Funding Constraints and Informational Efficiency*

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Abstract

We develop a tractable rational expectations model that allows for general price-dependent portfolio constraints and study a setting where constraints arise because of margin requirements. We argue that constraints affect and are affected by informational efficiency, leading to a novel amplification mechanism. A decline in wealth tightens constraints and reduces investors' incentive to acquire information, lowering price informativeness. Lower informativeness, in turn, increases the risk borne by financiers who fund trades, leading them to further tighten constraints faced by investors. This *information spiral* leads to significant increases in risk premium, return volatility and Sharpe ratio, as investors' wealth declines.

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1 Introduction

One of the basic tenets of financial economics is that market prices aggregate investors' information. The core of the argument is that investors acquire information about future asset values and trade on it, thereby impounding that information into price. This argument presupposes that investors have incentives to acquire information and the capacity to trade on it, where each of these factors is crucially affected by investors' ability to fund their trades. Thus, an important question arises: how do funding constraints faced by investors affect price informativeness? Conversely, since lower informativeness might have an effect on the financier's risk of funding a trade, another important question is: how does price informativeness affect the tightness of funding constraints? Answering these questions requires a model in which price informativeness and funding constraints are jointly determined in equilibrium. Our paper develops such a model and examines its implications for asset pricing.

The main challenge in studying the interplay between funding constraints and informational efficiency is that most noisy rational expectation equilibrium (REE) models, which are instrumental in analyzing informational efficiency, cannot accommodate constraints in a tractable manner.[1] Our first contribution is, developping a tractable REE model with *general portfolio constraints* that can depend on prices. We then apply our methodology to study a model in which portfolio constraints arise because of margin requirements set by financiers. Our second contribution is to show that investors' funding both affects and is affected by informational efficiency, which leads to a novel amplification mechanism that we call the *information spiral*. This mechanism implies that the risk premium, conditional volatility of returns and Sharpe ratio rise significantly as investors' wealth falls.

We consider a canonical CARA-Normal REE model in which investors first acquire information and then, trade in order to profit from their private signals about the risky asset's fundamental value and also to hedge their endowment shocks. The novelty is that we allow

[1]Two noteworthy exceptions are Yuan (2005) and Nezafat, Schroder, and Wang (2017); these authors analyze borrowing constraints and short-sale constraints, respectively.

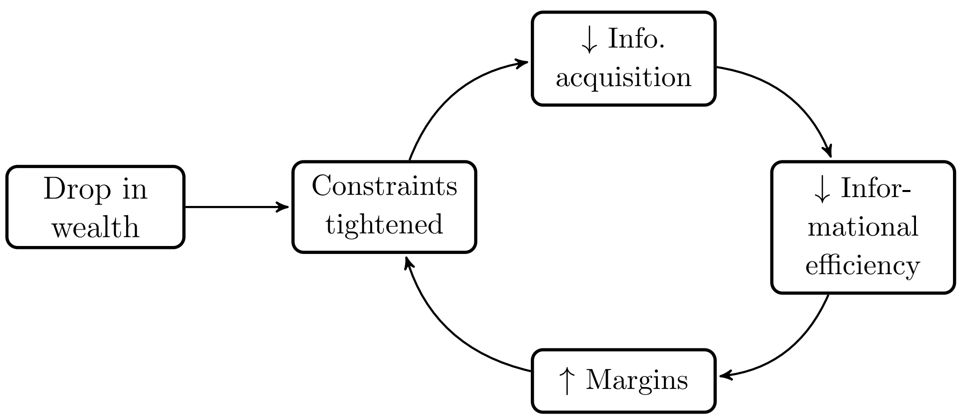
for general portfolio constraints: investors can trade up only to some maximal long and short positions of the risky asset, and these portfolio constraints can depend on price. This general price-dependent specification of portfolio constraints subsumes many types of real-world trading constraints (e.g., short-sale constraints, borrowing constraints, margin requirements). Without constraints, the model is standard: (i) an investor's demand is linear in his private signal, the endowment shock, and the price; (ii) the equilibrium price itself is linear in the fundamental value and aggregate endowment shock; and (iii) investors' initial wealth is irrelevant for asset prices.

Under portfolio constraints, the financial market equilibrium is as follows. (i) Investors' *desired* demand (i.e., the amount they would like to trade) is still linear, but their *actual* demand is the desired demand truncated to the maximal long or short positions. (ii) Although the price function may not be linear, it is informationally equivalent to a linear combination of the fundamental value and the aggregate endowment shock; hence, inference remains tractable. (iii) Investors' initial wealth matters for asset prices provided that it affects constraints. With the methodology of solving equilibrium with constraints at hand, we turn to study the paper's primary concern: the reinforcing interaction between constraints and informational efficiency.

We begin with an analysis of how constraints affect informational efficiency. Without further specifying the source or form of constraints, we show that they hinder such efficiency. It is intuitive that, when constraints become tighter, investors must take smaller positions and so profit less on their private information. Anticipating the reduced scope for profit, they acquire less information ex ante. As investors acquire less information, the price becomes less informative about asset fundamentals in equilibrium. And to the extent that investors' wealth relaxes their constraints, a wealth effect emerges in our model despite investors' absolute risk aversion being constant: lower wealth impedes information acquisition, and hence, reduces informational efficiency.

Next we study the reverse channel of informational efficiency affecting constraints. Motivated by real-world margin requirements, we follow Brunnermeier and Pedersen (2009) in assuming that investors finance their positions through collateralized borrowing from financiers who require margins that control their value-at-risk (VaR).[2] We show that lower informational efficiency leads to tighter margins. Here, it is intuitive that, when prices are less informative, the price tracks fundamentals less closely, which implies a greater risk of the trade they finance, leading them to set higher margins. When we combine these analyses, our model yields two key implications. First, tighter funding constraints reduce the information acquired by investors, which reduces informational efficiency; second, reduced informational efficiency leads to higher margins, which tightens investors' constraints. This interdependence gives rise to an information-based amplification mechanism, illustrated in Figure 1, that we call the *information spiral*.

Figure 1: Amplification mechanism



There are two key implications of this information spiral. First, a negative shock to investors' wealth is amplified and causes larger changes in asset prices than in a model with fixed signal quality and/or a fixed margin requirement. A drop in investors' wealth tightens their constraints, discouraging acquisition of information. The resulting lower informational efficiency in turn causes financiers to set higher margin requirements, further tightening in-

[2] Our main results are robust to alternative risk-based margins, such as tail value-at-risk (TVaR) and expected shortfall (ES).

vestors' constraints. We show that, owing to this amplification mechanism, such a shock leads to increases in risk premium, return volatility and Sharpe ratio when investors' wealth is low, which we interpret as a crisis period. These results match empirical observations made during crisis periods.[3] Although the literature has proposed other amplifying mechanisms for the effect of wealth shocks, ours is distinct in this sense: it works via the interaction between the informational efficiency of financial markets and the funding constraints of investors.

The information spiral's second key implication is that it provides a source for strategic complementarities in investors' decisions to acquire information: a reduction in information acquired by others makes price less informative, which increases the margin requirements faced by the investor and induces him to acquire less information. Furthermore, this complementarity effect occurs when investors' wealth is low, i.e., in a crisis period.

This paper makes several methodological contributions. We present and solve a REE model with general portfolio constraints and compute the marginal value of information for an investor facing these constraints in closed-form.[4] In our main application we consider constraints arising from margin requirements, but one can also utilize our methodology to study other types of constraints.[5]

Related Literature

This paper lies at the intersection of various strands of literature. It shares the emphasis of seminal studies that address the role played by financial markets in aggregating and disseminating information, which include Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980) and Diamond and Verrecchia (1981). In these models, it is generally assumed that investors can

[3]Financial crises, such as the hedge fund crisis of 1998 and the 2007–2008 subprime crisis, have several common characteristics: risk premia rise, the conditional volatility of asset prices rises, and the Sharpe ratio rises.

[4]By using stochastic calculus, we compute the marginal value of information for an investor facing general portfolio constraints directly, without first calculating the value of information and then differentiating it with respect to investor's choice of signal precision.

[5]In Appendix C, we study the case in which investors have some risky assets as initial wealth, which gives rise to a form of borrowing constraints as in Yuan (2005).

borrow or lend freely at the riskless rate—in other words, there are no funding constraints. We contribute to this literature by developing an REE model that incorporates general portfolio constraints. Some particular types of portfolio constraints have been examined before: Yuan (2005), Venter (2015), Yuan (2006) study REE models with borrowing constraints, short-sale constraints, and both constraints, respectively. Albagli, Hellwig, and Tsyvinski (2011) derive various asset-pricing implications in a model with risk-neutral investors, exogenous portfolio constraints and exogenous information. Our work differs from these papers in that we study investors' information acquisition problem and focus on the interplay between the tightness of constraints and the equilibrium informational efficiency.

Closely related to our work is Nezafat et al. (2017), who focus on how short-sale constraints affect information production and asset prices. While similar in spirit, our paper differs in two important dimensions. First, our methodology extends their work to explore price-dependent constraints of a more general nature, allowing us to consider constraints resulting from risk-based margin requirements. Second, and more importantly, in our paper informational efficiency affects constraints, which is not present in their paper.

Our work is related to the literature on information acquisition in REE models. Grossman and Stiglitz (1980), Verrecchia (1982), Peng and Xiong (2006), Van Nieuwerburgh and Veldkamp (2009) study financial investor's information acquisition problem in the case of no funding constraints. Peress (2004) and Breugem and Buss (2017) use approximation and numerical methods, respectively, to investigate the effect of investors' wealth on information acquisition in a setting with investors who exhibit constant relative risk aversion (CRRA). Our tractable model also features wealth effects, despite the investors having constant absolute risk aversion, because investors' wealth relaxes their funding constraints. Moreover, we can derive all our results analytically in the "crisis" limit when investors' wealth is small without relying on approximations. Since our paper speaks to the evidence in crises, when changes in equilibrium quantities are highly non-linear, not relying on approximations is important.

In addition, we contribute to the literature on strategic complementarities in information acquisition, for example, Veldkamp (2006); Hellwig and Veldkamp (2009); Garcia and Strobl (2011); Ganguli and Yang (2009); Goldstein and Yang (2015); Avdis (2016); Huang (2015) and Dow, Goldstein, and Guembel (2017). In particular, the focus of complementarity between two groups of agents via information acquisition and trading is shared in Goldstein and Yang (2015). In Goldstein and Yang (2015), when one group of investors learns less about (and trades less aggressively on) one component of fundamentals, the price becomes less informative about this component. This increases uncertainty for the other group, which in turn, learns less about (and trades less aggressively on) another component. As a result, acquiring information about the two components of fundamentals is a strategic complement. We view our analysis as complementary to theirs: at the aggregate level, our paper shares the feature that more learning by one group of agents (investors in our model) reduces the uncertainty faced by the other group (financiers in our model), whose response (more financing) in turn reinforces learning by the first group. However, there are important differences in the underlying mechanism. In our model, financiers do not produce information. The complementarity comes from financiers' funding decisions to investors. Moreover, complementarity in our paper arises even without multiple (learnable) components of fundamentals.

Except for Dow et al. (2017), the main distinguishing feature of our model is that complementarities arise in bad times, and therefore, our results have business-cycle predictions. While the mechanism in Dow et al. (2017) also generates complementarities during bad times, our paper differs from theirs in two dimensions. First, the predictions are different because bad times mean that investors, or the financial sector in general, have low wealth in our paper while in theirs bad times mean low productivity in the real sector. Second, the amplification mechanism acts through firm managers' learning to make real investment decisions in Dow et al. (2017), whereas in ours, it is via financiers' funding decisions.

Our paper is also related to the literature on secondary financial markets as a source of information for decision makers; see Bond, Edmans, and Goldstein (2012) for a survey. We contribute to this literature by studying how financiers can use the information in prices to set their margins, and we find that lower informational efficiency leads to tighter margins.

Finally, our work contributes to the literature on the effect of investors' wealth and the associated amplification mechanisms. For example, Xiong (2001) and Kyle and Xiong (2001) study wealth constraints as amplification and spillover mechanisms, respectively. Gromb and Vayanos (2002, 2017) develop an equilibrium model of arbitrage trading with margin constraints to explain contagion. Brunnermeier and Pedersen (2009) examine how funding liquidity and market liquidity reinforce each other. He and Krishnamurthy (2011, 2013) and Brunnermeier and Sannikov (2014) study how declines in an intermediary's capital reduce her risk-bearing capacity and lead to higher risk premia and conditional volatility; see also He and Krishnamurthy (2018) for a survey of the topic. None of these papers studies the interaction of investors' wealth (or constraints) and informational efficiency, which is the crux of our paper.

The rest of our paper is organized as follows. In Section 2, we solve for the financial market equilibrium and the value of information in an REE model with general portfolio constraints. Section 3 introduces margin requirements and shows how funding constraints affect—and are affected by—informational efficiency. In Section 4, we explore the implications of our information spiral for asset prices. After summarizing our predictions in Section 5, we discuss the robustness of the results in Section 6 and conclude in Section 7. Appendix A contains proofs of propositions in the main text. Appendix B considers an alternative model with classical noise traders to show that the information spiral continues to hold. Appendix C studies the case in which investors have risky asset as initial wealth. Appendix D outlines financiers' problem and provides microfoundation for the use of VaR-based margin. Appendix E studies a model with an alternative specification for VaR margin.

2 An REE model with general portfolio constraints

In this section we develop a model with general portfolio constraints. In Section 3, we will apply our model to study constraints that arise from margin requirements.

2.1 Setup

There are three dates (i.e., $t \in \{0, 1, 2\}$) and two assets. The risk-free asset has exogenous (net) return normalized to zero. The payoff (fundamental value) of the risky asset is $f = v + \theta$ (which is paid at date 2), where v is the learnable (i.e., information about which can be acquired) component of fundamentals, $v \sim N(0, \tau_v^{-1})$ and θ is the unlearnable component of fundamentals, $\theta \sim N\left(0, \tau_\theta^{-1}\right)$ and is independent of v. The aggregate supply of the asset is assumed to be constant 1 unit. The economy is populated by a unit continuum of investors, indexed by $i \in [0, 1]$, with identical CARA preferences over terminal wealth with CARA parameter γ. There is also a competitive market maker with CARA preferences over terminal wealth with CARA parameter γ_m.[6] Investors acquire information at $t = 0$ and trade the risky asset with the market maker at $t = 1$. All agents consume at $t = 2$.

Investors trade the risky asset for hedging and profit reasons. Specifically, at date 2, each investor receives a random, non-tradable, and non-pledgeable endowment b_i, which has a payoff that is correlated with the unlearnable component of the risky asset's payoff, θ. We assume that the endowment is given by $b_i = e_i\theta$. The coefficient e_i measures the sensitivity of the endowment shock to the payoff of the risky asset and is known to the investor at $t = 1$. Hereafter, we will refer to e_i as the endowment shock of investor i. Finally, the investor i's endowment shock e_i has systematic and idiosyncratic components: $e_i = z + u_i$. Both components are normally distributed and independent of v and θ, with $z \sim N\left(0, \tau_z^{-1}\right)$ and $u_i \sim N(0, \tau_u^{-1})$. Moreover, idiosyncratic shocks u_i are independent across investors and independent of z. This

[6]This specification of market maker nests two benchmarks. When $\gamma_m = 0$, our market maker is risk-neutral, as in Vives (1995). When $\gamma_m = \infty$, the market maker does not trade, hence, our model is equivalent to a model without a market maker.

formulation implies that there is uncertainty about the aggregate endowment shock z, which will create noise in the price.

At date 1, each investor i receives a signal $s_i = v + \epsilon_i$, where the ϵ_i are independent across investors with $\epsilon_i \sim N(0, \tau_{\varepsilon_i}^{-1})$. The precision of his private signal τ_{ϵ_i} is optimally chosen by investor i at date 0, subject to an increasing, twice continuously differentiable cost function $C(\tau_{\epsilon_i} - \underline{\tau_\epsilon})$. We assume that this cost function is identical for all investors and $C(x) = 0$ for $x = 0$. That is, there is no cost of acquiring information with quality below $\underline{\tau_\epsilon} > 0$.[7] When forming their expectations about the fundamental, investors use all the information available to them. The information set of investor i at time 1 is $\mathcal{F}_i = \{p, s_i, e_i\}$, where p is the equilibrium price at time 1. A competitive market maker faces no endowment shocks and receives no signals about the asset payoff. Hence, the market maker's information set at time 1 is $\mathcal{F}_m = \{p\}$.

Constraints. The investors in our model—but not the market maker—are subject to the funding constraints described here.[8] Given the price p, the minimum and maximum positions that an investor can take are $a(p)$ and $b(p)$, respectively, with $a(p) < 0 < b(p)$. The functions $a(p)$ and $b(p)$ may depend on investors' initial wealth W_0 and other aggregate equilibrium variables, such as volatility of returns. In short: at date 1, investors solve the problem

$$\begin{aligned} \max_{x_i(p,s_i,e_i)} \quad & E[-\exp(-\gamma W_i) \mid p, s_i, e_i], \\ \text{subject to} \quad & a(p) \leq x_i(p, s_i, e_i) \leq b(p), \\ \text{where} \quad & W_i = W_0 + x_i(v + \theta - p) + e_i\theta. \end{aligned} \tag{1}$$

The equation above states that the terminal wealth of investor i is the sum of his initial wealth, the profit or loss from trading the risky asset, and his endowment.

[7] This is equivalent to assuming that investors are endowed with information with quality $\underline{\tau_\epsilon}$ and only the incremental information is subject to a cost $C(\cdot)$. This assumption is technical and $\underline{\tau_\epsilon}$ can be arbitrary small: we need it to make sure that τ_ϵ is bounded away from zero in equilirbium, even when investors are fully constrained.

[8] We assume the market maker is unconstrained because our focus is on the interplay between investors' constraints and informational efficiency. Nevertheless, our model remains tractable if the market maker is also subject to constraints.

Similarly, the market maker solves

$$\max_{x_m(p)} \quad E[-\exp(-\gamma_m W_m) \mid p], \tag{2}$$
$$\text{where} \quad W_m = W_{0,m} + x_m(v + \theta - p).$$

Finally, the equilibrium price is set to clear the market as follows:

$$\int x_i(p, s_i, e_i)di + x_m(p) = 1. \tag{3}$$

We proceed to solve the model via backward induction. We postpone the discussion of modelling choices to Section 2.2.3, after we have demonstrated the tractability of our model. In Section 2.2 we characterize the financial market equilibrium at $t = 1$ for given investors information-acquisition decisions made at $t = 0$. In Section 2.3, we solve for the investor's optimal information acquisition decision.

