\gamma_m}{\tau_m} \left(\frac{\pi_1 W_0}{m^-(p)^2} + \frac{\pi_3 W_0}{m^+(p)^2}\right) c_p^m}{\pi_2 c_\phi + c_\phi^m + \frac{\gamma_m}{\tau_m} \left(\frac{\pi_1 W_0}{m^-(p)^2} + \frac{\pi_3 W_0}{m^+(p)^2}\right) c_\phi^m}.$$

Clearly, the derivative above is always positive, which means that the equilibrium function g(p) is invertible. Thus, for each fundamental ϕ there exists a unique p clearing the market. The initial condition for the ODE above can be found by clearing the market for a particular price, for example, p = 0.

We now examine specialists' incentives to acquire information when nonpecialists' risk aversion is small.

Proposition 18. Consider an economy in which all investors face portfolio constraints $a(p) \leq 0 \leq b(p)$ such that $\forall \gamma_m \geq 0$, there exists a generalised linear financial market equilibrium with a monotone function g(p). Next, consider an alternative economy with tighter constraints. That is, investors face portfolio constraints $\hat{a}(p)$ and $\hat{b}(p)$ where $a(p) \leq \hat{a}(p) \leq 0 \leq \hat{b}(p) \leq b(p)$ for all p, such that $\forall \gamma_m \geq 0$, and there exists a generalised linear financial market equilibrium with a monotone function $\hat{g}(p)$. Suppose that in both economies investors choose precision τ_{ϵ} . Then there exists a $\overline{\gamma} > 0$ such that $\forall \gamma_m < \overline{\gamma}$, the marginal value of information decreases for all investors when constraints change from [a(p), b(p)] to $[\hat{a}(p), \hat{b}(p)]$.

Proof of Proposition 18. Denote $\mathcal{MVI}(a(p(\phi)), b(p(\phi)), p(\phi; \gamma_m))$ the marginal value of information when the constraints are given by functions a(p) and b(p) and the equilibrium price function is given by $p(\phi)$. Denote the new, tightened, contraints and the new price function by $\hat{a}(p)$, $\hat{b}(p)$ and $\hat{p}(\phi; \gamma_m)$, respectively. Note that since the information acquisition is held fixed, the sufficient statistic ϕ is the same across the two economies. Note also that

$$\lim_{\gamma_m \to 0} \left(\hat{p}(\phi; \gamma_m) - p(\phi; \gamma_m) \right) = 0.$$

This is because in both cases prices converge to $E[f|\phi]$.

Step 1. We prove that

$$\lim_{\gamma_m \to 0} \left(\mathcal{MVI}(\hat{a}(p(\phi)), \hat{b}(p(\phi)), \hat{p}(\phi; \gamma_m)) - \mathcal{MVI}(a(p(\phi)), b(p(\phi)), p(\phi; \gamma_m)) \right) < 0.$$
(81)

We write the expression in the brackets as follows

$$\mathcal{MVI}(\hat{a}(p(\phi)), \hat{b}(p(\phi)), \hat{p}(\phi; \gamma_m)) - \mathcal{MVI}(a(p(\phi)), b(p(\phi)), p(\phi; \gamma_m)) = \underbrace{\left\{ MVI(\hat{a}(p(\phi)), \hat{b}(p(\phi)), \hat{p}(\phi; \gamma_m)) - \mathcal{MVI}(\hat{a}(p(\phi)), \hat{b}(p(\phi)), p(\phi; \gamma_m)) \right\}_{\rightarrow 0 \text{ as } \gamma_m \rightarrow 0}}_{(MVI(\hat{a}(p(\phi)), \hat{b}(p(\phi)), p(\phi; \gamma_m)) - \mathcal{MVI}(a(p(\phi)), b(p(\phi)), p(\phi; \gamma_m)))} \right\}_{<0}$$

The first term (in the curly brackets) converges to 0, since $\hat{p}(\phi; \gamma_m)$ converges to $p(\phi; \gamma_m)$ pointwise. We now prove that the second term in parentheses is strictly negative in the limit as $\gamma \to 0$.