2.2 Financial market equilibrium at $t = 1$

We first solve for equilibrium in the unconstrained setting (i.e., when $a(p) = -\infty$ and $b(p) = \infty$), which was studied previously in Nezafat et al. (2017).[9] We review this setting here because it is an important benchmark in characterizing the equilibrium with constraints.

2.2.1 Unconstrained setting

Our first proposition characterizes the unconstrained equilibrium and its key features. Unless stated otherwise, proofs of all propositions in the main text are given in Appendix A. From here on, we use superscript "u" for variables characterizing the unconstrained setting. The corresponding variables without superscript are used for the constrained setting.

[9]See also Ganguli and Yang (2009) and Manzano and Vives (2011), who analyzed related settings.

Proposition 1. *(Financial market equilibrium without portfolio constraints) Suppose investors have identical signal precision* τ_ϵ *and* $\tau_u^2 \tau_\theta^2 < 3\gamma^2 (\tau_u + \tau_z) \tau_v$. *Then there exists a unique linear equilibrium in which the price is informationally equivalent to a statistic* $\phi^u = v - \frac{z}{\beta^u} = g_0^u + g_1^u p$. *The aggregate demand of investors and the market maker can be written as*

$$X^u(p, \phi) = c_0 + c_\phi \phi - c_p p \quad \textit{and} \quad x_m(p, \phi) = c_0^m + c_\phi^m \phi - c_p^m p,$$

respectively. The individual demand of investor i *can be written as follows:*

$$x_i^u = X^u + \xi_i, \quad \textit{where } \xi_i \sim \mathcal{N}(0, \sigma_\xi^2) \textit{ are i.i.d. across investors,}$$

and β^u *is the unique root* (β)*, which solves*

$$\beta^3 \gamma (\tau_u + \tau_z) - \beta^2 \tau_u \tau_\theta + \beta\gamma (\tau_\epsilon + \tau_v) - \tau_\theta \tau_\epsilon = 0. \tag{4}$$

Moreover, β^u *increases with* τ_ε, *the precision of investors' information. All the coefficients are reported in Appendix A.*

The analysis of unconstrained equilibrium highlights some important features of the model that will continue to hold in the constrained setting. We observe, first of all, that in equilibrium, price is informationally equivalent to a linear combination of the (learnable) fundamental payoff v and the aggregate endowment shock z. Second, the extent of fundamental information revealed by price is captured by an endogenous signal-to-noise ratio (β^u). More precisely, the conditional variance of the learnable fundamental decreases as β^u increases, as follows:

$$Var(v|p) = Var(v|\phi^u) = (\tau_v + (\beta^u)^2 \tau_z)^{-1}. \tag{5}$$

Hence, we refer to β^u as the *informational efficiency* of the market when investors are unconstrained in their trading. It is important to bear in mind that investors' information acquisition (higher signal precision τ_ϵ) improves the informational efficiency β^u of the market.

Note that the condition $\tau_u^2 \tau_\theta^2 < 3\gamma^2 (\tau_u + \tau_z) \tau_v$ is sufficient to guarantee the uniqueness of a linear equilibrium without constraints.[10] We shall proceed under the assumption that this condition continues to hold.

2.2.2 Constrained setting

We now impose the portfolio constraints $a(p)$ and $b(p)$ on the investor's problem. We posit and then verify that there exists a generalized linear equilibrium in the economy, which we define as follows.

Definition 1. *An equilibrium is generalized linear if there exists a function $g(p)$ and a scalar β, such that $\phi = v - \frac{z}{\beta}$ is informationally equivalent to price and is given by $\phi = g(p)$.*[11]

The ϕ and β defined here are the counterparts of ϕ^u and β^u in the economy without portfolio constraints. In a generalized linear equilibrium, the price function may be nonlinear, but the statistic ϕ is still linear in (v, z), and therefore, is normally distributed; therefore, the inference from price remains tractable. Since equation (5) holds in a generalized linear equilibrium, we continue using β to denote informational efficiency.

When there are constraints, the individual demand of investor i can be written as follows:

$$x_i(p, s_i, e_i) = \begin{cases} x_i^d(p, s_i, e_i), & \text{if } a(p) \le x_i^d(p, e_i, s_i) \le b(p), \\ b(p), & \text{if } x_i^d(p, s_i, e_i) > b(p), \\ a(p), & \text{if } x_i^d(p, s_i, e_i) < a(p), \end{cases}$$

[10] If the condition does not hold, there might be up to three roots for equation (4), which implies that there are at most three equilibria in the financial market. The multiplicity can arise because traders use information about their endowments to make inferences about the noise in the price (see Ganguli and Yang (2009) and Manzano and Vives (2011)). Since this source of multiplicity is well-understood in the literature, we do not analyze it here and instead focus on our amplification mechanism.

[11] We say that ϕ is informationally equivalent to price p if conditional distributions of $v|\phi$ and $v|p$ are the same. Our notion of a generalized linear equilibrium follows Breon-Drish (2015).

where $x_i^d(p, s_i, e_i)$ denotes investor i's desired demand, or the amount he would like to trade, in the absence of constraints.

To solve for the equilibrium with constraints, one needs to pin down the informational efficiency β, the function $g(p)$, and investors' desired demand $x_i^d(p, s_i, e_i)$. We do that in the following proposition.

Proposition 2. *(Financial market equilibrium with portfolio constraints) Suppose that investors face portfolio constraints and have identical signal precision τ_ϵ. Then there exists a unique pair $\{g(p), \beta\}$ that constitutes a generalized linear equilibrium in which informational efficiency $\beta = \beta^u$. Furthermore, investor i's desired demand $x_i^d(p, s_i, e_i)$ is equal to $x_i^u(p, s_i, e_i)$, where $x_i^u(p, s_i, e_i)$ is characterized in Proposition 1. The function $g(p)$ is determined as follows. For every p, $g(p)$ is the unique ϕ that solves*

$$X(p, \phi) + x_m(p, \phi) = 1;$$

here the demand $x_m(p, \phi)$ of market makers is given in Proposition 1 and the closed-form expression for investors' aggregate demand $X(p, \phi)$ is given in Appendix A. If both $a(p)$ and $b(p)$ are continuously differentiable, then $g(p)$ can be determined by solving the ordinary differential equation (ODE)

$$g'(p) = -\frac{\pi_1(p, g(p))a'(p) + \pi_3(p, g(p))b'(p) - \pi_2(p, g(p))c_p - c_p^m}{\pi_2(p, g(p))c_\phi + c_\phi^m} \tag{6}$$

subject to the boundary condition $g(0) = g_0$, where the constant g_0 is the unique solution to $X(0, g_0) + x_m(0, g_0) = 1$. The term $\pi_1(p, \phi) = \Phi\left(\frac{a(p) - X^u(p,\phi)}{\sigma_\xi}\right)$ is for the fraction of investors whose lower constraint binds, $\pi_3(p, \phi) = 1 - \Phi\left(\frac{b(p) - X^u(p,\phi)}{\sigma_\xi}\right)$ denotes the fraction of investors whose upper constraint binds, $\pi_2(p, \phi) = 1 - \pi_1(p, \phi) - \pi_3(p, \phi)$ is the fraction of unconstrained investors, and $\Phi(\cdot)$ stands for the cumulative distribution function (CDF) of a standard normal distribution.

Proposition 2 is our first main result establishing the existence of a tractable, generalized linear equilibrium in an REE model with portfolio constraints, even when price may be nonlinear. It also states that, for an *exogenously given signal precision* τ_ϵ, portfolio constraints are irrelevant for the informational efficiency ($\beta = \beta^u$). This result is the key to our model's tractability. Instead of solving for β in the complex model with constraints, we can solve the simpler unconstrained model. Nonetheless, it would be premature to conclude that constraints do not matter for informational efficiency: in Section 2.3, we show that constraints affect the amount of information acquired by investors at $t = 0$. It is when the signal precision τ_ϵ becomes *endogenous* that constraints affect informational efficiency.

The intuition behind our irrelevance result is as follows. In general, price informativeness is determined by aggregate trading intensity as well as aggregate hedging intensity, where trading (hedging) intensity is the sensitivity of investor's demand to her private signal (endowment shock). For a given signal precision, funding constraints affect trading intensity $\frac{\partial x_i}{\partial s_i}$: when constraints are tighter, an investor is more likely to be constrained, in which case her trading intensity is zero. As a result, as constraints tighten, the aggregate trading intensity $\int \frac{\partial x_i}{\partial s_i} di$ reduces. However, constraints also affect the hedging intensity $\frac{\partial x_i}{\partial e_i}$ in a similar way, so that the ratio of aggregate trading intensity to aggregate hedging intensity $\int \frac{\partial x_i}{\partial s_i} di / \int \frac{\partial x_i}{\partial e_i} di$ remains unchanged.[12]

Our irrelevance result is not only instrumental for the model's tractability but also sheds light on the way price aggregates information in an economy with portfolio constraints. In essence, this result underscores that, even with constraints, the aggregate demand of investors (and hence, the market-clearing price) still varies with and reflects fundamentals via changes in the *fractions* of constrained investors. Consider an improvement in the asset fundamental v (while fixing the endowment shock z), which leads investors to increase their demand for the risky asset. Although some investors cannot increase their demand owing to the upper portfolio

[12]The economic forces behind the irrelevance result is similar to the ones in Dávila and Parlatore (2017), who study the impact of various forms (quadratic, linear, or fixed) of trading cost on informational efficiency. We focus on the impact of general price-dependent portfolio constraints instead.

constraint, in aggregate, more (fewer) investors become constrained by a maximal long (short) position. Aggregate demand will increase and thereby reveal the improved asset fundamentals via a higher market-clearing price.

Besides the informational efficiency β, the other important equilibrium object $g'(p)$, which is given in equation (6), captures how much the statistic ϕ changes when the price p changes by a single unit. The numerator in (6) represents aggregate demand's price sensitivity, which derives from four sources. First is the fraction π_1 of investors constrained by the lower constraint, whose demand has price sensitivity $a'(p)$. Second, a similar effect applies for the fraction π_3 of investors for whom the upper constraint $b(p)$ binds. Third, there is a fraction π_2 of unconstrained investors whose demand has price sensitivity $\frac{\partial X^u}{\partial p} = -c_p$. The numerator's last term is the market maker's demand sensitivity to price, $\frac{\partial x_m}{\partial p} = -c_p^m$. The denominator of (6), which represents the sensitivity of aggregate demand to ϕ, can be interpreted similarly. Equation (6) clearly demonstrates that constraints affect the shape of the function $g(p)$. In general—and in contrast to standard CARA-Normal models—this function is nonlinear.[13]

2.2.3 Discussion of modelling choices

We view that tractability is a major advantage of our model because for instance, it allows us to study the interaction between investors' decision to acquire information and constraints. To keep the model tractable, we have made judicious choices of the modelling ingredients. These ingredients include: 1) the presence of a representative market maker, 2) noise coming from the investors' endowment shocks, and 3) the endowment shocks being correlated to the unlearnable component of the asset payoff. We discuss the role of each of these ingredients in the three remarks below.

Remark 1. *On market maker.* In the model, there is a competitive, unconstrained and unin-

[13]The fact that constraints affect the sensitivity $g'(p)$ distinguishes our setting from those in Dávila and Parlatore (2017) and Nezafat et al. (2017), where trading costs and short-sale constraints, respectively, are irrelevant not only for informational efficiency β but also for sensitivity $g'(p)$.

formed agent whom we call "market maker". The presence of a market maker brings several advantages. First, it ensures the existence of equilibrium in some special cases. One such case is when the investors are almost fully constrained to submit a demand of zero, the market maker is needed to hold the asset and clear the market. Second, it makes the model very flexible in incorporating new unconstrained and uninformed agents and hence greatly broadens the model's applications. Indeed, if we have two groups of uninformed investors with risk aversions γ_1 and γ_2 of measure μ_1 and $\mu_2 = 1 - \mu_1$, these two groups would be equivalent to a single investor with risk aversion $1/\gamma_m = \mu_1/\gamma_1 + \mu_2/\gamma_2$. We will make use of this property later, when we introduce financiers who may also participate in the financial market. Our financiers are unconstrained and uninformed, and therefore, will be aggregated (along with other unconstrained investors) into a single representative investor, the "market maker". Last but not least, having a market maker also generalizes the model and the results. By varying the risk-aversion coefficient of the market maker, our model nests a model with a risk-neutral market maker as in Vives (1995) (with $\gamma_m = 0$), a model with possibly heterogeneous uninformed investors (with $\gamma_m > 0$), and a model without a market maker (with $\gamma_m \to \infty$).

Remark 2. *On noise coming from endowment shocks.* The noise coming from endowment shocks is important for the tractability of our model which relies on the "irrelevance" result that the portfolio constraints do not affect the informativeness of price. An important property behind the irrelevance result is that investors who trade for information or hedging/liquidity motives are *ex-ante homogeneous*. Intuitively, constraints affect information-based trade and hedging-based trade symmetrically and hence the price informativeness remains unchanged. In order to keep the prices from fully revealing while at the same time maintaining ex-ante homogeneity among investors, we introduce endowment shocks to all investors.[14]

Remark 3. *On endowment shocks being correlated to the unlearnable component of the asset payoff.* We assume that the endowment shocks are correlated with the unlearnable component

[14]While irrelevance result is important for tractability of our model, it is not crucial for our key economic result, informational spiral. In Appendix B we present an alternative model with noise traders in which the irrelevance result does not hold, yet the informational spiral is robust.

of the asset payoff. The literature has considered two specifications about the endowment shocks: they are correlated to the learnable component of the payoff (as in Ganguli and Yang (2009) and Manzano and Vives (2011)) or they are correlated with the unlearnable component (as Nezafat et al. (2017)). We choose the latter because it has the advantage of ensuring uniqueness (under some condition) and existence of equilibrium in the financial market.[15] We would like to emphasize that this choice is not crucial for our results.[16]

2.3 Information acquisition at $t = 0$

Having solved for the financial market equilibrium at $t = 1$, we now study how portfolio constraints affect the incentives of investors to acquire information at $t = 0$. We maintain the assumption $a(p) \leq 0 \leq b(p)$ and say that constraints are *tightened* when $a(p)$ increases and/or $b(p)$ decreases. We start by deriving an expression for the marginal value of information under general portfolio constraints, after which we show that an investor's marginal value of information declines if the constraints are tightened.

At date 0, investor i decides on the optimal amount of information to acquire by solving the following problem:

$$\max_{\tau_{\epsilon_i}} \quad E\left[E\left[-e^{-\gamma\left(W_i - C\left(\tau_{\epsilon_i}\right)\right)}|\mathcal{F}_i\right]\right].$$

We first write down investor i's date-1 certainty equivalent $CE_{1,i} \equiv E[W_i|\mathcal{F}_i] - \frac{\gamma}{2}Var[W_i|\mathcal{F}_i]$, characterized in the following Lemma.

Lemma 1. *In a given financial market equilibrium, the date-1 certainty equivalent for investor*

[15] Ganguli and Yang (2009) and Manzano and Vives (2011) demonstrate that equilibrium may fail to exist or may not be unique, when endowment shocks are correlated with a learnable asset payoff and traders can use information about their endowment shocks to make inferences about the noise.

[16] In an earlier version of the paper, we showed that our results hold in the first specification as well.

i is

$$CE_{1,i} = \underbrace{W_0 + \frac{\gamma}{2\tau_i}\left(x_i^u\right)^2 - \frac{\gamma}{2\tau_\theta}e_i^2}_{\text{CE if investor } i \text{ was unconstrained}} \quad \underbrace{-\frac{\gamma}{2\tau_i}\left(x_i^u - x_i\right)^2}_{\text{Effect of constraints, } <0}, \tag{7}$$

where $\tau_i = Var(f|\mathcal{F}_i)$ and x_i (similarly x_i^u) denotes the demand of investor i if she is constrained (unconstrained).

Investor i's certainty equivalent at date 1 (given in equation 7) includes a negative term that captures the effect of constraints: the tighter the constraints the investor faces, the larger the "distance" between his desired demand and his actual demand, resulting in lower certainty equivalent.

We now write the investor's preferences at date 0 and solve his information acquisition problem. The investor's problem at $t = 0$ becomes

$$\max_{\tau_{\epsilon_i}} \quad E\left[-e^{-\gamma\left(CE_{1,i} - C\left(\tau_{\epsilon_i}\right)\right)}\right],$$

which is equivalent to

$$\max_{\tau_{\epsilon_i}} \quad CE_0\left(\tau_{\epsilon_i}\right) - C\left(\tau_{\epsilon_i}\right),$$

where the date-0 certainty equivalent CE_0 is the solution to $e^{-\gamma CE_0} = E[e^{-\gamma CE_{1,i}}]$.

We define the *marginal value of information* as $\text{MVI} \equiv \frac{\partial \text{CE}_0'(\tau_{\varepsilon_i})}{\partial \tau_{\varepsilon_i})}$. In the next proposition, we characterize this marginal value of information under general portfolio constraints and show that it declines when an investor's constraints tighten.

Proposition 3. *(Marginal value of information) The marginal value of information for an investor i choosing signal precision τ_{ϵ_i}, while others' signal precisions are τ_ϵ, is given by*

$$\text{MVI}(\tau_{\epsilon_i}, \tau_\epsilon) = \underbrace{\frac{\tau_i}{2\tau_{v,i}^2\gamma}}_{\text{MVI if investor } i \text{ was unconstrained}} \quad \underbrace{+\frac{\tau_i}{2\tau_{v,i}^2\gamma}\left(\frac{U_0^u(\tau_{\epsilon_i}, \tau_\epsilon)}{U_0(\tau_{\epsilon_i}, \tau_\epsilon)} - 1\right)}_{\text{Effect of constraints, } <0}, \tag{8}$$

where $\tau_{v,i} = Var(v|\mathcal{F}_i)$ is the total precision of investor i's information about the learnable component, $U_0^u(\tau_{\epsilon_i}, \tau_\epsilon) = E[-e^{-\gamma CE_{1,i}} \mathbb{I}_{x_i^u = x_i}]$ is the expectation of utility in the states when constraints do not bind, and $U_0(\tau_{\epsilon_i}, \tau_\epsilon) = E[-e^{-\gamma CE_{1,i}}]$ is date-0 expected utility.

The marginal value of information decreases when an individual investor's constraints become tighter, ceteris paribus.

Proposition 3 shows how portfolio constraints affect an investor's incentive to acquire information. If the investor was unconstrained, he has stronger incentive to acquire information, as the term capturing the effects of constraints in equations (8) is negative. It makes sense that an investor considers information valuable to the extent that he can profit from it.

Next, we study how the equilibrium information acquisition changes when the portfolio constraints of all investors become tighter. Tightening constraints for all investors is more complicated because the equilibrium price distribution will change due to the market maker's risk aversion, which in turn affects price-dependent constraints. If the market maker's risk aversion is not too large, then the changes in prices are second-order and we can prove that tightening the constraints for all investors reduces each investor's marginal value of information. This result is stated formally in the next proposition.

Proposition 4. *Consider an economy in which all investor face portfolio constraints $a(p) \leq 0 \leq b(p)$ such that $\forall \gamma_m \geq 0$, there exists a generalised linear financial market equilibrium with a monotone function $g(p)$. Next, consider an alternative economy with tighter constraints. That is, investors face portfolio constraints $\hat{a}(p)$ and $\hat{b}(p)$ where $a(p) \leq \hat{a}(p) \leq 0 \leq \hat{b}(p) \leq b(p)$ for all p, such that $\forall \gamma_m \geq 0$, there exists a generalised linear financial market equilibrium with a monotone function $\hat{g}(p)$. Then there exists a $\overline{\gamma} > 0$ such that $\forall \gamma_m < \overline{\gamma}$, the marginal value of information decreases for all investors when constraints change from $[a(p), b(p)]$ to $[\hat{a}(p), \hat{b}(p)]$.*

Proposition 4 illustrates one of the key forces of our mechanism: tighter constraints reduce investors' incentive to acquire information, and hence, the informational efficiency of prices. To close the model with the characterization of the effect of informational efficiency on the tightness of constraints, we need to further specify the nature of the portfolio constraints. In the rest of the paper, we focus on portfolio constraints that arise from margin requirements and study their interactions with informational efficiency.

3 Portfolio constraints arising from margin requirements

So far we have studied general, price-dependent portfolio constraints. In this section, we apply our model to study the constraints arising from margin requirements. In Section 4, we illustrate our model's implications for asset prices.

3.1 Margins and incentive to acquire information

Our notion of margin requirements is standard and closely follows Brunnermeier and Pedersen (2009). To build a long position in the risky asset, an investor can borrow from a financier at the risk-free rate, but he has to pledge a cash margin of $m^+(p) \geq 0$ per unit of asset to the financier as collateral. The investor can similarly establish a short position by providing, as collateral, a cash margin of $m^-(p)$ per unit of asset. Thus. investors face a funding constraint that the total margin on their positions cannot exceed their initial wealth as follows

$$m^-(p)[x_i]^- + m^+(p)[x_i]^+ \leq W_0,$$

where $[x_i]^-$ and $[x_i]^+$ are respectively the positive and negative parts of x_i.[17] We can rewrite the margin requirements in the form of portfolio constraints as

$$a(p) = -\frac{W_0}{m^-(p)}, \quad b(p) = \frac{W_0}{m^+(p)} \tag{9}$$

[17] Since the endowment b_i is not pledgeable, it cannot be used as a collateral to satisfy the margin requirements.

Equations (9) show that an investor faces tighter constraints when his initial wealth is lower and/or if the financier's margin requirements are higher. We shall delay until Section 3.2 a discussion of how the financier sets margins. For now, we simply assume that margins are independent of the price; we later prove that, in equilibrium, this is indeed the case.

We proceed with solving the model under margin requirements backwards. The financial market equilibrium at $t = 1$ is just a special case of Proposition 2, so the next result is a straightforward extension.

Corollary 1. *Suppose that investors have identical signal precisions τ_ϵ and face margin requirements that do not depend on prices, then there exists a unique generalized linear equilibrium in which informational efficiency $\beta = \beta^u$ and the function $g(p)$ that is monotone in p.*

Working backwards, we next characterize the equilibrium at $t = 0$, the information acquisition stage. In any symmetric equilibrium, investors acquire information until the marginal cost of doing so equals the marginal value of information.

Proposition 5. *Equilibrium information acquisition (τ_ϵ^*) at $t = 0$ satisfies*

$$C'(\tau_\epsilon^*) = \frac{\tau_i}{2\tau_{v,i}^2\gamma}\frac{U_0^u(\tau_\epsilon^*, \tau_\epsilon^*)}{U_0(\tau_\epsilon^*, \tau_\epsilon^*)}$$

In addition, the equilibrium precision τ_ϵ^* and equilibrium informational efficiency β in a stable equilibrium decrease when initial wealth W_0 drops and/or margins m^+ and m^- increase for all investors, if W_0 is small enough.[18]

Proposition 5 implies that wealth plays an important role in our model with constraints—in contrast to typical CARA-Normal models. As investors' initial wealth decreases, they become

[18] Our notion of stability is as in Manzano and Vives (2011) and Cespa and Foucault (2014) and is standard in game theory (see Fudenberg and Tirole (1991), Chapter 1, 1.2.5). We call an equilibrium *stable* if the fixed point determining equilibrium precision of investors' signals is stable. More specifically, we call an equilibrium *stable* if $|\tau'_{\epsilon_i}(\tau_\epsilon^*)| < 1$, where $\tau_{\epsilon_i}(\tau_\epsilon^*)$ is investor i's optimal choice of precision given that all other investors' precisions are equal to τ_ϵ^*. Numerically, we find that our equilibrium is always stable.

more constrained and hence acquire less information, reducing price informativeness in equilibrium. Similarly, an increase in the margins m^+ and m^- reduces price informativeness. While a small enough W_0 is needed for the proof, numerical simulations show that the results hold for all values of W_0 we tried. Thus, we view the requirement of a small W_0 as technical, and not restrictive for the economic mechanism.

3.2 Value-at-risk based margin requirements

Until now we have assumed that margins are fixed—in other words, they are not determined as part of the equilibrium. Here we assume that each financier sets her margin in order to control her value-at-risk, as in Brunnermeier and Pedersen (2009):

$$\begin{aligned} m^+(p) &= \inf\{m^+(p) \geq 0 : \Pr(p - f > m^+(p) \mid p) \leq 1 - \alpha\}, \\ m^-(p) &= \inf\{m^-(p) \geq 0 : \Pr(f - p > m^-(p) \mid p) \leq 1 - \alpha\}; \end{aligned} \tag{10}$$

here "Pr" stands for "probability". It says that the financier require the investors to set aside a minimum amount of cash (i.e., margin) large enough to cover, with probability α, the potential loss from trading. Furthermore, the financier is uninformed but can condition her margins on prices. As detailed in Brunnermeier and Pedersen (2009, Appendix A), this margin specification is motivated by the real-world margin constraints faced by hedge funds and the capital requirements imposed on commercial banks. In Appendix D, we provide a microfoundation for the use of VaR-based margin by financiers, when investors can default and financiers have to incur a cost to enforce repayment.

How the financiers are modelled will affect their evaluation of the VaR-based margins. We consider two possibilities. The first and our preferred way is to introduce the financiers as unconstrained and uninformed agents who can trade the risky asset. As discussed in Remark 1, the financiers in this case could be grouped with other unconstrained and uninformed agents as a representative market maker. Given their holding of the risky assets in equilibrium, the financiers assessment of risks of lending to investors is driven by their marginal utility. Therefore, the value-at-risk is evaluated under the risk-neutral measure induced by this marginal utility, which coincides with the risk-neutral measure of the market maker.[19] In Appendix D, we show how to construct this risk-neutral measure. We calculate margin this way in the main text and simply call it "VaR-based margin".

An alternative way of introducing financiers is to assume that they do not participate in the risky asset market and only exposed to aggregate risks (f and z) through their lending business. In this case, they evaluate the value-at-risk with the physical measure. We denote this case "VaR^P-margin". We study equilibrium with VaR^P-margins in Appendix E and show our main result, informational spiral, holds when γ_m is small enough. Appendix D microfounds both VaR- and VaR^P- margins.

3.3 Financial market equilibrium with VaR-based margins

We describe our financial market equilibrium with VaR-based margin constraints as follows. (i) Financiers set their margin requirements according to (10), given a conjectured price function, computing the probabilities under the risk-neutral measure. (ii) Investors and the market maker choose their optimal demand given the margin requirements and the conjectured price function. (iii) In equilibrium, the conjectured price function is consistent with market clearing. As before, we take the precisions of investors' signals as given.

Proposition 6. *(Financial market equilibrium under VaR-based margin requirements) If portfolio constraints are of the form of margin requirements, as in equation (9) and if margins are determined by value-at-risk, as in (10) evaluated under the risk-neutral measure, then there exists a unique generalized linear equilibrium in which the function $g(p)$ is as characterized by*

[19] In a similar spirit, Ait-Sahalia and Lo (2000) propose to use state price density implied in asset prices to estimate the economic value of value-at-risk.

Corollary 1 and the equilibrium margins are given by

$$m^+ = m^- = \Phi^{-1}(\alpha)\sqrt{(\tau_v + \beta^2\tau_z)^{-1} + \tau_\theta^{-1}}. \tag{11}$$

Consequently, for a given investors' wealth W_0, if informational efficiency (β) decreases then the margins $(m^+$ and m^- both$)$ increase. This implies that the lower constraint $($i.e., $a)$ increases and the upper constraint $($i.e., $b)$ decreases. In other words: as informational efficiency declines, constraints become tighter.

The proposition above establishes the uniqueness of the equilibrium with VaR-based margin requirements. Moreover, in this unique equilibrium, a decrease in informational efficiency leads to higher margins and tighter constraints, which is the key result in this subsection. The intuition is as follows. When informational efficiency is lower, the price tracks fundamentals less closely. This increases the risk of financing a trade for financiers, who in turn demand higher margins for financing this trade. This leads to higher margins and tighter constraints. Moreover, Corollary 2 implies that the effect of informational efficiency on margins increases with α, the risk-tolerance of financiers.

Corollary 2. *The strength of the effect of a drop in β on margins increases with α i.e., $\frac{\partial^2 m}{\partial\alpha\partial\beta} < 0$.*

We will end this subsection with couple of remarks about the results.

Remark 4. *Informational efficiency affects constraints even if financiers do not learn from prices.* We emphasize that this section's results do not rely on financiers learning from prices. Indeed, one can compute the *unconditional* variance of returns under risk-neutral measure as

$$Var^Q[f-p] = E^Q[Var^Q[f-p|p]] + Var^Q[E^Q[f-p|p]] = E^Q[Var^Q[f-p|p]].$$

In the above equation we used the fact that $p = E^Q[f|p]$. It follows from normality of $f|p$ under the risk-neutral measure (Lemma 15 in Appendix D), that the conditional variance $Var^Q[f-p|p]$

is constant, and therefore equal to the unconditional variance $Var^Q[f-p]$. This implies that the financier will set the same margins irrespective of whether (or not) she learns from prices.

Remark 5. *Alternative risk-based margins.* Our result that margins increase when informational efficiency falls holds also for alternative risk-based margins, such as tail value-at-risk (TVaR) and expected shortfall (ES). This is because all these risk measures depend on the conditional distribution of the loss $p-f$ ($f-p$) for a long (short) position given p. Under the risk-neutral measure this distribution is normal with mean zero and variance $Var^Q[f|p]$. Hence, the distribution is parameterized by a single parameter, $Var^Q[f|p]$. Since VaR, ES and TVaR are all monotone in $Var^Q[f|p]$, it follows that results in this section are robust to using these alternative risk-based margins.

3.4 Information spiral

In Section 3.1, we undertook a partial equilibrium analysis and argued that given margins, tighter funding constraints (e.g., reductions in wealth) lead to lower informational inefficiency because investors acquire less information (Proposition 5). In Section 3.3, we argued that, for a given level of wealth, lower informational efficiency leads to higher margins (Proposition 6). Putting these two results together yields the amplification loop that we call the information spiral (see Figure 1, in Section 1, for an illustration). The main implication of this spiral is that small changes in the underlying funding constraints lead to sharp reductions in information acquisition, and hence, in informational efficiency.

3.4.1 Effect of wealth on informational efficiency and margins

As illustrated in Figure 1, the information spiral amplifies a negative shock to investor wealth into a decrease in informational efficiency (β) and an increase in margin requirements (m^+ and m^-). The results are direct consequences of Propositions 5 and 6 and we summarize them in the following corollary.

Corollary 3. *If W_0 is small enough, a decrease in investor wealth W_0 decreases informational efficiency β and increases VaR-based margins m^+, m^-.*

Wealth effect exists in our model, even investors have constant absolute risk aversion, because it relaxes portfolio constraints. The effect is then reinforced in two steps: investors with more relaxed portfolio constraints acquire more information , and hence, informational efficiency is improved. The improved informational efficiency further relaxes portfolio constraints, because financiers have less value-at-risk and require a lower margin.[20]

3.4.2 Complementarity in information acquisition

Our mechanism underpins a novel strategic complementarity effect in investors' decisions to acquire information. An investor's incentive to acquire information decreases when other investors acquire less information because of the VaR-based margins. As less information is acquired by other investors, the price becomes less informative about the asset fundamentals; hence, financiers set higher VaR-based margins and so, with a tightened funding constraint, the investor values information less.

As standard in REE models (e.g., Grossman and Stiglitz (1980)), there is also a substitutability effect in information acquisition: when other investors acquire more information, price is more informative about fundamentals, and hence, there is less incentive for an investor to acquire private information. The question then is: when does the complementarity effect dominates the substitutability effect? Since the complementarity effect arises due to constraints, which are more likely to bind when investors have lower wealth, we expect the information acquisition by investors to be strategic complements when investor wealth is low. This conjecture is verified by the proposition below.

Proposition 7. *Consider an investor i choosing his signal precision τ_{ϵ_i}, while others' signal*

[20] As in Proposition 5, we view the requirement of a small W_0 as technical and the result holds numerically for a wide range of values of W_0 we have tried.

precisions are τ_ϵ. For W_0 small enough we have that $\frac{\partial \mathrm{MVI}\left(\tau_{\epsilon_i},\tau_\epsilon;W_0\right)}{\partial \tau_\epsilon} > 0$, in a range of τ_{ϵ_i} and τ_ϵ characterised by the condition (30) in the proof.

The proposition above shows that for low enough wealth, investors' decisions to acquire information are strategic complements in a range of precisions τ_ϵ and τ_{ϵ_i}. This range of precisions, as shown in the proof of the Proposition, is characterized by the condition (30) which is equivalent to

$$\frac{d}{d\tau_\epsilon}\log\left(\frac{W_0}{m\left(\tau_\epsilon\right)}\right) > -\frac{d}{d\tau_\epsilon}\log\left(\underbrace{\frac{\tau_i}{2\tau_{v,i}^2\gamma}}_{\text{MVI absent constraints}}\right). \tag{12}$$

The left-hand side of (12) represents the force behind the strategic complementary in our model: when other investors acquire more information, the margins for the investor of interest are lower and his constraints W_0/m are relaxed, giving him incentive to acquire more information as well. The strength of this force is captured by the elasticity of tightness of constraints, W_0/m, with respect to τ_ϵ. The right-hand side of (12) summarizes the classic Grossman-Stiglitz substitutability effect, which would have been the only effect at work in the absence of constraints. The strength of this effect is captured by the elasticity of $\frac{\tau_i}{2\tau_{v,i}^2\gamma}$ (i.e., the marginal value of information without constraints) with respect to τ_ϵ. The condition (12) simply requires the complementarity effect operating through changes in constraints to be stronger than the Grossman-Stiglitz substitutability effect.

4 Asset-pricing implications

In this section we derive the implications of a decline in investor wealth on the risky asset's equilibrium risk premium and return volatility. The main result in this section is that a drop in wealth, ceteris paribus, leads to an increase in the risk premium, return volatility, and Sharpe ratio, when investors' wealth W_0 is small.

4.1 Risk premium

We start by analyzing how the initial wealth of investors affects the risk premium. The conditional risk premium is formally defined as $rp(p) = E[f - p|p]$. Since an econometrician measures the unconditional risk premium, we will focus on it. It is given by

$$\bar{rp}(W_0, \tau_\epsilon) \equiv E[f - p].$$

The change in risk premium in response to a change in the investors' wealth can be decomposed as follows:

$$\frac{d\bar{rp}(W_0, \tau_\epsilon)}{dW_0} = \underbrace{\frac{\partial \bar{rp}}{\partial W_0}}_{\text{Direct Effect}} + \underbrace{\frac{\partial \bar{rp}}{\partial \tau_\epsilon}\frac{\partial \tau_\epsilon}{\partial W_0}}_{\text{Indirect Effect}} \tag{13}$$

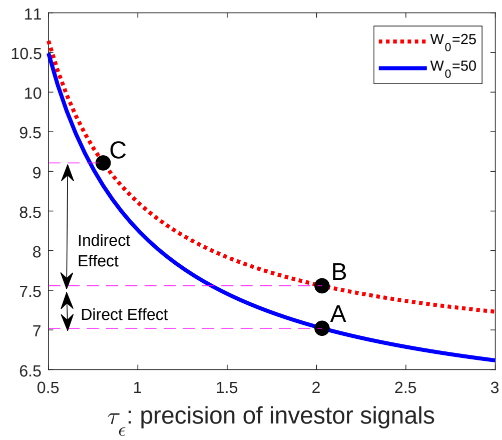
The first term in the right-hand side of equation (13) captures the direct effect that a change in investors' wealth has on the risk premium; the second term captures the indirect effect resulting from investors' endogenous information acquisition decisions. We establish that both the direct effect and the indirect effects are negative (the later under the restriction of W_0 small enough) in the Proposition below.

Proposition 8. *The direct effect of wealth on risk premium is negative, i.e.* $\frac{\partial \bar{rp}}{\partial W_0} < 0$. *For* W_0 *small enough, the indirect effect is also negative, i.e.,* $\frac{\partial \bar{rp}}{\partial \tau_\epsilon}\frac{\partial \tau_\epsilon}{\partial W_0} < 0$.