Note that when we compute that term, we keep the price function fixed, as if price function is independent of portfolio constraints. We write the expression for the marginal value of information as

$$\mathcal{MVI} = \frac{\tau_i}{2\tau_{v,i}^2 \gamma} \frac{U_0^u}{U_0}.$$
(82)

The only term affected by constraints is $\frac{U_0^u}{U_0}$. Consider first the nominator $U_0^u = E[-e^{-\gamma CE_1}\mathbb{I}(x_i^u = x_i)] = E[-e^{-\left(W_0 + \frac{\gamma}{2\tau_i}(x_i^u)^2 - \frac{\gamma}{2\tau_\theta}e_i^2\right)}\mathbb{I}(x_i^u = x_i)]$. As constraints tighten, only the $\mathbb{I}(x_i^u = x_i)$ changes: we keep the price function fixed, therefore the desired demands x_i^u are the same. The term U_0^u increases (becomes less negative) as constraints become tighter. Recall that specialists get negative utility; as constraints become tighter, they get it in fewer states of the world. The denominator U_0 decreases (becomes more negative) as with constraints, the certainty equivalent $CE_{1,i}$ in all states weakly decreases. Thus, the ratio decreases as constraints become tighter.

Step 2. There exists a $\overline{\gamma} > 0$ such that $\forall \gamma_m < \overline{\gamma}$, the marginal value of information decreases for all specialists when constraints change from [a(p), b(p)] to $[\hat{a}(p), \hat{b}(p)]$.

Denote the limit in (81) by λ and the difference between the marginal values of information across the two economies as $\Delta \mathcal{MVI}(\gamma_m)$. By the epsilon-delta definition of a limit (see, e.g., Kolmogorov and Fomm (1975)) it follows that for any $\epsilon > 0$, there exists $\delta > 0$ such that for all γ_m such that $|\gamma_m| < \delta$, we have $|\Delta \mathcal{MVI}(\gamma_m) - \lambda| < \epsilon$. Taking $\epsilon = -\lambda$ we get that for all γ_m such that $\gamma_m < \delta$, $\Delta \mathcal{MVI}(\gamma_m) < 0$. Thus, the desired statement holds for $\overline{\gamma} = \delta$.

Value of information with a risk-averse nonspecialist

Here, we study how the incentives of specialists to acquire information at t = 0 are affected by general portfolio constraints in the case of risk-averse nonspecialists. The goal of this section is to generalize Proposition 18. When nonspecialists are risk-averse, tightening constraints for all specialists is complicated because the equilibrium price distribution will change, which affects price-dependent constraints.

Proposition 19. The following results hold:

- 1. Suppose all specialists are unconstrained. Once finite constraints a(p) and b(p) are introduced for all specialists, the marginal value of information decreases for all of them.
- 2. Suppose that all specialists face portfolio constraints a(p) and b(p). Once all specialists are constrained to hold 0 positions in the asset the marginal value of information decreases for all of them.
- 3. Suppose there exists an equilibrium with monotone function g(p). There exists $\overline{\gamma_m}$ such that for all $\gamma_m < \overline{\gamma_m}$ as constraints tighten, specialists acquire less information.

Proof. Parts 1. and 2. follow directly from Proposition 3 and the fact that $0 < \frac{U_0^u}{U_0} < 1$. Part 3 follows from Proposition 18.

The part 1 of the proposition above demonstrates that when constraints a(p) and |b(p)| are tightened from infinity to some finite positive numbers, marginal value of information for all specialists decreases. Similarly, part 2 demonstrates that when constraints a(p) and |b(p)| are tightened from some finite positive numbers to zero, the marginal value of information decreases as well. Part 3 of the proposition considers general changes in the constraints. It shows that if risk aversion of the nonspecialist is below some threshold, specialists acquire less information as their constraints tighten. All these results yield confidence that the Proposition 18 holds with risk-averse nonspecialist. Even though we cannot prove the statement of Proposition 18 beyond the case of γ_m small enough, we verify it numerically.

In Figure 11 we plot the marginal value of information, against the constraints specialists face. This figure is typical: similar numerical results hold over the entire range of parameters. We record the following observation for future reference.

Observation Suppose that all specialists face portfolio constraints a(p) and b(p) and that the nonspecialist is risk-averse. If constraints become tighter for all specialists, that is, a(p) increases and b(p) decreases, the marginal value of information decreases for all specialists.