Figure 2 plots the unconditional risk premium in our model against τ_ϵ, the precision of investors' signals for two different levels of wealth. Point A in the figure corresponds to the equilibrium risk premium with high wealth level ($W_0 = 50$), and consider a negative shock to investors' wealth. With decreased wealth, constraints become tighter and investors' capacity to go long or short the asset is diminished, which is similar to the effect of lowering their risk-bearing capacity (i.e., increasing their risk aversion). Therefore, the risk premium rises. This argument implies that absent the information acquisition channel (i.e., holding τ_ϵ fixed), the wealth drop would cause an increase in risk premium that corresponds to the move from the

Figure 2: Risk premium

The figure plots risk premium as a function of precision of investors signal for different levels of wealth: $W_0 = 25$ (dotted line) and $W_0 = 50$ (solid line). Other parameter values are set to: $\tau_u = \tau_z = 1$, $\tau_v = 0.01$, $\tau_\theta = 0.5$, $\underline{\tau_\epsilon} = 0.75$, $\gamma = \gamma_m = 3$ and $\alpha = 0.99$. We assume that the cost function is $C(\tau_\epsilon) = k_0\tau_\epsilon^2$ where $k_0 = 0.0066$.



graph's solid line (corresponding to high wealth level) to its dotted line (corresponding to low wealth level)—that is, from point A to point B. This reflects the direct effect in equation (13). Moreover, because of the information spiral, investors in equilibrium acquire less information (lower τ_ϵ), which leads to an additional increase in risk premium; this increase corresponds to the move from point B to point C along the dotted line, which is the indirect effect given in (13). Thus the effect of a decline in wealth on risk premium is amplified by the information acquisition channel, so the equilibrium moves from point A in the graph all the way to point C.

4.2 Return volatility

We next examine the risky asset's return volatility. The variance of returns can be written as

$$\mathcal{V}(W_0, \tau_\epsilon) \equiv Var[f - p] \tag{14}$$

$$= \underbrace{\left(\frac{\gamma_m}{\tau_m}\right)^2 Var[x_m(p)]}_{\text{Variance of conditional risk premium}} + \underbrace{(\tau_v + \beta^2 \tau_z)^{-1} + \tau_\theta^{-1}}_{\text{Conditional variance of payoff } f} \tag{15}$$

where we used law of total variance to go from the first equation to the second. As before, the change in return variance in response to a change in the investors' wealth can be decomposed as follows:

$$\frac{d\mathcal{V}(W_0, \tau_\epsilon)}{dW_0} = \underbrace{\frac{\partial \mathcal{V}}{\partial W_0}}_{\text{Direct Effect}} + \underbrace{\frac{\partial \mathcal{V}}{\partial \tau_\epsilon} \cdot \frac{\partial \tau_\epsilon}{\partial W_0}}_{\text{Indirect Effect}} . \tag{16}$$

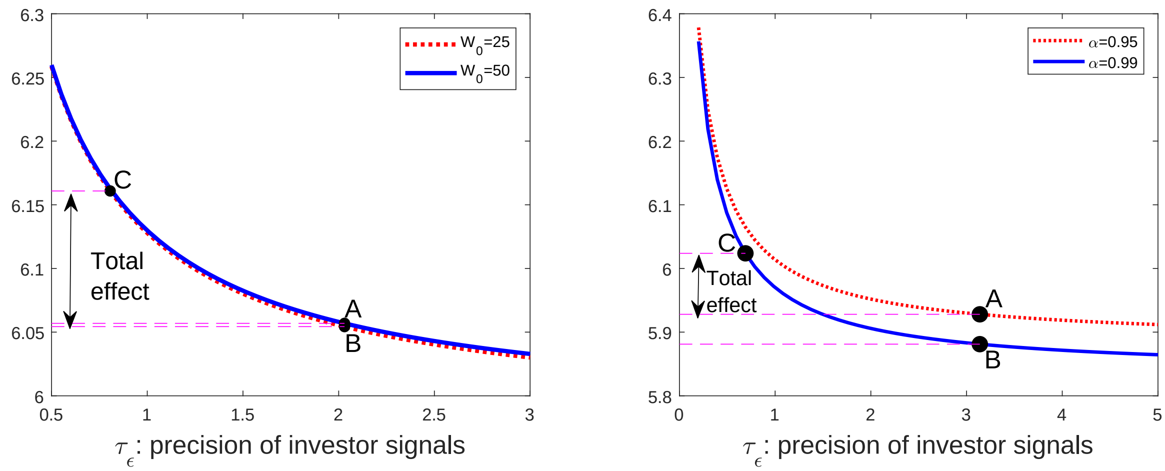
Since the price is nonlinear, the preceding expression for the variance of returns cannot be further simplified. However for W_0 small enough, we prove that the indirect effect is negative and that the direct effect is second-order, so that the overall effect of wealth on volatility is negative.

Proposition 9. *For W_0 small enough, the overall effect of wealth on volatility is negative, i.e., $\frac{d\mathcal{V}(W_0,\tau_\epsilon)}{dW_0} < 0$.*

Panel (a) of Figure 3 plots return volatility against the precision of investors' signals (τ_ϵ), for two levels of wealth. Once again, point A is an equilibrium with high wealth level and then imagine reducing wealth. If τ_ϵ is held fixed then we can see that, as wealth declines, there is less volatility (corresponding to the move from point A to point B). The intuition follows from equation (15). With decreasing wealth, the second and third terms do not change when τ_ϵ is fixed but the first term decreases because investor demand is then less volatile; this is the direct effect. Yet, investors acquire less information when they are constrained, so there is an increase in volatility corresponding to the move from point B to point C. Thus, the indirect

Figure 3: Return volatility

The figure plots return volatility as a function of precision of investors signal for different levels of wealth, $W_0 = 25$ (dotted line) and $W_0 = 50$ (solid line) (panel (a)) and different levels of value-at-risk confidence level α, $\alpha = 0.95$ (dotted line) and $\alpha = 0.99$ (solid line) (panel (b)). Other parameter values are set to: $\tau_u = \tau_z = 1$, $\tau_v = 0.01$, $\tau_\theta = 0.5$, $\underline{\tau_\epsilon} = 0.75$, $\gamma = \gamma_m = 3$. We assume that the cost function is $C(\tau_\epsilon) = k_0\tau_\epsilon^2$ where $k_0 = 0.0066$.



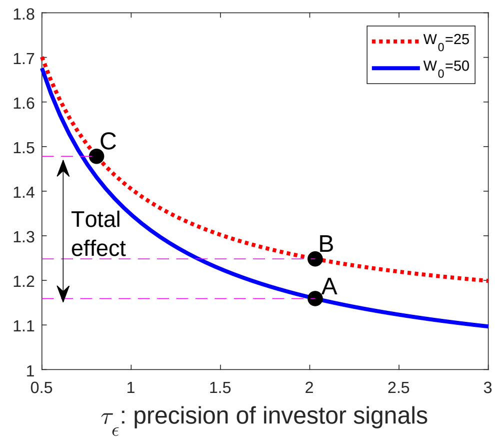
(a) (b)

effect (which operates through the information acquisition channel) may end up dominating, which means that volatility increases overall as wealth declines. For small wealth, proposition 9 implies that indirect effect always dominates direct effect.

We also examine the effects of margin requirements (as measured by VaR confidence level α) on volatility. It has long been argued that tighter margin requirements stabilize prices. The reasoning is that tighter margin requirements curb investors' positions, thereby limiting the price impact of their information and liquidity shocks. Panel (b) of Figure 3 illustrates the effect of margin requirements on volatility. As margin constraints tighten (α increases), volatility indeed drops—when information acquisition choices are held fixed—as we move from point A to point B; this outcome confirms the conventional wisdom. With tighter constraints, however, investors acquire less information, so return volatility may increase when funding requirements are stricter (and thus we move from point B to point C on the graph). In this way, our model complements the results in Wang (2015) by giving an alternative information-

Figure 4: Sharpe ratio

The figure plots Sharpe ratio as a function of precision of investors signal for different levels of wealth, $W_0 = 25$ (dotted line) and $W_0 = 50$ (solid line). Other parameter values are set to: $\tau_u = \tau_z = 1$, $\tau_v = 0.01$, $\tau_\theta = 0.5$, $\underline{\tau_\epsilon} = 0.75$, $\gamma = \gamma_m = 3$ and $\alpha = 0.99$. We assume that the cost function is $C(\tau_\epsilon) = k_0 \tau_\epsilon^2$ where $k_0 = 0.0066$.



based explanation for the positive association between tightening margin requirements and increased return volatility.

4.3 Sharpe ratio

Our final asset pricing implication is about the risky asset's Sharpe ratio, defined as $SR = \frac{E[f-p]}{\sqrt{Var[f-p]}}$. We argued previously that, when τ_ϵ is fixed, the risk premium rises and volatility falls (from point A to point B in both plots) as the wealth of investors declines. These movements imply that falling wealth will cause the Sharpe ratio to rise (the direct effect). We also argued that, with endogenous information acquisition, both the risk premium and return volatility rise; as a consequence, the indirect effect cannot be signed in general. However for W_0 small enough, we prove that the indirect effect is also negative so that the overall effect is negative.

Proposition 10. *When W_0 is small enough the overall effect of wealth on Sharpe ratio is negative, i.e.* $\frac{dSR}{dW_0} < 0$.

Figure 4 depicts a case in which the direct effect (from A to B) and the indirect effect (from B to C) are in the same direction, thus amplifying the wealth shock's effect.

5 Empirical Predictions

In this section, we use the results of our model to generate novel, testable predictions. The main predictions consider the interaction between the two key variables of the model, margin requirements and informational efficiency (Predictions 1 and 2), and how these two variables are affected by changes in investors' wealth, or net worth, a key parameter of the model (Predictions 3 and 4).

First, our model implies that an exogenous decrease in margin requirements should lead to an increase in informational efficiency of prices (Proposition 5). Testing this prediction directly presents a number of empirical challenges, including finding exogenous shocks to margins and identifying proxies for informativeness of price. One source of exogenous shocks to margin requirement could come from policy changes. For instance, Jylha (2018) argues that the New York Stock Exchange's portfolio Margining Pilot Program of 2005–2007 was an exogenous shock that relaxed the margin constraints of index options but not of individual equity options. While there are many proxies for price informativeness in the empirical literature, the ones suggested by Bai, Philippon, and Savov (2016) and Dávila and Parlatore (2018) are perhaps the closest to our notion. They capture the extent to which asset prices reflect fundamentals (cashflows). Our model predicts that following the pilot Program of 2005-2007, the informativeness of prices of index options should increase more than that of individual equity options.

Second, an exogenous drop in informational efficiency should result in an increase in margin requirements (Proposition 6). In order to test this prediction, we need an exogenous shock to informational efficiency for some stocks. The shutting down of a broker (Kelly and Ljungqvist, 2012) or the merger of brokers (Hong and Kacperczyk, 2010) could be shocks to the activity of analysts, and hence, to price informativeness. Our model suggests that such a shock would lead to higher margin requirements. Moreover, the magnitude of this mechanism is likely to vary over time. For instance, Corollary 2 implies that the impact of an informational efficiency shock on margin requirements should be stronger when the financiers are less risk-tolerant to investors' trading loss. Financiers in reality may become effectively less risk-tolerant due to new regulations. For example, Boyarchenko, Eisenbach, Gupta, Shachar, and Van Tassel (2018) provides evidence that the post-crisis regulations make global systemically important banks less willing to finance hedge funds' arbitrage activities, suggesting that these banks are less risk-tolerant. Our model suggests that in the post-crisis regulations era, margin requirement should become more sensitive to informational efficiency of prices.

Third, our model predicts that a negative shock to the investors' net worth would reduce informational efficiency of prices and increase margin requirements (Corollary 3). One proxy for investors' net worth would be the market equity capital ratio of the New York Fed's primary dealers, which is shown in He, Kelly, and Manela (2017) to have important asset-pricing implications.[21] Our model implies that exogenous negative shocks to equity capital ratio of primary dealers should lead to decreases in price informativeness and increases in margin requirements.

Finally, the model implies that the effect of investors' net worth on informational efficiency and margin requirements are non-linear. A shock to the investors' net worth should have a negligible impact when the net worth is high because the constraints are unlikely to bind. In contrast, when the net worth is low, i.e., in bad times., a complementary in information acquisition can emerge, resulting in a larger impact (Proposition 7).

6 Discussion and Extensions

We have made several assumptions in the analysis for tractability and to highlight the under-

[21]Another related proxy is broker-dealer leverage. Adrian, Etula, and Muir (2014) has shown that it can explain the cross-section of stock and bond returns.

lying mechanism in the clearest manner. In this section, we show the robustness of our main results in alternative environments and discuss the additional implications we derive in these alternative environments.

1. *Noise trading:* In our baseline model, we assume that noise in prices comes from investors' hedging needs . This assumption makes the model tractable due to the irrelevance result: given a fixed quality of private information, the informational efficiency of prices is independent of the constraints investors face. In Appendix B, we consider an alternative setting in which the noise in prices comes from classic noise traders. We assume that the noisy supply of assets is exogenous, and, in particular, is not affected by constraints. We show that the information spiral manifests itself through a new channel. In the alternative setup, tightening the funding constraints of informed investors reduce aggregate trading intensity but not the noisy supply. This hurts price informativeness, even for a given quality of private information, via the *information aggregation* function of price. The reduction in price informativeness leads to an increase in margins, as in our baseline model. Hence, the information spiral is still present in this setting, even when the irrelevance result does not hold.

2. *Endowment of risky assets:* In the baseline model, we assume that investors have cash as initial endowment. In Appendix C, we instead assume that investors are endowed with some risky assets and show that the information spiral continues to hold in this economy.

3. *VaR under the physical measure:* In the baseline model, we assume that the financiers use the risk-neutral measure to compute the value at risk.[22] The advantage of using the risk-neural measure is that the VaR-based margins will be independent of price level. In Appendix E, we study the case in which the financiers use the physical measure to evaluate risk. There, margins will depend on the price level. We show that when the market

[22] In Appendix D, we microfound the use of risk-neutral and physical measures by the financiers under different setup.

maker's risk aversion is not too high, all the results associated with the information spiral continue to hold.

7 Conclusion

In this paper we developed a tractable REE model with general portfolio constraints, and we applied our methodology to study a canonical REE model with margin constraints. We argued that funding constraints affect and are affected by informational efficiency, leading to a novel amplification mechanism that we call the information spiral. This spiral implies that the risk premium, conditional return volatility and Sharpe ratio each rise disproportionately as investors' wealth declines. The information spiral also generates complementaries in the investors' acquisition of information during crises (i.e., when investor wealth is low). These results imply a new, information-based rationale for why the wealth of investors is important. Our analysis also yields novel testable predictions.

While many papers describe amplification mechanisms for amplification over the business cycle, ours is different because it involves changes in price informativeness and the interaction with constraints. Given the important role financial markets play in aggregating and disseminating information, as argued by Bond et al. (2012), our mechanism could have significant real implications.

Appendices

A Proofs

A.1 Proof of Proposition 1

Proof. (Proposition 1) At time 1, the first order condition for investor i solving problem (1) is given by

$$x_i = \frac{\tau}{\gamma}\left(E\left[v|\mathcal{F}_i\right] - p - \gamma e_i \tau_\theta^{-1}\right), \quad \text{where} \quad \tau^{-1} = Var\left[v + \theta|\mathcal{F}_i\right].$$

Similarly, the first order condition for the market maker solving problem (2) is

$$x_m = \tau_m \frac{E\left[v|\phi\right] - p}{\gamma_m}, \quad \text{where} \quad \tau_m^{-1} = Var\left[v + \theta|\phi\right].$$

Using Bayes's rule for jointly normal random variables, we can write

$$E\left[v|\mathcal{F}_i\right] = \frac{\tau_\epsilon s_i + \beta^2\left(\tau_u + \tau_z\right)\phi + \beta\tau_u e_i}{\tau_\epsilon + \beta^2\left(\tau_u + \tau_z\right) + \tau_v} \quad \text{and} \quad \frac{1}{\tau} = \frac{1}{\tau_\epsilon + \beta^2\left(\tau_u + \tau_z\right) + \tau_v} + \frac{1}{\tau_\theta},$$

$$E\left[v|\phi\right] = \frac{\beta^2\tau_z}{\beta^2\tau_z + \tau_v}\phi \quad \text{and} \quad \frac{1}{\tau_m} = \frac{1}{\beta^2\tau_z + \tau_v} + \frac{1}{\tau_\theta}.$$

Substituting these into the market clearing condition (3), we get

$$\frac{\tau}{\gamma}\left(\frac{\tau_\epsilon v + \beta^2\left(\tau_u + \tau_z\right)\phi + \beta\tau_u z}{\tau_\epsilon + \beta^2\left(\tau_u + \tau_z\right) + \tau_v}\right) - \frac{\tau}{\tau_\theta}z + \frac{\tau_m}{\gamma_m}\frac{\beta^2\tau_z\phi}{\beta^2\tau_z + \tau_v} = p\left(\frac{\tau}{\gamma} + \frac{\tau_m}{\gamma_m}\right).$$

One can express equilibrium price $p = p(v, z)$ from the above equation. Since it can only depend on v and z through $\phi = v - \frac{1}{\beta}z$, it must be true that $\frac{\partial p}{\partial v}/\frac{\partial p}{\partial z} = -\beta$. This implies that β satisfies:

$$\beta^3\gamma\left(\tau_u + \tau_z\right) - \beta^2\tau_u\tau_\theta + \beta\gamma\left(\tau_\epsilon + \tau_v\right) - \tau_\theta\tau_\epsilon = 0. \tag{17}$$

It can be seen from the above equation that the solution to it is always positive and there exists at least one solution. The solution is unique if the first derivative of the above polynomial does not change sign. The first derivative of the above equation is given by

$$3\beta^2\gamma\left(\tau_u + \tau_z\right) - 2\beta\tau_u\tau_\theta + \gamma\left(\tau_\epsilon + \tau_v\right).$$

At $\beta = 0$, the slope is positive and the slope is always positive if the above equation has no roots. This is true if and only if

$$\tau_u^2\tau_\theta^2 < 3\gamma^2\left(\tau_u+\tau_z\right)\left(\tau_\epsilon+\tau_v\right).$$

Using implicit differentiation of (17), β increases in τ_ϵ if and only if $\tau_\theta - \beta\gamma > 0$, which always holds for β solving equation (17).

Finally, we provide expressions for the coefficients mentioned in the proposition. Since the aggregate demand of investors and market makers can depend on v only through ϕ, we find

$$c_\phi = \frac{\tau}{\gamma}\frac{\partial E[v|\mathcal{F}_i]}{\partial v} = \frac{\tau}{\gamma}\left(\frac{\tau_\epsilon+\beta^2\left(\tau_u+\tau_z\right)}{\tau_\epsilon+\beta^2\left(\tau_u+\tau_z\right)+\tau_v}\right), \tag{18}$$

$$c_\phi^m = \frac{\tau_m}{\gamma_m}\frac{\partial E[v|\phi]}{\partial v} = \frac{\tau_m}{\gamma_m}\frac{\beta^2\tau_z}{\beta^2\tau_z+\tau_v}. \tag{19}$$

Similarly,

$$c_p = \frac{\tau}{\gamma},\ \ c_p^m = \frac{\tau_m}{\gamma_m}.$$

Finally,

$$\xi_i = \frac{\tau}{\gamma}\left(\frac{\tau_\epsilon\epsilon_i+\beta\tau_u u_i}{\tau_\epsilon+\beta^2\left(\tau_u+\tau_z\right)+\tau_v} - \gamma u_i\tau_\theta^{-1}\right),$$

$$\sigma_\xi^2 = \left(\frac{\tau}{\gamma}\right)^2\left(\frac{\tau_\epsilon+\left(\beta\tau_u-\gamma\tau_\theta^{-1}\right)^2\tau_u^{-1}}{\left(\tau_\epsilon+\beta^2\left(\tau_u+\tau_z\right)+\tau_v\right)^2}\right).$$

The coefficients g_0^u and g_1^u can be expressed throgh the above coefficients as follows:

$$g_0^u = \frac{1-c_0-c_0^m}{c_\phi+c_\phi^m},\ \ g_1^u = \frac{c_p+c_p^m}{c_\phi+c_\phi^m}.$$

The coefficients c_0 and c_0^m are both zero. ∎

A.2 Proof of Proposition 2

Proof. (Proposition 2) We first define a function $T(x;a,b)$ that truncates its argument x to the interval $[a,b]$:

$$T(x;a,b) = \begin{cases} x, & \text{if } a \le x \le b, \\ b, & \text{if } x > b, \\ a, & \text{if } x < a. \end{cases} \tag{20}$$

Conjecture that there exists a generalized linear equilibrium with informational efficiency β. Investor i's demand can then be written as

$$x_i = T\left(x_i^d; a(p), b(p)\right).$$

Moreover, as in the proof of Proposition 1, one can find investor i's *desired demand* x_i^d as

$$x_i^d = X^d + \xi_i^d,$$

where *aggregate desired demand* X^d is

$$X^d = \frac{\tau}{\gamma}\left(\frac{\tau_\epsilon v + \beta^2\left(\tau_u + \tau_z\right)\phi + \beta\tau_u z}{\tau_\epsilon + \beta^2\left(\tau_u + \tau_z\right) + \tau_v}\right) - \frac{\tau}{\tau_\theta}z - p\frac{\tau}{\gamma}$$

and the idiosyncratic part of the desired demand is

$$\xi_i^d = \frac{\tau}{\gamma}\left(\frac{\tau_\epsilon \epsilon_i + \beta\tau_u u_i}{\tau_\epsilon + \beta^2\left(\tau_u + \tau_z\right) + \tau_v} - \gamma u_i \tau_\theta^{-1}\right).$$

By the exact law of large numbers, one can write the aggregate demand of investors as

$$X = \int x_i di = E_{\xi_i}\left[T\left(X^d + \xi_i^d; a(p), b(p)\right)\right].$$

For a given price p, the aggregate demand X is an increasing (and thus invertible) function of the aggregate desired demand X^d. Therefore, given p, one can compute X^d, from which one can express $\eta(\phi, v, z) \equiv \frac{\tau}{\gamma}\left(\frac{\tau_\epsilon v + \beta^2(\tau_u + \tau_z)\phi + \beta\tau_u z}{\tau_\epsilon + \beta^2(\tau_u + \tau_z) + \tau_v}\right) - \frac{\tau}{\tau_\theta}z$. Thus, the price in the constrained economy is informationally equivalent to η. However, in the generalized linear equilibrium the price must be informationally equivalent to ϕ. For this to hold we need $-\frac{\partial\eta(\phi,v,z)}{\partial v}/\frac{\partial\eta(\phi,v,z)}{\partial z} = \beta$, which is equivalent to equation (17) that characterizes the informational efficiency in the unconstrained economy. Thus, $\beta = \beta^u$ and $x_i^d = x_i^u$. Moreover, for the aggregate demand of investors we can write

$$X = X(\phi, p) = E_{\xi_i}\left[T\left(X^u(\phi, p) + \xi_i; a(p), b(p)\right)\right],$$

where $X^u(\phi, p)$ and ξ_i are characterized in Proposition 1.

We now prove that for every p there exists unique $\phi = g(p)$ such that market clears. Indeed, the market clearing can be written as

$$X(\phi, p) + c_0^m - c_p^m p + c_\phi^m \phi = 1.$$

For a given p, aggregate investors' demand $X(\phi, p)$ is increasing in ϕ. Thus, there is at most one solution. At least one solution exists by the Intermediate Value Theorem. The aggregate demand at $+\infty(-\infty)$ is equal to $+\infty(-\infty)$, thus at some intermediate point aggregate demand has to be equal to 1.

We now compute a closed-form expression for the aggregate demand of investors $X(\phi, p)$. It can be split into three parts. For a fraction π_1 of investors the lower constraint $a(p)$ will bind. They contribute $\pi_1(\phi, p)a(p)$ to the aggregate demand. Similarly, a fraction π_3 of investors for whom the upper constraint $b(p)$ binds. They contribute $\pi_3(\phi, p)b(p)$. Finally a fraction π_2 will be unconstrained. They contribute $\pi_2 \cdot (X^u + E[\xi_i | (\xi_i + X^u) \in [a(p), b(p)]])$. Using the standard results for the mean of truncated normal distribution, the last term can be further simplified to

$$\pi_2 E[\xi_i | (\xi_i + X^u) \in [a(p), b(p)]] = \sigma_\xi \left(\Phi' \left(\frac{a(p) - X^u}{\sigma_\xi} \right) - \Phi' \left(\frac{b(p) - X^u}{\sigma_\xi} \right) \right)$$

where $\Phi(\cdot)$ and $\Phi'(\cdot)$ stands for the cumulative distribution function (CDF) and probability density function (PDF) of a standard normal distribution. Combining all of the terms we get

$$X(\phi, p) = \pi_1 a(p) + \pi_3 b(p) + \pi_2 X^u + \sigma_\xi \left(\Phi' \left(\frac{a(p) - X^u}{\sigma_\xi} \right) - \Phi' \left(\frac{b(p) - X^u}{\sigma_\xi} \right) \right).$$

Now we determine the fractions π_1, π_2 and π_3. The fraction of investors constrained by the lower constraint, π_1, is given by

$$\pi_1(p, \phi) = \mathrm{P}\left(x_i < a(p)\right) = \mathrm{P}\left(X^u(p, \phi) + \xi_i < a(p)\right) = \Phi \left(\frac{a(p) - X^u(p, \phi)}{\sigma_\xi} \right)$$

The expressions for π_2 and π_3 can be derived analogously:

$$\pi_3(\phi, p) = 1 - \Phi \left(\frac{b(p) - X^u(p, \phi)}{\sigma_\xi} \right),$$

$$\pi_2(\phi, p) = 1 - \pi_1 - \pi_3.$$

Finally, we find the expression for the function $g'(p)$. Differentiating the market-clearing condition implicitly, we have

$$g'(p) = -\frac{\frac{\partial}{\partial p}(X(p, \phi) + x_m(p))}{\frac{\partial}{\partial \phi}(X(p, \phi) + x_m(p))}. \tag{21}$$

For the numerator, we have

$$\frac{\partial}{\partial p}(X(p, \phi) + x_m(p)) = \pi_1 a'(p) + \pi_3 b'(p) - \pi_2 c_p - c_p^m.$$

For the denominator, we have

$$\frac{\partial}{\partial \phi}(X(p, \phi) + x_m(p)) = c_\phi^m + \pi_2 c_\phi.$$

Substituting these expressions into (21) gives us the desired result. ■

A.3 Proof of Lemma 1

Proof. (Lemma 1) The date-1 certainty equivalent solves

$$-\exp(-\gamma CE_{1,i}) = E\left[-\exp(-\gamma(W_0 + x_i(v+\theta-p) + e_i\theta|\mathcal{F}_i\right].$$

The certainty equivalent at time 1 can thus be written as

$$CE_{1,i} = W_0 + x_i(E[v|\mathcal{F}_i] - p) - \frac{\gamma}{2\tau_{v,i}}x_i^2 - \frac{\gamma}{2\tau_\theta}(x_i+e_i)^2.$$

where $\tau_{v,i}^{-1} = var(v|\mathcal{F}_i)$. Next, we note that

$$x_i^u = \frac{\tau_i}{\gamma}(E[v|\mathcal{F}_i] - p - \gamma e_i\tau_\theta^{-1}) \Rightarrow E[v|\mathcal{F}_i] - p = \frac{\gamma}{\tau_i}x_i^u + \frac{\gamma}{\tau_\theta}e_i$$

where x_i^u is her demand in the unconstrained economy. Substituting this into the certainty equivalent, we get

$$CE_1 = -\frac{\gamma}{2\tau_i}(x_i^u - x_i)^2 + W_0 + \frac{\gamma}{2\tau_i}(x_i^u)^2 - \frac{\gamma}{2\tau_\theta}e_i^2.$$

■

A.4 Proof of Proposition 3

Proof. (Proposition 3) For compactness of notation, we denote investor i's precision of signal by t,

$$t \equiv \tau_{\epsilon_i}.$$

The key to computing MVI is to understand how the time-1 certainty equivalent $CE_{1,i}$ depends on the realizations of random variables $\{s_i = v + \epsilon_i, e_i, p\}$ and how investor i's choice of precision t affects the distribution of these variables. Clearly, only the distribution of ϵ_i is affected by the choice of t. To emphasize the latter fact we will use the notation $\epsilon_i(t)$. The key step in the proof is to substitute $\epsilon_i(t) = \frac{1}{t}B_t$, where B_t is a Brownian motion that is independent of all other random variables in the model. Indeed, such a substitution is valid, as ex-ante both $\epsilon_i(t)$ and $\frac{1}{t}B_t$ have the same distribution, $N(0, 1/t)$. Hence, computing $E[-e^{-\gamma CE_{1,i}}]$, with or without substitution of $\epsilon_i(t) = \frac{1}{t}B_t$, will produce the same result. With the substution at hand, to emphasize the fact that $CE_{1,i}$ depends on t only through the dependence of the distribution of B_t on t we will write $CE_{1,i} = CE_{1,i}(B_t)$. The advantage of substitution we've made is that now we can utilize Ito's lemma to compute $dCE_{1,i}(B_t)$.[23] Note that

[23] There is a technicality here. Ito's lemma is applicable to $CE_{1,i}(B_t)$ that is C^2 in B_t. However, our function is only C^1 in B_t. One can also show that it is convex, which makes the Ito-Tanaka-Meyer rule applicable (see, e.g. Cohen and Elliott (2015), p. 352, and also Björk (2015) p. 18 for a more light reading). Since $CE_{1,i}$ is C^1, the local time terms in the Ito-Tanaka-Meyer rule disappear and we can write the Ito rule in the usual way.

marginal value of information (MVI) is given by

$$\text{MVI} = -\frac{\frac{d}{dt}E[e^{-\gamma CE_{1,i}}]}{\gamma e^{-\gamma CE_0}}.$$

We proceed as follows:

$$\frac{d}{dt}E[-e^{-\gamma CE_{1,i}}] = -E\left[\frac{de^{-\gamma CE_{1,i}(B_t)}}{dt}\right].$$

We use Ito's lemma to compute

$$de^{-\gamma CE_{1,i}(B_t)} = -\gamma e^{-\gamma CE_{1,i}(B_t)} dCE_{1,i} + \frac{\gamma^2}{2} e^{-\gamma CE_{1,i}(B_t)} dCE_{1,i}^2.$$

The expression for $dCE_{1,i}$ and $dCE_{1,i}^2$ depends on whether constraints bind for the investor i. The latter depends on realizations of random variables in his information set $\mathcal{F}_i = \{v + \frac{1}{t}B_t, e_i, \phi\}$.

We first consider the situation where agent i is unconstrained ex-post i.e., where $\{v+\frac{1}{t}B_t, e_i, \phi\}$ are such that $x_i^u \in (a(p), b(p))$. Then,

$$CE_{1,i} = \frac{\tau_i}{2\gamma}\left(v_i - p - \gamma e_i \tau_\theta^{-1}\right)^2 + \text{terms that do not depend on } t,$$

where we denoted

$$v_i = E[v|\mathcal{F}_i] = \frac{t\left(v + \frac{1}{t}B_t\right) + \beta^2(\tau_u + \tau_z)\phi + \beta\tau_u e_i}{\tau_{v,i}},$$

$$\tau_{v,i} = Var[v|\mathcal{F}_i] = t + \beta^2(\tau_u + \tau_z) + \tau_v.$$

Differentiating $CE_{1,i}$ we get

$$dCE_{1,i} = \frac{d\tau_i}{2\gamma}\left(v_i - p - \gamma e_i \tau_\theta^{-1}\right)^2 + \frac{\tau_i}{\gamma}\left(v_i - p - \gamma e_i\tau_\theta^{-1}\right)dv_i + \frac{\tau_i}{2\gamma}(dv_i)^2,$$

$$(dCE_{1,i})^2 = \left(\frac{\tau_i}{\gamma}\left(v_i - p - \gamma e_i\tau_\theta^{-1}\right)\right)^2 (dv_i)^2. \tag{22}$$

We now differentiate v_i and $\tau_i = \left(\frac{1}{\tau_{v,i}} + \frac{1}{\tau_\theta}\right)^{-1}$ to get

$$d\tau_i = \left(\frac{\tau_i}{\tau_{v,i}}\right)^2 dt, \;\; dv_i = v\frac{dt}{\tau_{v,i}} + \frac{dB_t}{\tau_{v,i}} - \frac{dt}{\tau_{v,i}}v_i, \;\; (dv_i)^2 = \left(\frac{dB_t}{\tau_{v,i}}\right)^2 = \frac{dt}{\tau_{v,i}^2}.$$

We now compute $E\left[de^{-\gamma CE_{1,i}(B_t)}\right]$. We use the law of iterated expectations and write $E\left[de^{-\gamma CE_{1,i}(B_t)}\right] = E\left[E_t\left[de^{-\gamma CE_{1,i}(B_t)}\right]\right]$, where we introduced notation

$$E_t[\cdot] = E\left[\cdot|v + 1/t\cdot B_t, e_i, \phi\right].$$

Moreover, we can write

$$E_t\left[de^{-\gamma CE_{1,i}(B_t)}\right] = -\gamma e^{-\gamma CE_{1,i}(B_t)}E_t[dCE_{1,i}] + \frac{\gamma^2}{2}e^{-\gamma CE_{1,i}(B_t)}E_t[dCE_{1,i}^2]. \tag{23}$$

Note that since $E_t[v] = v_i$ and $E_t[dB_t] = 0$, we have $E_t[dv_i] = 0$. Hence,

$$E_t[dCE_{1,i}] = \frac{d\tau_i}{2\gamma}\left(v_i - p - \gamma e_i\tau_\theta^{-1}\right)^2 + \frac{\tau_i}{2\gamma}\frac{dt}{\tau_{v,i}^2}. \tag{24}$$

Substituting (22) and (24) into (23), we get

$$E_t\left[de^{-\gamma CE_{1,i}(B_t)}\right] = -e^{-\gamma CE_1}\frac{\tau_i}{2}\frac{dt}{\tau_{v,i}^2}.$$

We now consider the situation where agent i is constrained by lower bound i.e., where $\{v + \frac{1}{t}B_t, e_i, \phi\}$ are such that $x_i^u < a(p)$. Then,

$$CE_{1,i} = a(p)\left(v_i - p - \gamma e_i\tau_\theta^{-1}\right) - \frac{\gamma}{2\tau_i}a(p)^2.$$

Differentiating this expression, we get

$$dCE_{1,i} = a(p)dv_i + \frac{\gamma}{2\tau_i^2}a(p)^2 d\tau_i.$$

We therefore get

$$E_t\left[dCE_{1,i}\right] = \frac{\gamma}{2}a(p)^2\left(\frac{1}{\tau_{v,i}}\right)^2 dt.$$

$$E_t\left[(dCE_{1,i})^2\right] = a(p)^2\left(dv_i\right)^2 = a(p)^2\frac{dt}{\tau_{v,i}^2}.$$

Substituting the above two equations into (23), we get:

$$E_t\left[de^{-\gamma CE_{1,i}(B_t)}\right] = 0.$$

Proceeding analogously for the case $\{v + \frac{1}{t}B_t, e_i, \phi\}$ are such that $x_i^u > b(p)$ and combining the results, we obtain

$$\begin{aligned} E_t\left[de^{-\gamma CE_{1,i}(B_t)}\right] &= \begin{cases} -e^{-\gamma CE_{1,i}}\frac{\tau_i}{2}\frac{dt}{\tau_{v,i}^2}, & \text{if } x_i^u \in (a(p), b(p)), \\ 0, & \text{otherwise,} \end{cases} \\ &= -e^{-\gamma CE_{1,i}}\frac{\tau_i}{2}\frac{dt}{\tau_{v,i}^2}\mathbb{I}(x_i^u = x_i). \end{aligned}$$

For marginal value of information (MVI), we finally get

$$\text{MVI} = -\frac{\frac{dU_0}{dt}}{\gamma U_0} = \frac{\tau_i}{2\tau_{v,i}^2} \frac{E\left[e^{-\gamma CE_{1,i}} \mathbb{I}(x_i^u = x_i)\right]}{E\left[e^{-\gamma CE_{1,i}}\right]}.$$

The result that the marginal value of information decreases when individual investors constraints become tighter, holding everything else fixed, can be proved exactly as the Proposition 4, the proof of which we present below. ■

A.5 Proof of Proposition 4

Proof. (Proposition 4) Denote $MVI(a(p(\phi)), b(p(\phi)), p(\phi; \gamma_m))$ the marginal value of information when the constraints are given by functions $a(p)$ and $b(p)$ and the equilirbium price function is given by $p(\phi)$. Denote the new, tightened, contraints and the new price funciton by $\hat{a}(p)$, $\hat{b}(p)$ and $\hat{p}(\phi; \gamma_m)$, respectively. Note that $\hat{p}(\cdot)$ depends on γ_m. Note that

$$\lim_{\gamma_m \to 0} \left(\hat{p}(\phi; \gamma_m) - p(\phi; \gamma_m)\right) = 0.$$

This is because in both cases prices converge to $E[f|\phi]$. It suffices to prove that

$$\lim_{\gamma_m \to 0} \Big(MVI(\hat{a}(p(\phi)), \hat{b}(p(\phi)), \hat{p}(\phi; \gamma_m)) - MVI(a(p(\phi)), b(p(\phi)), p(\phi; \gamma_m)) \Big) < 0 \ .$$

Write the expression in the brackets as follows

$$\begin{gathered} MVI(\hat{a}(p(\phi)), \hat{b}(p(\phi)), \hat{p}(\phi; \gamma_m)) - MVI(a(p(\phi)), b(p(\phi)), p(\phi; \gamma_m)) = \\ \underbrace{\Big\{ MVI(\hat{a}(p(\phi)), \hat{b}(p(\phi)), \hat{p}(\phi; \gamma_m)) - MVI(\hat{a}(p(\phi)), \hat{b}(p(\phi)), p(\phi; \gamma_m)) \Big\}}_{\to 0 \text{ as } \gamma_m \to 0} + \\ \underbrace{\Big(MVI(\hat{a}(p(\phi)), \hat{b}(p(\phi)), p(\phi; \gamma_m)) - MVI(a(p(\phi)), b(p(\phi)), p(\phi; \gamma_m)) \Big)}_{<0} \end{gathered}$$

The first term (in the curly brackets) converges to 0, since $\hat{p}(\phi; \gamma_m)$ converges to $p(\phi; \gamma_m)$ pointwise. We now prove that the second term in round brackets is strictly negative in the limit as $\gamma \to 0$

Note that when we compute that term, we keep the price function fixed, as if price function is independent of portfolio constraints. We write the expression for the marginal value of information as

$$\text{MVI} = \frac{\tau_i}{2\tau_{v,i}^2 \gamma} \frac{U_0^u}{U_0}. \tag{25}$$

The only term affected by constraints is $\frac{U_0^u}{U_0}$. Consider first the nominator: $U_0^u = E[-e^{-\gamma CE_1} \mathbb{I}(x_i^u =$

$x_i)] = E[-e^{-\left(W_0+\frac{\gamma}{2\tau_i}(x_i^u)^2-\frac{\gamma}{2\tau_\theta}e_i^2\right)}\mathbb{I}(x_i^u = x_i)]$. As constraints tighten, only the $\mathbb{I}(x_i^u = x_i)$ changes: we keep the price function fixed, therefore the desired demands x_i^u are the same. The term U_0^u increases (becomes less negative) as constraints become tighter: recall that investors get negative utility; as constraints become tighter, they get it in fewer states of the world. The denominator U_0 decreases (becomes more negative) as with constraints the certainty equivalent $CE_{1,i}$ in all states weakly decreases. Thus, the ratio decreases as constraints become tighter. ■

A.6 Proof of Corollary 1

Proof. (Corollary 1) Since constraints do not depend on prices we write our ODE for $g(p)$ as follows

$$g'(p) = \frac{\pi_2(p,g(p))c_p + c_p^m}{\pi_2(p,g(p))c_\phi + c_\phi^m} > 0,$$

from which the monotonicity of $g(p)$ follows. The rest follows directly from Proposition 2. ■

A.7 Proof of Proposition 5.