In the rest of this appendix, we focus on how margin requirements change with informational efficiency.

Figure 11: Value of information

The plot shows the marginal value of information as a function of specialists' wealth. Other parameters are $\tau_v = 1$; $\tau_z = 1$; $\gamma = 3$; $\alpha = 0.99$. All other parameters are chosen to be 1.



Approximation of the price function $p(\phi)$

In general, the equilibrium price function is nonlinear (as a function of fundamental ϕ) in the economy with margin constraints. This is because, the fractions of specialists who are constrained or not vary with ϕ . In order to facilitate additional analysis and interpretation of the results, we approximate the equilibrium price function with a piece-wise linear price function with three linear parts. The idea is that as ϕ increases from low to high value regions, demands of *almost all* specialists change from being constrained by the margin on short position $m^{-}(\cdot)$, to being unconstrained, and to being constrained by the margin on long position $m^{+}(\cdot)$.³⁵

Consider the case with some $\phi < \phi^-$ such that most specialists' demand are constrained by the margin on short position, that is, $X^u \approx -\frac{W_0}{m^-(p)} = -\frac{W_0}{\left[\frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}} + rp(p)\right]^+}$. With the nonspecialist's demand $x_m = \frac{\tau_m}{\gamma_m} rp(p)$, the market clearing condition becomes

the market clearing condition becomes

$$-\frac{W_0}{\left[\frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}} + rp(p)\right]} + \frac{\tau_m}{\gamma_m} rp(p) = 1 \quad \text{for } rp(p) > -\frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}}.$$

Denote rp^- as the risk premium that satisfies the above equation. It is easy to check that the risk premium rp^- in this case is unique and does not depend on price. Finally, since the nonspecialist's demand is a proportional to risk premium and thus also a constant in price this case, the equilibrium price must adjust with respect to the fundamental ϕ such that the nonspecialist's demand stays constant. That is, from $rp^- = \frac{\gamma_m}{\tau_m} x_m = \frac{\gamma_m}{\tau_m} (c_0^m + c_{\phi}^m \phi - c_p^m p)$, we find the linear price function $p(\phi) = -\frac{\tau_m}{\gamma_m c_p^m} rp^- + \frac{c_0^m}{c_p^m} + \frac{c_{\phi}^m}{c_p^m} \phi$ for $\phi < \phi^-$.

One can characterize linear pricing functions with similar procedures for the case of unconstrained specialists in the intermediate range of fundamental $\phi \in (\phi^-, \phi^+)$ and the case of specialists' demand being constrained by the margin on long margin for the high range of $\phi > \phi^+$. The boundary values of ϕ^- and ϕ^+ are pinned down by imposing continuity on the approximated price functions. We summarize the result in the following lemma.

Lemma 10. (An approximated price function.) The equilibrium price function $p(\phi)$ can be approximated by three linear price functions for three different scenarios about specialists' demand: i) constrained by the

³⁵For any fundamental ϕ , there are always constrained specialists and unconstrained specialists, thanks to dispersed idiosyncratic shocks on specialists' endowment and signal. Even at extreme ϕ , there are a measure non-zero of specialists who are unconstrained, which makes the aggregate demand vary with fundamentals. Importantly, such arbitrarily small variations in aggregate demand allows the nonspecialist to learn about the fundamental and clear the market with a market-clearing price.

margin on short position $m^{-}(p)$; ii) unconstrained; iii) constrained by the margin on long position $m^{+}(p)$. The approximated price function $\hat{p}(\phi)$ is