We begin with the following Lemma.

Lemma 2. *For W_0 small enough, $\frac{\partial}{\partial A}\text{MVI}\left(\tau_{\epsilon_i},\tau_\epsilon;A\right) > 0$, where $A = W/m$ is the size of the constraint.*

Proof. (Lemma 2) We need to show that $\lim_{W_0\to 0}\frac{\partial}{\partial A}\text{MVI}\left(\tau_{\epsilon_i},\tau_\epsilon;A\right) > 0$. We know that

$$\text{MVI}\left(\tau_{\epsilon_i},\tau_\epsilon;A\right) = \frac{\tau_i}{2\tau_{v,i}^2\gamma}\frac{U_0^u}{U_0},$$

where $U_0^u(\tau_{\epsilon_i},\tau_\epsilon) = E[-e^{-\gamma CE_{1,i}}\mathbb{I}_{x_i^u=x_i}]$ is the expected utility in the states where agent i is unconstrained and $U_0(\tau_{\epsilon_i},\tau_\epsilon) = E[-e^{-\gamma CE_{1,i}}]$ is date-0 expected utility. We write

$$\lim_{W_0\to 0}\frac{\partial}{\partial A}\text{MVI}\left(\tau_{\epsilon_i},\tau_\epsilon;A\right) = \frac{\tau_i}{2\tau_{v,i}^2\gamma}\left(\lim_{W_0\to 0}\frac{\frac{\partial}{\partial A}U_0^u}{U_0} - \lim_{W_0\to 0}\frac{U_0^u\frac{\partial}{\partial A}U_0}{U_0^2}\right).$$

We will prove that $\lim_{W_0\to 0}\frac{\frac{\partial}{\partial A}U_0^u}{U_0} > 0$ and that $\lim_{W_0\to 0}\frac{\partial}{\partial A}U_0 < \infty$, which combined with $\lim_{W_0\to 0}U_0^u = 0$ implies that $\lim_{W_0\to 0}\frac{U_0^u\frac{\partial}{\partial A}U_0}{U_0^2} = 0$. We first compute $\frac{\partial\left(-U_0^u\right)}{\partial A}$:

$$\begin{aligned}\frac{\partial\left(-U_0^u\right)}{\partial A} &= \frac{\partial}{\partial A}E\left[e^{-\gamma CE_{1,i}}\mathbb{I}\left(x_i^u\in[-A;A]\right)\right] = \\ &= E\left[e^{-\gamma CE_{1,i}}\left(\delta\left(x_i^u - A\right)+\delta\left(x_i^u + A\right)\right)\right] + E\left[\frac{\partial}{\partial A}\left(e^{-\gamma CE_{1,i}}\right)\mathbb{I}\left(x_i^u\in[-A;A]\right)\right],\end{aligned}$$

where $\delta(\cdot)$ denotes Dirac's delta function. For the first term above we have

$$\lim_{W_0 \to 0} E\left[e^{-\gamma CE_{1,i}} \left(\delta\left(x_i^u - A\right) + \delta\left(x_i^u + A\right)\right)\right] = 2E\left[e^{-\gamma CE_{1,i}} \delta\left(x_i^u\right)\right] > 0.$$

We now consider the second term and prove that its' limit is zero as $W_0 \to 0$. Note that in the unconstrained region

$$CE_{1,i} = W_0 + \frac{\gamma}{2\tau_i}\left(x_i^u\right)^2 - \frac{\gamma}{2\tau_\theta} e_i^2 \implies \frac{\partial CE_{1,i}}{\partial A} = 1 + \frac{\gamma}{\tau_i} x_i^u \frac{\partial x_i^u}{\partial A}.$$

The unconstrained demand is given by

$$x_i^u = c_\phi \phi - c_p l(\phi; A) + \xi_i,$$

and depends on A only through $l(\phi, A) \equiv g^{-1}(\phi)$ (i.e., the inverse of $g(p)$ for a given W_0). Therefore

$$\frac{\partial x_i^u}{\partial A} = -c_p l_A(\phi; A).$$

We can write now

$$\begin{aligned}
\left| E\left[\frac{\partial}{\partial A}\left(e^{-\gamma CE_{1,i}}\right) \mathbb{I}\left(x_i^u \in [-A; A]\right)\right]\right| &< E\left[\gamma e^{-\gamma CE_{1,i}} \left|\frac{\partial CE_{1,i}}{\partial A}\right| \mathbb{I}\left(x_i^u \in [-A; A]\right)\right] \\
&< \gamma E\left[\left|\frac{\partial CE_{1,i}}{\partial A}\right| \mathbb{I}\left(x_i^u \in [-A; A]\right)\right] \\
&< \gamma E\left[\left(1 + \frac{\gamma}{\tau_i} c_p \left|x_i^u\right| \left|l_W\right|\right) \mathbb{I}\left(x_i^u \in [-A; A]\right)\right] \\
&< \gamma E\left[\left(1 + \frac{\gamma}{\tau_i} c_p A \frac{1}{c_p^m}\right) \mathbb{I}\left(x_i^u \in [-A; A]\right)\right] \\
&= \gamma \left(1 + \frac{\gamma}{\tau_i} c_p a \frac{1}{c_p^m}\right) E\left[\mathbb{I}\left(x_i^u \in [-A; A]\right)\right].
\end{aligned}$$

In the above we have used that $\left|x_i^u\right| \mathbb{I}\left(x_i^u \in [-A; A]\right) < A$ and that $\left|l_A(\phi; A)\right| < \frac{1}{c_p^m}$. We prove the last statement below. Indeed, the inverse of $g(p)$, $l(\phi)$ solves

$$X^{agg}(l(\phi), \phi; A) = 1$$

we thus have that

$$X_{W_0}^{agg}(l(\phi; A), \phi; A) + X_p^{agg}(l(\phi; A), \phi; A) \frac{\partial l(\phi; A)}{\partial A} = 0.$$

Computing the derivatives of aggregate demand and expressing $\frac{\partial l(\phi;A)}{\partial A}$ yields

$$\frac{\partial l(\phi;A)}{\partial A} = \frac{(\pi_3 - \pi_1)}{c_p^m + \pi_2 c_p}$$

It is clear from above that

$$\left|\frac{\partial l(\phi;A)}{\partial A}\right| < \frac{1}{c_p^m}.$$

Since

$$\lim_{W_0 \to 0} E\left[\mathbb{I}\left(x_i^u \in [-A;A]\right)\right] = 0,$$

we have that

$$\lim_{W_0 \to 0} E\left[\frac{\partial}{\partial A}\left(e^{-\gamma CE_{1,i}}\right)\mathbb{I}\left(x_i^u \in [-A;A]\right)\right] = 0.$$

We now prove that $\lim_{W_0 \to 0} \frac{\partial}{\partial A} U_0 < \infty$. Recall that

$$CE_{1,i} = \frac{\gamma}{2\tau_i}\left(x_i^u\right)^2 - \frac{\gamma}{2\tau_\theta} e_i^2 - \frac{\gamma}{2\tau_i}\left(x_i^u - x_i\right)^2.$$

Therefore,

$$\begin{aligned}\frac{\partial CE_{1,i}}{\partial A} &= \frac{\gamma}{\tau_i} x_i^u \frac{\partial x_i^u}{\partial A} - \frac{\gamma}{\tau_i}\left(x_i^u - x_i\right)\left(\frac{\partial x_i^u}{\partial A} - \frac{\partial x_i}{\partial A}\right) \\ &= \frac{\gamma}{\tau_i} x_i \frac{\partial x_i^u}{\partial A} + \frac{\gamma}{\tau_i}\left(x_i^u - x_i\right)\frac{\partial x_i}{\partial A}.\end{aligned}$$

Note that, as was shown before:

$$\left|\frac{\gamma}{\tau_i} x_i \frac{\partial x_i^u}{\partial A}\right| < \frac{\gamma}{\tau_i} c_p A \frac{1}{c_p^m}.$$

For the other term we have

$$\begin{aligned}\left|\frac{\gamma}{\tau_i}\left(x_i^u - x_i\right)\frac{\partial x_i}{\partial A}\right| &< \frac{\gamma}{\tau_i}\left|x_i^u\right| \\ &< \frac{\gamma}{\tau_i}\left(c_\phi|\phi| + c_p|l(\phi;A)| + |\xi_i|\right) \\ &< \frac{\gamma}{\tau_i}\left(c_\phi|\phi| + c_p\frac{1}{c_p^m}|\phi| + |\xi_i|\right).\end{aligned}$$

In the above, we have used that $|l(\phi;A)| < \frac{1}{c_p^m}|\phi|$. Indeed, this claim is true as long as $l(0,W_0) < 0$ for W_0 small enough. To see this fact, write market-makers demand

$$x_m = c_\phi^m \phi - c_p^m p = -c_p^m l(0,W_0) = 1 - X > 1 - A.$$

Therefore, $c_p^m l(0,W_0) < -1 + \frac{W_0}{m}$ which is negative for W_0 small enough (and hence for A small enough). We now write, similar to the previous part of the proof

$$\begin{aligned}
\left|E\left[\frac{\partial}{\partial A}\left(e^{-\gamma CE_{1,i}}\right)\right]\right| &< E\left[\gamma e^{-\gamma CE_{1,i}}\left|\frac{\partial CE_{1,i}}{\partial A}\right|\right] \\
&< \gamma E\left[\left|\frac{\partial CE_{1,i}}{\partial A}\right|\right] \\
&< \gamma E\left[\left(1+\frac{\gamma}{\tau_i}c_p\left|x_i^u\right|\left|l_w\right|+\frac{\gamma}{\tau_i}\left|x_i^u\right|\right)\right] \\
&< \gamma E\left[\left(1+\frac{\gamma}{\tau_i}c_p A\frac{1}{c_p^m}+\frac{\gamma}{\tau_i}\left(c_\phi\left|\phi\right|+c_p\frac{1}{c_p^m}\left|\phi\right|+\left|\xi_i\right|\right)\right)\right] \\
&< \gamma E\left[\left(1+c_1\left|v\right|+c_2\left|z\right|+c_3\left|\epsilon_i\right|+c_4\left|u_i\right|\right)\right] \\
&= \gamma+c_1E\left[\left|v\right|\right]+c_2E\left[\left|z\right|\right]+c_3E\left[\left|\epsilon_i\right|\right]+c_4E\left[\left|u_i\right|\right]
\end{aligned}$$

where c_1, c_2, c_3 and c_4 are some finite constants. All four expectations above exist: these are the means of folded normal distributions. Hence,

$$\left|E\left[\frac{\partial}{\partial A}\left(e^{-\gamma CE_{1,i}}\right)\right]\right| < \infty.$$

Summarizing all the above

$$\lim_{W\to 0}\frac{\partial}{\partial A}\mathrm{MVI}\left(\tau_{\epsilon_i},\tau_\epsilon;W_0\right)=\frac{\tau_i}{2\tau_{v,i}^2\gamma}\left(\underbrace{\lim_{W\to 0}\frac{\frac{\partial}{\partial A}U_0^u}{U_0}}_{>0}-\underbrace{\lim_{W\to 0}\frac{\overbrace{U_0^u}^{\to 0}\overbrace{\frac{\partial}{\partial A}U_0}^{\text{limit}<\infty}}{U_0^2}}_{=0}\right)>0.$$

■

Proof. (Proposition 5) We prove that in a stable equilibrium $\frac{d\tau_\epsilon^*}{dm^+}<0$ and $\frac{d\beta}{dm^+}<0$. The claims for m^- can be proved analogously. Given that other investors choose precision τ_ϵ^*, it is optimal for an investor i to choose τ_{ϵ_i} such that:

$$C'(\tau_{\epsilon_i})=\mathrm{MVI}(\tau_{\epsilon_i},\tau_\epsilon^*),$$

$$C''(\tau_{\epsilon_i})-\mathrm{MVI}_1(\tau_{\epsilon_i},\tau_\epsilon^*)>0.$$

The first (second) equation above corresponds to the first (second) order condition in investor i's optimization problem and $\mathrm{MVI}_k(\cdot,\cdot)$ denotes the derivative of $\mathrm{MVI}(\cdot,\cdot)>0$ with respect to its' k-th argument. Differentiating the first equation above implicitly, we get

$$\tau_{\epsilon_i}'(\tau_\epsilon^*)=\frac{\mathrm{MVI}_2(\tau_{\epsilon_i},\tau_\epsilon^*)}{C''(\tau_{\epsilon_i})-\mathrm{MVI}_1(\tau_{\epsilon_i},\tau_\epsilon^*)}. \tag{26}$$

In a symmetric equilibrium $\tau_{\epsilon_i} = \tau_\epsilon^*$, therefore:

$$C'(\tau_\epsilon^*) = \mathrm{MVI}(\tau_\epsilon^*, \tau_\epsilon^*),$$

$$C''(\tau_\epsilon^*) - \mathrm{MVI}_1(\tau_\epsilon^*, \tau_\epsilon^*) > 0.$$

Moreover, since in a stable equilibrium $|\tau'_{\epsilon_i}(\tau_\epsilon^*)| < 1$, from (26) we also have

$$C''(\tau_\epsilon^*) - \mathrm{MVI}_1(\tau_\epsilon^*, \tau_\epsilon^*) - \mathrm{MVI}_2(\tau_\epsilon^*, \tau_\epsilon^*) > 0. \tag{27}$$

To calculate $\frac{d\tau_\epsilon^*}{dm^+}$ we differentiate $C'(\tau_\epsilon^*(m^+)) = \mathrm{MVI}(\tau_\epsilon^*(m^+), \tau_\epsilon^*(m^+); m^+)$ with respect to m^+ to get

$$\frac{d\tau_\epsilon^*}{dm^+} = \frac{\mathrm{MVI}_3(\tau_\epsilon^*, \tau_\epsilon^*)}{C''(\tau_\epsilon^*) - \mathrm{MVI}_1(\tau_\epsilon^*, \tau_\epsilon^*) - \mathrm{MVI}_2(\tau_\epsilon^*, \tau_\epsilon^*)}.$$

It follows from Lemma 2 that $\mathrm{MVI}_3(\tau_\epsilon^*, \tau_\epsilon^*) < 0$. Combining it with (27) we get $\frac{d\tau_\epsilon^*}{dm^+} < 0$. To see that $\frac{d\beta}{dm^+} < 0$, note that β still satisfies equation (4) and β decreases as investors acquire less information. ■

A.8 Proof of Proposition 6

Proof. (Proposition 6) The existence and uniqueness of equilibrium follows from Corollary 1. We now derive the formula for m^+ margin. It solves

$$\Pr^Q\left(p - f > m^+ | p\right) = 1 - \alpha.$$

Lemma 15 implies that $p - f$ given p is distributed normally with mean zero and variance $Var[f|p]$ under risk-neutral measure. Therefore, one can write

$$\begin{aligned} \Pr^Q\left(p - f > m^+ | p\right) &= \Pr^Q\left(\frac{p-f}{\sqrt{Var[f|p]}} > \frac{m^+}{\sqrt{Var[f|p]}} | p\right) \\ &= 1 - \Phi\left(\frac{m^+}{\sqrt{Var[f|p]}}\right) \end{aligned}$$

Analogous argument is true for m^- margin. Therefore, the margins are given by

$$m^+ = m^- = \frac{\Phi^{-1}(\alpha)}{\sqrt{Var[f|p]}}$$

where $Var[f|p]^{-1} = \frac{1}{\tau_v + \beta^2\tau_z} + \frac{1}{\tau_\theta}$. As informational efficiency (β) decreases, margins increase. This implies that the constraint $a(p) = -\frac{W_0}{m^+(p)}$ decreases and the constraint $b(p) = \frac{W_0}{m^-(p)}$ increases. This implies that constraints tighten as informational efficiency decreases. ■

A.9 Proof of Corollary 2

Proof. (Corollary 2) Differentiating (11), we get that

$$\frac{\partial m}{\partial \tau_\epsilon} = \frac{\partial m}{\partial \beta}\frac{\partial \beta}{\partial \tau_\epsilon} = -\frac{\Phi^{-1}(\alpha)}{(\tau_v+\beta^2\tau_z)^2\sqrt{(\tau_v+\beta^2\tau_z)^{-1}+\tau_\theta^{-1}}}\underbrace{\frac{\partial \beta}{\partial \tau_\epsilon}}_{>0,\text{ does not depend on }\alpha}. \tag{28}$$

Differentiating (28) with respect to α we get

$$\frac{\partial^2 m}{\partial \alpha \partial \tau_\epsilon} = -\frac{1}{(\tau_v+\beta^2\tau_z)^2\sqrt{(\tau_v+\beta^2\tau_z)^{-1}+\tau_\theta^{-1}}}\underbrace{\frac{\partial \beta}{\partial \tau_\epsilon}}_{>0}\underbrace{\frac{\partial \Phi^{-1}(\alpha)}{\partial \alpha}}_{>0} < 0.$$

■

A.10 Proof of Corollary 3

Proof. (Corollary 3) The effect of W_0 on β can be written as

$$\frac{d\beta}{dW_0} = \underbrace{\frac{\partial \beta}{\partial W_0}}_{\text{Direct Effect}} + \underbrace{\frac{\partial \beta}{\partial \tau_\epsilon}\cdot\frac{\partial \tau_\epsilon}{\partial W_0}}_{\text{Indirect Effect}}$$

The irrelevance result implies that the direct effect is zero. The indirect effect is positive because $\frac{\partial \beta}{\partial \tau_\epsilon} > 0$ according to the definition of β and $\frac{\partial \tau_\epsilon}{\partial W_0} > 0$ from Proposition 5. Finally, using the result in Proposition 6, we have $\frac{dm^+}{dW_0} = \frac{\partial m^+}{\partial \beta}\frac{\partial \beta}{\partial W_0} < 0$. ■

A.11 Proof of Proposition 7

The proof uses the following two Lemmas.