$$\hat{p}(\phi) = \begin{cases} \frac{1}{c_p^m} (-\frac{\tau_m}{\gamma_m} rp^- + c_0^m + c_\phi^m \phi) & \text{for } \phi < \phi^- \\ \frac{c_0 + c_0^m - 1}{c_p + c_p^m} + \frac{c_\phi + c_\phi^m}{c_p + c_p^m} \phi & \text{for } \phi \in [\phi^-, \phi^+] \\ \frac{1}{c_p^m} (-\frac{\tau_m}{\gamma_m} rp^+ + c_0^m + c_\phi^m \phi) & \text{for } \phi > \phi^+ \end{cases}$$
where $rp^- = \frac{\gamma_m - k\tau_m + \sqrt{\gamma_m^2 + k^2 \tau_m^2 + 2k\gamma_m \tau_m + 4\gamma_m \tau_m W_0}}{2\tau_m}, \ rp^+ = \frac{\gamma_m + k\tau_m - \sqrt{\gamma_m^2 + k^2 \tau_m^2 - 2k\gamma_m \tau_m + 4\gamma_m \tau_m W_0}}{2\tau_m} \text{ with } k = \frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}}, \ \text{and } \phi^j = \frac{-\frac{\tau_m}{\gamma_m c_p^m} rp^j + \frac{c_0^m - c_0^m - 1}{c_p + c_p^m}}{\frac{c_0^{+} + c_p^m}{c_p} - \frac{c_0^{+} + c_p^m}{c_p}}{\frac{c_0^{+} + c_p^m}{c_p} - \frac{c_0^{+} + c_p^m}{c_p}}{\frac{c_0^{+} + c_p^m}{c_p} - \frac{c_0^{+} + c_p^m}{c_p}}} \text{ for } j = \{-, +\}, \ \text{and } rp^- > rp^+ \ \text{and } \phi^- < \phi^+. \end{cases}$

Note that the price function is more sensitive to fundamental at the intermediate values of ϕ when most specialists' demands are not restricted by the margin constraints. Intuitively, this is because the specialists can adjust their demand and thus impound more information about fundamentals into price.

Margins with a risk-averse nonspecialist

In this section, we provide conditions for Proposition 4 and Remark 4 from the main text to still hold with a risk-averse nonspecialist. Assume that each financier sets her margin in order to control her VaR, as in Brunnermeier and Pedersen (2009). Given fundamentals v and z, the margins are given by

$$m^{+}(p) = [m_{n}^{+}(p)]^{+} = \left[\frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_{m}}} - rp(p)\right]^{+}$$
 and $m^{-}(p) = \left[\frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_{m}}} + rp(p)\right]^{-}$.

Proposition 20. For a given specialists' wealth W_0 , when informational efficiency (β) decreases, margins increase for all price realizations if

$$\gamma_m < \frac{\sqrt{\tau_m} \Phi^{-1}(\alpha)}{2}$$

and W_0 and σ_{ξ} are small. This implies that, as informational efficiency drops, constraints become tighter.

Proof. Using the approximated price function, which is valid, given that σ_{ξ} is small, we can write the risk premium as

$$rp(p) = \begin{cases} rp^{-} & \text{for } g(p) < \phi^{-} \\ \frac{\gamma_{m}}{\tau_{m}} \left(\frac{c_{p}^{m}}{c_{p} + c_{p}^{m}} + \frac{c_{p}c_{\phi}^{m} - c_{\phi}c_{p}^{m}}{c_{p} + c_{p}^{m}} g(p) \right) & \text{for } g(p) \in [\phi^{-}, \phi^{+}] \\ rp^{+} & \text{for } g(p) > \phi^{+}. \end{cases}$$
(83)

Let $k = \frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}}$ and $l = \frac{\gamma_m}{\tau_m}$. Taking the derivative of margins with respect to τ_m , we get

$$\frac{\partial m^{+}(p)}{\partial \tau_{m}} = \begin{cases} \frac{\partial m_{n}^{+}(p)}{\partial \tau_{m}} = -\frac{\Phi^{-1}(\alpha)}{2\sqrt{\tau_{m}^{3}}} - \frac{\partial rp(p)}{\partial \tau_{m}} & \text{if } \frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_{m}}} > rp(p) \\ 0 & \text{otherwise} \end{cases}$$
(84)

$$\frac{\partial m^{-}\left(p\right)}{\partial \tau_{m}} = \begin{cases} \frac{\partial m_{n}^{-}\left(p\right)}{\partial \tau_{m}} = -\frac{\Phi^{-1}\left(\alpha\right)}{2\sqrt{\tau_{m}^{3}}} + \frac{\partial rp\left(p\right)}{\partial \tau_{m}} & \text{if } \frac{\Phi^{-1}\left(\alpha\right)}{\sqrt{\tau_{m}}} > -rp\left(p\right) \\ 0 & \text{otherwise} \end{cases}$$
(85)

where $\frac{\partial k}{\partial \tau_m} = -\frac{k}{2\tau_m}, \frac{\partial l}{\partial \tau_m} = -\frac{l}{\tau_m}$ and

$$\begin{split} \frac{\partial m_n^+(p)}{\partial \tau_m} &= \frac{\partial}{\partial \tau_m} \left[k - rp\left(p\right) \right] \\ &= \begin{cases} \frac{\partial}{\partial \tau_m} \left[k - rp^- \right] & \text{for } g\left(p\right) < \phi^- \\ \frac{\partial}{\partial \tau_m} \left[k - \frac{\gamma_m}{\tau_m} \left(\frac{c_p^m}{c_p + c_p^m} + \frac{c_p c_\phi^m - c_\phi c_p^m}{c_p + c_p^m} g\left(p\right) \right) \right] & \text{for } g\left(p\right) \in [\phi^-, \phi^+] \\ \frac{\partial}{\partial \tau_m} \left[k - rp^+ \right] & \text{for } g\left(p\right) > \phi^+ \end{cases} \\ &= \begin{cases} \frac{2l - 3k}{4\tau_m} + \frac{(2l + k)(l + k) + 4W_0 l}{4\tau_m \sqrt{l^2 + k^2 + 2kl + 4lW_0}} & \text{for } g\left(p\right) < \phi^- \\ -\frac{k}{2\tau_m} + \frac{\frac{\gamma}{\gamma} \frac{\partial \tau_m}{\partial \tau_m} + \frac{1}{\tau_m}}{\left(\frac{\tau}{\gamma} + \frac{1}{\gamma}\right)^2} & \text{for } g\left(p\right) \in [\phi^-, \phi^+] \\ \frac{2l - k}{4\tau_m} - \frac{(2l - k)(l - k) + 4lW_0}{4\tau_m \sqrt{l^2 + k^2 - 2kl + 4lW_0}} & \text{for } g\left(p\right) > \phi^+. \end{cases} \end{split}$$

The third term in the above expression is negative if 2l - k < 0. For W_0 small enough, even the first term in the above expression is negative. Similarly,

$$\begin{split} \frac{\partial m_n^-(p)}{\partial \tau_m} &= \frac{\partial}{\partial \tau_m} \left[k + rp\left(p \right) \right] = \begin{cases} \frac{\partial}{\partial \tau_m} \left[k + rp^- \right] & \text{for } g\left(p \right) < \phi^- \\ \frac{\partial}{\partial \tau_m} \left[k + \frac{\gamma_m}{\tau_m} \left(\frac{c_p^m}{c_p + c_p^m} + \frac{c_p c_\phi^m - c_\phi c_p^m}{c_p + c_p^m} g\left(p \right) \right) \right] & \text{for } g\left(p \right) \in [\phi^-, \phi^+] \\ \frac{\partial}{\partial \tau_m} \left[k + rp^+ \right] & \text{for } g\left(p \right) > \phi^+ \end{cases} \\ &= \begin{cases} -\frac{3k}{4\tau_m} - \frac{l}{2\tau_m} - \frac{2l^2 + k^2 + 3kl + 4W_0 l}{4\tau_m \sqrt{l^2 + k^2 + 2kl + 4lW_0}} & \text{for } g\left(p \right) < \phi^- \\ -\frac{k}{2\tau_m} - \frac{\frac{1}{\gamma} \frac{\partial \sigma_m}{\partial \tau_m} + \frac{1}{\tau_m}}{\left(\frac{\tau}{\gamma} + \frac{1}{l} \right)^2} & \text{for } g\left(p \right) < \phi^- \\ -\frac{k}{4\tau_m} - \frac{l}{2\tau_m} + \frac{2l^2 + k^2 - 3kl + 4lW_0}{4\tau_m \sqrt{l^2 + k^2 - 2kl + 4lW_0}} & \text{for } g\left(p \right) > \phi^+ \end{cases} \end{split}$$

Note that the first term and second terms in the above expression are always negative. If W_0 is small enough, even the third term is negative. This implies that $m^-(p)$ always decreases with informational efficiency. So the sufficient conditions for the margins to decrease with informational efficiency is 2l < k and small enough wealth.