Lemma 3. *One can write $x_i^u = c_v(\tau_\epsilon)v + c_z(\tau_\epsilon)z + c_p(\tau_\epsilon)p(v,z;\tau_\epsilon) + c_u(\tau_\epsilon)u_i + c_\epsilon(\tau_\epsilon)\epsilon_i$, where the coefficients in the expression as well as their derivatives with respect to τ_ϵ are finite for every finite $\tau_\epsilon > \underline{\tau_\epsilon}$. Moreover, for W_0 small enough, $\frac{d}{d\tau_\epsilon}x_i^u$ is finite for every finite v, z, u_i and ϵ_i.*

Proof. It is straightforward to express

$$c_v(\tau_\epsilon) = \frac{\tau(\tau_\epsilon)}{\gamma}\left(\frac{\tau_\epsilon + \beta(\tau_\epsilon)^2(\tau_u+\tau_z)}{\tau_\epsilon + \beta(\tau_\epsilon)^2(\tau_u+\tau_z)+\tau_v}\right),$$

$$c_z(\tau_\epsilon) = -\frac{1}{\beta(\tau_\epsilon)}c_v(\tau_\epsilon),$$

$$c_p = \frac{\tau(\tau_\epsilon)}{\gamma},$$

$$c_u(\tau_\epsilon) = \frac{\tau(\tau_\epsilon)}{\gamma}\left(\frac{\beta(\tau_\epsilon)\tau_u}{\tau_\epsilon + \beta(\tau_\epsilon)^2(\tau_u+\tau_z)+\tau_v} - \gamma\tau_\theta^{-1}\right),$$

$$c_\epsilon(\tau_\epsilon) = \frac{\tau(\tau_\epsilon)}{\gamma}\frac{\tau_\epsilon}{\tau_\epsilon + \beta(\tau_\epsilon)^2(\tau_u+\tau_z)+\tau_v}.$$

It can be directly examined that these coefficients as well as their derivatives with respect to τ_ϵ are finite for every finite $\tau_\epsilon > \underline{\tau_\epsilon}$. We can write

$$\frac{d}{d\tau_\epsilon}x_i^u = c_v'(\tau_\epsilon)v + c_z'(\tau_\epsilon)z + c_p'(\tau_\epsilon)p + c_u'(\tau_\epsilon)u_i + c_\epsilon'(\tau_\epsilon)\epsilon_i + c_p(\tau_\epsilon)\frac{dp}{d\tau_\epsilon}.$$

All terms above are finite for every finite v, z, u_i and ϵ_i (we show in the proof of Lemma 4 that $\frac{dp}{d\tau_\epsilon} < \infty$ for small enough W_0). ■

Lemma 4. $\lim_{W_0\to 0}\frac{dX}{d\tau_\epsilon} = 0$.

Proof. The aggregate demand of investors changes with τ_ϵ because: (1) constraints $A(\tau_\epsilon) = W/m(\tau_\epsilon)$ change (2) price function $p(v,z;A(\tau_\epsilon),\tau_\epsilon)$ changes (both directly and through changes in constraints) and (3) τ_ϵ affects X directly. Correspondingly, we can write

$$X = X(v,z,p(v,z;A(\tau_\epsilon),\tau_\epsilon);A(\tau_\epsilon),\tau_\epsilon)$$

and

$$\frac{dX}{d\tau_\epsilon} = \left(\frac{\partial X}{\partial A} + \frac{\partial X}{\partial p}\frac{\partial p}{\partial A}\right)A'(\tau_\epsilon) + \frac{\partial X}{\partial p}\frac{\partial p}{\partial \tau_\epsilon} + \frac{\partial X(v,z)}{\partial \tau_\epsilon}.$$

Consider

$$\frac{\partial X(v,z)}{\partial\tau_\epsilon} = E\left[\left(c_v'(\tau_\epsilon)v + c_z'(\tau_\epsilon)z + c_p'(\tau_\epsilon)p + c_u'(\tau_\epsilon)u_i + c_\epsilon'(\tau_\epsilon)\epsilon_i\right)\mathbb{I}\left(x_i^u \in [-A;A]\right)|v,z\right].$$

From the above, it is clear that $\lim_{W_0\to 0}\frac{\partial X(v,z)}{\partial\tau_\epsilon} = 0$. We now compute $\frac{\partial p}{\partial\tau_\epsilon}$. Differentiating market clearing condition implicitly we get

$$\frac{\partial p}{\partial\tau_\epsilon} = \frac{\frac{\partial X^{agg}}{\partial\tau_\epsilon}}{c_p^m + \pi_2 c_p} = \frac{\overbrace{\frac{\partial X}{\partial\tau_\epsilon}}^{\to 0} + \frac{\partial x_m}{\partial\tau_\epsilon}}{c_p^m + \pi_2 c_p}.$$

It is clear that

$$\lim_{W_0\to 0}\frac{\partial p}{\partial\tau_\epsilon} = \frac{c_\phi^m(\tau_\epsilon)'\left(v - \frac{1}{\beta(\tau_\epsilon)}z\right) + \frac{c_\phi^m(\tau_\epsilon)}{\beta(\tau_\epsilon)^2}\beta'(\tau_\epsilon)z - c_p^m(\tau_\epsilon)'p}{c_p^m} < \infty.$$

On the other hand,

$$\lim_{W\to 0}\frac{\partial X}{\partial p}=\lim_{W\to 0}\pi_2 c_p=0,$$

hence

$$\lim_{W\to 0}\frac{\partial X}{\partial p}\frac{\partial p}{\partial \tau_\epsilon}=0.$$

We now consider $\frac{\partial X}{\partial A}$:

$$\frac{\partial X}{\partial A}=\pi_3-\pi_1<\infty,$$

On the other hand

$$\lim_{W\to 0}\frac{\partial X}{\partial p}=\lim_{W\to 0}\pi_2 c_p=0.$$

We finally compute $\frac{\partial p}{\partial A}$. Differentiating the market-clearing condition implicitly we get

$$\frac{\partial p}{\partial A}=\frac{\pi_3-\pi_1}{c_p^m+\pi_2 c_p}\Rightarrow\lim_{W\to 0}\frac{\partial p}{\partial A}<\infty.$$

Therefore

$$\lim_{W_0\to 0}\left(\frac{\partial X}{\partial A}+\frac{\partial X}{\partial p}\frac{\partial p}{\partial A}\right)A'(\tau_\epsilon)=\lim_{W_0\to 0}\left(-\underbrace{\left(\frac{\partial X}{\partial A}+\frac{\partial X}{\partial p}\frac{\partial p}{\partial A}\right)\frac{m(\tau_\epsilon)'}{m(\tau_\epsilon)^2}}_{<\infty}W_0\right)=0.$$

■

Proof. (Proposition 7) We know that

$$\mathrm{MVI}\left(\tau_{\epsilon_i},\tau_\epsilon;W_0\right)=\frac{\tau_i}{2\tau_{v,i}^2\gamma}\frac{U_0^u}{U_0},$$

where $U_0^u(\tau_{\epsilon_i},\tau_\epsilon)=E[-e^{-\gamma CE_{1,i}}\mathbb{I}_{x_i^u=x_i}]$ and $U_0(\tau_{\epsilon_i},\tau_\epsilon)=E[-e^{-\gamma CE_{1,i}}]$ is date-0 expected utility. Since

$$\frac{\partial\log\mathrm{MVI}(\tau_{\epsilon_i},\tau_\epsilon;W_0)}{\partial\tau_\epsilon}=\frac{1}{\mathrm{MVI}(\tau_{\epsilon_i},\tau_\epsilon;W_0)}\frac{\partial\mathrm{MVI}\left(\tau_{\epsilon_i},\tau_\epsilon;W_0\right)}{\partial\tau_\epsilon}$$

and $\mathrm{MVI}(\tau_{\epsilon_i},\tau_\epsilon;W_0)>0$, we may instead prove that $\frac{\partial\log\mathrm{MVI}(\tau_{\epsilon_i},\tau_\epsilon;W_0)}{\partial\tau_\epsilon}>0$. We write

$$\frac{d\log\mathrm{MVI}(\tau_{\epsilon_i},\tau_\epsilon;W_0)}{d\tau_\epsilon}=\frac{d}{d\tau_\epsilon}log\left(\frac{\tau_i}{2\gamma\tau_{v,i}^2}\right)+\frac{d\log U_0^u}{d\tau_\epsilon}-\frac{d\log U_0}{d\tau_\epsilon}.$$

Consider the third term, $\frac{d\log U_0}{d\tau_\epsilon}$. Recall that

$$CE_{1,i}=W_0+\frac{\gamma}{2\tau_i}\left(x_i^u\right)^2-\frac{\gamma}{2\tau_\theta}e_i^2-\frac{\gamma}{2\tau_i}\left(x_i^u-x_i\right)^2.$$

Hence,

$$\begin{aligned}\frac{dCE_{1,i}}{d\tau_\epsilon} &= 1 + \frac{\gamma}{\tau_i} x_i^u \frac{dx_i^u}{d\tau_\epsilon} - \frac{\gamma}{\tau_i}\left(x_i^u - x_i\right)\left(\frac{dx_i^u}{d\tau_\epsilon} - \frac{dx_i}{d\tau_\epsilon}\right) + \frac{\gamma}{2}\left(\left(x_i^u\right)^2 - \left(x_i^u - x_i\right)^2\right)\frac{d}{d\tau_\epsilon}\left(\frac{1}{\tau_i}\right) \\ &= 1 + \frac{\gamma}{\tau_i} x_i \frac{dx_i^u}{d\tau_\epsilon} + \frac{\gamma}{\tau_i}\left(x_i^u - x_i\right)\frac{dx_i}{d\tau_\epsilon} + \frac{\gamma}{2}\left(\left(x_i^u\right)^2 - \left(x_i^u - x_i\right)^2\right)\frac{d}{d\tau_\epsilon}\left(\frac{1}{\tau_i}\right).\end{aligned}$$

Note that,

$$\lim_{W_0 \to 0} \underbrace{x_i}_{\to 0} \underbrace{\frac{dx_i^u}{d\tau_\epsilon}}_{<\infty} = 0,$$

where we have used Lemma 3. Consider $x_i^u > x_i$. For the third term, $\frac{\gamma}{\tau_i}\left(x_i^u - x_i\right)\frac{dx_i}{d\tau_\epsilon}$, we have (recall that $x_i = W_0/m\left(\tau_\epsilon\right)$ in that case)

$$\begin{aligned}\left|\frac{\gamma}{\tau_i}\left(x_i^u - x_i\right)\frac{dx_i}{d\tau_\epsilon}\right| &< \frac{\gamma}{\tau_i}\left|x_i^u\right|\left|\frac{dx_i}{d\tau_\epsilon}\right| \\ &= \frac{\gamma}{\tau_i}\underbrace{\left|x_i^u\right|}_{<\infty}\frac{W_0}{m^2}m'(\tau_\epsilon) \\ &\to 0 \text{ as } W_0 \to 0.\end{aligned}$$

Similar argument shows that this term goes to 0 for $x_i^u < x_i$. For $x_i^u = x_i$ this term is zero. Thus,

$$\lim_{W_0 \to 0} \frac{\gamma}{\tau_i}\left(x_i^u - x_i\right)\frac{dx_i}{d\tau_\epsilon} = 0.$$

Since

$$\lim_{W \to 0}\left(\left(x_i^u\right)^2 - \left(x_i^u - \underbrace{x_i}_{\to 0}\right)^2\right) = 0$$

we have that

$$\lim_{W \to 0} \frac{\gamma}{2}\underbrace{\left(\left(x_i^u\right)^2 - \left(x_i^u - x_i\right)^2\right)}_{\to 0}\underbrace{\frac{d}{d\tau_\epsilon}\left(\frac{1}{\tau_i}\right)}_{<\infty} = 0.$$

Thus,

$$\lim_{W_0 \to 0} \frac{d\log U_0}{d\tau_\epsilon} = 0.$$

We now consider the second term

$$\frac{d\log\left(-U_0^u\right)}{d\tau_\epsilon} = \frac{1}{-U_0^u}\frac{d\left(-U_0^u\right)}{d\tau_\epsilon}.$$

We multiply both terms in the above by $m(\tau_\epsilon)/(2W_0)$ and take the limit as $W_0 \to 0$. For $U_0^u \cdot \frac{m(\tau_\epsilon)}{2W_0}$

one can write

$$-U_0^u \cdot \frac{m(\tau_\epsilon)}{2W_0} = E\left[e^{-\gamma CE_{1,i}} \frac{\mathbb{I}\left(x_i^u \in \left[-\frac{W_0}{m(\tau_\epsilon)}; \frac{W_0}{m(\tau_\epsilon)}\right]\right)}{\frac{2W_0}{m(\tau_\epsilon)}}\right] \to E\left[e^{-\gamma CE_{1,i}} \delta(x_i^u)\right] \text{ as } W_0 \to 0.$$

In the above $\delta(\cdot)$ denotes Dirac's delta function. For $\frac{m(\tau_\epsilon)}{2W_0} \frac{d\left(-U_0^u\right)}{d\tau_\epsilon}$ we write

$$\begin{aligned} \frac{m(\tau_\epsilon)}{2W_0} \frac{d}{d\tau_\epsilon} U_0^u(\tau_{\epsilon_i}, \tau_\epsilon) = E\left[\gamma e^{-\gamma CE_{1,i}} \frac{dCE_{1,i}}{d\tau_\epsilon} \frac{\mathbb{I}\left(x_i^u \in \left[-\frac{W_0}{m(\tau_\epsilon)}; \frac{W_0}{m(\tau_\epsilon)}\right]\right)}{\frac{2W_0}{m(\tau_\epsilon)}}\right] - \\ \frac{m(\tau_\epsilon)}{2W_0} \frac{d}{d\tau_\epsilon}\left(\frac{W_0}{m(\tau_\epsilon)}\right) E\left[e^{-\gamma CE_{1,i}} \delta\left(x_i^u - \frac{W_0}{m(\tau_\epsilon)}\right) + e^{-\gamma CE_{1,i}} \delta\left(x_i^u + \frac{W_0}{m(\tau_\epsilon)}\right)\right]. \end{aligned} \tag{29}$$

The limit of the first term in (29) is

$$\lim_{W_0 \to 0} E\left[\gamma e^{-\gamma CE_{1,i}} \frac{dCE_{1,i}}{d\tau_\epsilon} \frac{\mathbb{I}\left(x_i^u \in \left[-\frac{W_0}{m(\tau_\epsilon)}; \frac{W_0}{m(\tau_\epsilon)}\right]\right)}{\frac{2W_0}{m(\tau_\epsilon)}}\right] = E\left[\gamma e^{-\gamma CE_{1,i}} \frac{dCE_{1,i}}{d\tau_\epsilon} \delta(x_i^u)\right].$$

Note that in the unconstrained region

$$\begin{aligned} \frac{dCE_{1,i}}{d\tau_\epsilon} &= \frac{d}{d\tau_\epsilon}\left(\frac{\gamma}{2\tau_i}(x_i^u)^2\right) \\ &= \frac{\gamma (x_i^u)^2}{2} \underbrace{\frac{d}{d\tau_\epsilon}\left(\frac{1}{\tau_i}\right)}_{<\infty} + \frac{\gamma}{\tau_i} x_i \underbrace{\frac{d}{d\tau_\epsilon}(x_i^u)}_{<\infty}. \end{aligned}$$

Therefore,

$$E\left[\gamma e^{-\gamma CE_{1,i}} \frac{dCE_{1,i}}{d\tau_\epsilon} \delta(x_i^u)\right] = 0.$$

The limit of the second term in (29) is simply

$$\begin{aligned} \lim_{W_0 \to 0} \frac{m(\tau_\epsilon)}{2W_0} \frac{d}{d\tau_\epsilon}\left(\frac{W_0}{m(\tau_\epsilon)}\right) E\left[e^{-\gamma CE_{1,i}} \delta\left(x_i^u - \frac{W_0}{m(\tau_\epsilon)}\right) + e^{-\gamma CE_{1,i}} \delta\left(x_i^u + \frac{W_0}{m(\tau_\epsilon)}\right)\right] = \\ \frac{d}{d\tau_\epsilon} \log\left(\frac{W_0}{m(\tau_\epsilon)}\right) E\left[e^{-\gamma CE_{1,i}} \delta(x_i^u)\right]. \end{aligned}$$

Combining the above results we get

$$\frac{d \log\left(-U_0^u\right)}{d\tau_\epsilon} = \frac{d}{d\tau_\epsilon} \log\left(\frac{W_0}{m(\tau_\epsilon)}\right).$$

Thus,

$$\frac{d\log\mathrm{MVI}(\tau_{\epsilon_i},\tau_\epsilon;W_0)}{d\tau_\epsilon}\to\frac{d}{d\tau_\epsilon}\log\left(\frac{W_0}{m\left(\tau_\epsilon\right)}\right)+\frac{d}{d\tau_\epsilon}log\left(\frac{\tau_i}{2\gamma\tau_{v,i}^2}\right)\text{ as }W_0\to 0.$$

The statement of the proposition then follows provided that $\tau_{\epsilon,i}$ and τ_ϵ are such that

$$\frac{d}{d\tau_\epsilon}\log\left(\frac{W_0}{m\left(\tau_\epsilon\right)}\right)>-\frac{d}{d\tau_\epsilon}log\left(\frac{\tau_i}{2\gamma\tau_{v,i}^2}\right).$$

The latter inequality is equivalent to

$$\frac{1}{2}\frac{\tau_m\tau_z}{\left(\tau_v+\beta^2\tau_z\right)^2}>\frac{\left(\tau_u+\tau_z\right)\left(2\tau_{v,i}-\tau_i\right)}{\left(\tau_{\epsilon,i}+\beta(\tau)^2\left(\tau_u+\tau_z\right)+\tau_v\right)^2}. \tag{30}$$

■

A.12 Proof of Proposition 8

We split the proof in two parts. We first show that the direct effect is negative. Then we show that the indirect effect is negative for W_0 small enough.