Below we examine the conditions of the proposition above, for which Remark 4 from the main text still holds.

Proposition 21. Suppose that the conditions of Proposition 20 hold. Then margins increase when informational efficiency drops, even when the financier does not learn from prices.

Proof. We prove the proposition for m^+ , the margin on a long position. The proof for m^- is analogous and is omitted for brevity.

When financiers cannot condition margins on prices, the margins are determined by the following equation:

$$\alpha = E\left[\Phi\left(\sqrt{\tau_m}(m^+ + rp(p))\right)\right].$$
(86)

We prove this proposition by contradiction. Denote $m_n^+(p) = \frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}} - rp(p)$ (the margins the financier would set if he would be able to condition on prices). Suppose that m_n^+ decreases as informational efficiency drops. One can write

$$\begin{split} \sqrt{\tau_m} \left(m^+ + rp(p) \right) &= \sqrt{\tau_m} \left(m^+ - m_n^+(p) + m_n^+(p) - rp(p) \right) \\ &= \sqrt{\tau_m} \left(m^+ - m_n^+(p) + \frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}} \right) \\ &= \sqrt{\tau_m} \left(m^+ - m_n^+(p) \right) + \Phi^{-1}(\alpha). \end{split}$$

We assume that the conditions of Proposition 20 hold, therefore, $m_n^+(p)$ increases for all p as informational efficiency drops. At the same time, $\sqrt{\tau_m}$ decreases and we assumed that m^+ drops as well. This implies that

 $E\left[\Phi\left(\sqrt{\tau_m}(m^+ + rp(p))\right)\right]$ drops when informational efficiency drops. A contradiction with (86).

The following corollary to Propositions 19 and 20 generalizes Corollary ??.

Corollary 2. Suppose that the conditions of Proposition 20 hold and γ_m is small enough. Then a decrease in specialist wealth W_0 decreases informational efficiency β and increases VaR-based margins m^+, m^- .

Proof. Follows directly from Propositions 19 and 20. ■

Intuitively, information efficiency decreases with wealth (Proposition 19); when information efficiency drops, margins rise (Proposition 20).

Micro-founding VaR^{P} -margin

Here, we provide a microfoundation for situations where risk should be evaluated under physical measure.

We assume that financiers do not participate in financial markets and are exposed to only idiosyncratic shocks. Given this, the wealth from financing activity is not correlated with their other wealth, and the financier is effectively risk-neutral with respect to income from financing. The financier is endowed with a large but finite amount of cash. We assume that the the financier gets a gross return of 1 when lending to the specialists but only $(1 - \epsilon)$ when investing in the risk-free asset.

We also assume that the specialists' date-2 wealth is not pledgeable and the financier has to pay a cost to enforce the specialist to repay with date-2 wealth. Therefore, for every unit of asset that the specialist has invested, he can transfer the asset and some cash m^+ to the financier's account as a collateral. Effectively, the financier is lending an amount $(p - m^+)$ to the specialist while holding the asset as collateral.

At t = 2, the specialist has to repay $(p - m^+)$ to get the asset dividend f back. If the dividend is more than the promised repayment, that is, $f > p - m^+$, costly enforcement is not needed because the financier can just take the repayment from the dividend of the asset which is at his custody. If instead the dividend from the asset is less than the promised repayment, we assume the financier has to pay an enforcement cost kper dollar lent to force the specialists to pay with his date-2 wealth. In sum, upon observing p, a competitive financier chooses a cash margin m^+ so that he is indifferent between lending to the specialists and investing in the risk-free asset

$$x_i(p - m^+) - Pr(x_i f < x_i(p - m^+)|p) k x_i(p - m^+) \ge x_i(p - m^+)(1 - \epsilon)$$
(87)

After simplification, we have

$$Pr(p-f > m^+|p) \le \frac{\epsilon}{k} \tag{88}$$

which coincides with the VaR margin constraint in (8) when $\frac{\epsilon}{k}$ is replaced with $1 - \alpha$.