A.12.1 The direct effect

The proof follows a sequence of lemmas. Consider the ODE for the function $g(p)$

$$g'(p)=h(p,g(p))\equiv\frac{\pi_2(p,g(p))c_p+c_p^m}{\pi_2(p,g(p))c_\phi+c_\phi^m}>0, \tag{31}$$

where $\pi_2(p,\phi)=\Phi\left(\frac{W_0/m-X^u(p,\phi)}{\sigma_\xi}\right)-\Phi\left(\frac{-W_0/m-X^u(p,\phi)}{\sigma_\xi}\right)$.

Lemma 5. *The function $h(p,\phi)$ is an even function, i.e. $h(p,\phi)=h(-p,-\phi)$.*

Proof. We prove that $\pi_2(p,\phi)=\pi_2(-p,-\phi)$ from which the claim follows. Note that $X_u(p,\phi)=c_\phi\phi-c_pp=-X_u(-p,-\phi)$. Then

$$\begin{aligned}
\pi_2(-p,-\phi) &= \Phi\left(\frac{W_0/m - X^u(-p,-\phi)}{\sigma_\xi}\right) - \Phi\left(\frac{-W_0/m - X^u(-p,-\phi)}{\sigma_\xi}\right) \\
&= \Phi\left(\frac{-\left(-W_0/m - X^u(p,\phi)\right)}{\sigma_\xi}\right) - \Phi\left(\frac{-\left(W_0/m - X^u(p,\phi)\right)}{\sigma_\xi}\right) \\
&= 1 - \Phi\left(\frac{-W_0/m - X^u(p,\phi)}{\sigma_\xi}\right) - \left(1 - \Phi\left(\frac{W_0/m - X^u(p,\phi)}{\sigma_\xi}\right)\right) \\
&= \Phi\left(\frac{W_0/m - X^u(p,\phi)}{\sigma_\xi}\right) - \Phi\left(\frac{-W_0/m - X^u(p,\phi)}{\sigma_\xi}\right).
\end{aligned}$$

■

Lemma 6. *The solution to the ODE (31) with boundary condition $g(0) = 0$ is an odd function, i.e. is such that $g(p) + g(-p) = 0$ $\forall p$.*

Proof. We prove the statement by contradiction. Denote by $\hat{p}$ the largest p such that for all $p \leq \hat{p}$ we have $g(p) + g(-p) = 0$. Consider $g(\hat{p} + dp) = g(\hat{p}) + h\left(\hat{p}, g\left(\hat{p}\right)\right) dp$. On the other hand

$$\begin{aligned}
g(-\hat{p} - dp) &= g(-\hat{p}) - h\left(-\hat{p}, g\left(-\hat{p}\right)\right) dp \\
&= -g(\hat{p}) - h\left(-\hat{p}, -g\left(\hat{p}\right)\right) dp \\
&= -\left(g(\hat{p}) + h\left(\hat{p}, g\left(\hat{p}\right)\right) dp\right) \\
&= -g(\hat{p} + dp),
\end{aligned}$$

whcich means that $\hat{p}$ can be increased to $\hat{p} + dp$, which contradicts the fact that $\hat{p}$ is the largest. ■

Lemma 7. *The solution to the ODE (31) with boundary condition $g(0) > 0$ is such that $g(p)+g(-p) > 0$ $\forall p$. The solution to the ODE (31) with boundary condition $g(0) < 0$ is such that $g(p) + g(-p) < 0$ $\forall p$.*

Proof. We prove the first statement, the second is proved analogously. Denote the solution to the ODE (31) with a bounday condition $g(0) = 0$ by $g_0(p)$. We have shown in the lemma above that $g_0(p)$ is odd.

Any solution to the ODE (31) with a boundary condition $g(0) > 0$ is a function $g(p)$ that is always above the function $g_0(p)$. Hence

$$g(p) + g(-p) > g_0(p) + g_0(-p) = 0.$$

■

Lemma 8. *In equilibrium, $g(0) > 0$.*

Proof. The aggregate demand of investors is given by

$$X(p,\phi) = -\pi_1(p,\phi)\frac{W_0}{m} + \pi_3(p,\phi)\frac{W_0}{m} + \pi_2(p,\phi)X^u(p,\phi) +$$
$$\sigma_\xi \left(\Phi' \left(\frac{-W_0/m - X^u(p,\phi)}{\sigma_\xi} \right) - \Phi' \left(\frac{W_0/m - X^u(p,\phi)}{\sigma_\xi} \right) \right).$$

It can be shown that $X(0,0) = 0$. Denote the aggregate demand $X_{agg}(p,\phi) = X(p,\phi) + c_\phi \phi - c_p p$. The value $g(0)$ is ϕ^* such that $X_{agg}(0,\phi^*) = 1$. Since $X_{agg}(0,0) = 0$ and $\frac{\partial}{\partial \phi} X_{agg}(p,\phi) = c_\phi^m + \pi_2 c_\phi > 0$, we have that $g(0) > 0$. ■

Lemma 9. *Suppose that $f(x)$ is a positive even function and $l(x)$ is such that $l(x) + l(-x) < 0$. Then $\int_{-\infty}^{\infty} f(x) l(x) dx < 0$.*

Proof. Given symmetry the integral can be written as

$$\int_0^\infty f(x) \left(l(x) + l(-x) \right) dx > 0.$$

■

Lemma 10. *$X^u(p, g(p)) + X^u(-p, g(-p)) < 0$.*

Proof. $X^u(-p, g(-p)) = -\left(c_p p + c_\phi g(-p) \right) < -\left(c_p p - c_\phi g(p) \right) = -X^u(p, g(p))$. ■

Lemma 11. *For any $a > 0$ the function $k(x) = \Phi(x+a) - \Phi(x-a)$ decreases (increases) in x for $x > 0 (< 0)$. Moreover suppose that $x + y < 0$ and $x < 0$, then $k(x) < k(y)$.*

Proof. By symmetry $\Phi(x+a) - \Phi(x-a)$ attains unique maximum at $x = 0$, hence it decreases to the right of it and increases to the left of it. The second claim follows from symmetry of $k(x)$. ■

Lemma 12. *$\pi_2(p, g(p)) > \pi_2(-p, g(-p))$ for p such that $X^u(p, g(p)) > 0$. Vice versa, $\pi_2(p, g(p)) < \pi_2(-p, g(-p))$ for p such that $X^u(p, g(p)) < 0$.*

Proof. Consider the case $X^u(p, g(p)) > 0$. In that case $-X^u(-p, g(-p)) > X^u(p, g(p)) > 0$ (Lemma 10)

$$\pi_2(-p, g(-p)) = \Phi\left(\frac{W_0/m - X^u(-p, g(-p))}{\sigma_\xi} \right) - \Phi\left(\frac{-W_0/m - X^u(-p, g(-p))}{\sigma_\xi} \right)$$

Applying Lemma 11 we get

$$\begin{aligned} \pi_2(-p, g(-p)) &< \Phi\left(\frac{W_0/m + X^u(p, g(p))}{\sigma_\xi} \right) - \Phi\left(\frac{-W_0/m - X^u(p, g(p))}{\sigma_\xi} \right) \\ &= \pi_2(p, g(p)) \end{aligned}$$

Consider the case $X^u(p, g(p)) < 0$. In that case $X^u(p, g(p)) + X^u(-p, g(-p)) < 0$ (Lemma 10). Applying Lemma 11 (second claim) we get

$$\begin{aligned}\pi_2(-p, g(-p)) &< \Phi\left(\frac{W_0/m + X^u(p, g(p))}{\sigma_\xi}\right) - \Phi\left(\frac{-W_0/m - X^u(p, g(p))}{\sigma_\xi}\right)\\ &= \pi_2(p, g(p))\end{aligned}$$

■

Lemma 13. $\pi_3(p, g(p)) - \pi_1(p, g(p)) + \pi_3(-p, g(-p)) - \pi_1(-p, g(-p)) < 0.$

Proof. One can write

$$\begin{aligned}\pi_3(-p, g(-p)) - \pi_1(-p, g(-p)) &= 1 - \Phi\left(\frac{W_0/m - X^u(-p, g(-p))}{\sigma_\xi}\right) - \Phi\left(\frac{-W_0/m - X^u(-p, g(-p))}{\sigma_\xi}\right)\\ &= \Phi\left(\frac{-W_0/m + X^u(-p, g(-p))}{\sigma_\xi}\right) - \Phi\left(\frac{-W_0/m - X^u(-p, g(-p))}{\sigma_\xi}\right)\\ &< \Phi\left(\frac{-W_0/m - X^u(p, g(p))}{\sigma_\xi}\right) - \Phi\left(\frac{-W_0/m + X^u(p, g(p))}{\sigma_\xi}\right)\\ &= -\left(\pi_3(p, g(p)) - \pi_1(p, g(p))\right)\end{aligned}$$

■

Lemma 14. $\frac{\partial g(p;W)}{\partial W} + \frac{\partial g(-p;W)}{\partial W} < 0$ *and* $\frac{\partial l(\phi;W)}{\partial W} + \frac{\partial l(-\phi;W)}{\partial W} > 00$

Proof. Write

$$\frac{\partial g(p;W)}{\partial W} = -\frac{(\pi_3 - \pi_1)/m}{c_\phi^m + \pi_2 c_\phi} \equiv \frac{n(p)}{d(p)},$$

where

$$n(p) = -\left(\pi_3(p, g(p)) - \pi_1(p, g(p))\right)$$

and

$$d(p) = c_\phi^m + \pi_2(p, g(p))c_\phi.$$

We have that

$$\frac{n(p)}{d(p)} + \frac{n(-p)}{d(-p)} = \frac{\overbrace{n(-p) + n(p)}^{>0}}{d(-p)} - n(p)\left(\frac{d(p) - d(-p)}{d(-p)d(p)}\right)$$

Note also that $n(p)$ is < 0 (> 0) iff $X(p, g(p)) > 0$ (< 0), which by Lemma 13 implies that $\text{sign}(n(p)) = -\text{sign}(d(p) - d(-p))$. Therefore $n(p)\left(d(p) - d(-p)\right) < 0$. The second statement can be proven analogously. ■

Proposition 11. $\frac{\partial}{\partial W_0}\overline{rp} < 0.$

Proof. Using the Lemma 14, we get:

$$\frac{\partial}{\partial W_0}\overline{rp} = -\frac{\partial}{\partial W_0}E[p] = -\int_0^\infty \frac{\partial}{\partial W_0}(l(\phi)+l(-\phi))f_\phi(\phi)d\phi < 0.$$

■

A.12.2 The indirect effect

Proposition 12. *The indirect effect is $\frac{\partial \overline{rp}}{\partial \tau_\epsilon}\frac{d\tau_\epsilon}{dW}$ is negative for W small enough.*

Proof. The risk premium can be written as

$$\overline{rp} = \frac{Var[f|p]}{\gamma_m}(1-E[X]).$$

For $\frac{\partial \overline{rp}}{\partial \tau_\epsilon}$ we write

$$\frac{\partial \overline{rp}}{\partial \tau_\epsilon} = \underbrace{\frac{(1-E[X])}{\gamma_m}}_{\to 1/\gamma_m}\underbrace{\frac{\partial Var[f|p]}{\partial \tau_\epsilon}}_{<0} - \frac{Var[f|p]}{\gamma_m}\frac{\partial E[X]}{\partial \tau_\epsilon}.$$

It follows from Lemma 4 that $\lim_{W_0\to 0}\frac{\partial E[X]}{\partial \tau_\epsilon} = 0$, thus:

$$\lim_{W_0\to 0}\frac{\partial \overline{rp}}{\partial \tau_\epsilon} = \frac{\partial \overline{rp}}{\partial \tau_\epsilon} = \frac{1}{\gamma_m}\underbrace{\frac{\partial Var[f|p]}{\partial \tau_\epsilon}}_{<0}.$$

The indirect effect is $\frac{\partial \overline{rp}}{\partial \tau_\epsilon}\frac{d\tau_\epsilon}{dW_0}$ and we already know that for small enough W_0, $\frac{d\tau_\epsilon}{dW_0} > 0$. ■

A.13 Proof of Proposition 9

Proof. Using the law of total variance we write

$$Var(f-p) = Var(f-p|p) + Var\left(E\left[f-p|p\right]\right).$$

For the first term, we already know that

$$\frac{d}{dW_0}Var(f-p|p) = \frac{d}{dW_0}Var(f|p) < 0,$$

for W_0 small enough. For the second term, we write

$$E\left[f-p|p\right] = \gamma_m Var[f|p](1-X).$$

Therefore,

$$Var\left(E\left[f-p|p\right]\right)=\left(\gamma_m Var[f|p]\right)^2 Var\left(X\right).$$

Taking the derivative,

$$\frac{d}{dW_0}Var\left(E\left[f-p|p\right]\right)=\underbrace{\frac{d}{dW_0}\left(\gamma_m Var[f|p]\right)^2}_{<0} Var\left(1-X\right)+\left(\gamma_m Var[f|p]\right)^2\left(\frac{d}{dW_0}Var(X)\right).$$

Since $\lim_{W_0\to 0} Var(1-X)=0$, we have that

$$\lim_{W_0\to 0}\frac{d}{dW_0}\left(\gamma_m Var[f|p]\right)^2 Var\left(1-X\right)=0.$$

For $\frac{d}{dW_0}Var(X)$ note that

$$\lim_{W_0\to 0} Var(X)=0,$$

Therefore,

$$\lim_{W_0\to 0}\frac{d}{dW_0}Var(X)=\lim_{W_0\to 0}\frac{Var(X)-\lim_{W_0\to 0}Var(X)}{W_0}=\lim_{W_0\to 0}\frac{Var(X)}{W_0}.$$

Note that

$$0\leq Var(X)\leq E[X^2]\leq\frac{W_0^2}{m^2},$$

Therefore,

$$0\leq\lim_{W_0\to 0}\frac{Var(X)}{W_0}\leq\lim_{W_0\to 0}\frac{W_0}{m^2}=0.$$

Combining all the above we have

$$\lim_{W_0\to 0}\frac{d}{dW_0}Var\left(E\left[f-p|p\right]\right)=0.$$

so that

$$\lim_{W_0\to 0}\frac{d\mathcal{V}}{dW_0}=\frac{d}{dW_0}Var(f|p)<0.$$

■

A.14 Proof of Proposition 10

Proof. By definition $SR=\frac{\overline{rp}}{\sqrt{\mathcal{V}}}$. Therefore,

$$\frac{dSR}{dW_0}=\frac{\frac{d\overline{rp}}{dW_0}\sqrt{\mathcal{V}}+\frac{1}{2\sqrt{\mathcal{V}}}\frac{d\mathcal{V}}{dW_0}\overline{rp}}{\mathcal{V}}.$$

Note that

$$\lim_{W_0 \to 0} \overline{rp} = \lim_{W_0 \to 0} \frac{Var[f|p]}{\gamma_m}(1 - E[X]) = \frac{Var[f|p]}{\gamma_m}.$$

$$\lim_{W_0 \to 0} \mathcal{V} = Var[f|p].$$

For the derivatives, we have shown that

$$\frac{d\overline{rp}}{dW_0} < \frac{\partial \overline{rp}}{\partial \tau_\epsilon} \frac{d\tau_\epsilon}{dW_0}.$$

the inequality is true since there is also a direct effect, which is negative. Moreover, we have shown in the proof of Proposition 8 that

$$\lim_{W_0 \to 0} \frac{\partial \overline{rp}}{\partial \tau_\epsilon} = \frac{1}{\gamma_m} \frac{dVar[f|p]}{d\tau_\epsilon}.$$

Similarly, we have shown in the proof of Proposition 9 that

$$\begin{aligned} \lim_{W_0 \to 0} \frac{d\mathcal{V}}{dW_0} &= \lim_{W_0 \to 0} \frac{d}{dW_0} Var(f|p) \\ &= \lim_{W_0 \to 0} \frac{dVar[f|p]}{d\tau_\epsilon} \frac{d\tau_\epsilon}{dW_0}. \end{aligned}$$

Combining all of the above we get

$$\lim_{W_0 \to 0} \frac{dSR}{dW_0} < \underbrace{\frac{d\tau_\epsilon}{dW_0}}_{>0} \underbrace{\frac{dVar[f|p]}{d\tau_\epsilon}}_{<0} \underbrace{\frac{\frac{1}{\gamma_m}\sqrt{Var[f|p]} + \frac{1}{2\sqrt{Var[f|p]}} \frac{Var[f|p]}{\gamma_m}}{Var[f|p]}}_{>0} < 0$$

■

B A Setup with Noise Traders

In our baseline setting, we assume that noise in prices comes from aggregate hedging needs. It contributes to the tractability of our baseline setting due to the irrelevance result: given exogenous private information, informational efficiency of prices is independent of the constraints investors face. Thus, the informational efficiency can be found by solving for equilibrium in the unconstrained economy. What matters for the irrelevance result is that constraints affect not only the informed demand, but also the demand from hedging needs of the investors.

In this Appendix, we eliminate the hedging needs of investors and assume that the noise in prices comes from classic noise traders. We assume that the noise is exogenous, in particular, it is not affected by constraints. The question we answer here is "will the information spiral continue to hold in this setting?". We argue that the answer is positive. Moreover, the setting in this section highlights a novel channel for the interaction between constraints and information efficiency. We call this channel, the information aggregation channel: as constraints tighten, noise is unaffected by constraints whereas informed trading is more constrained; thus, the price informativeness decreases, even with exogenous information.

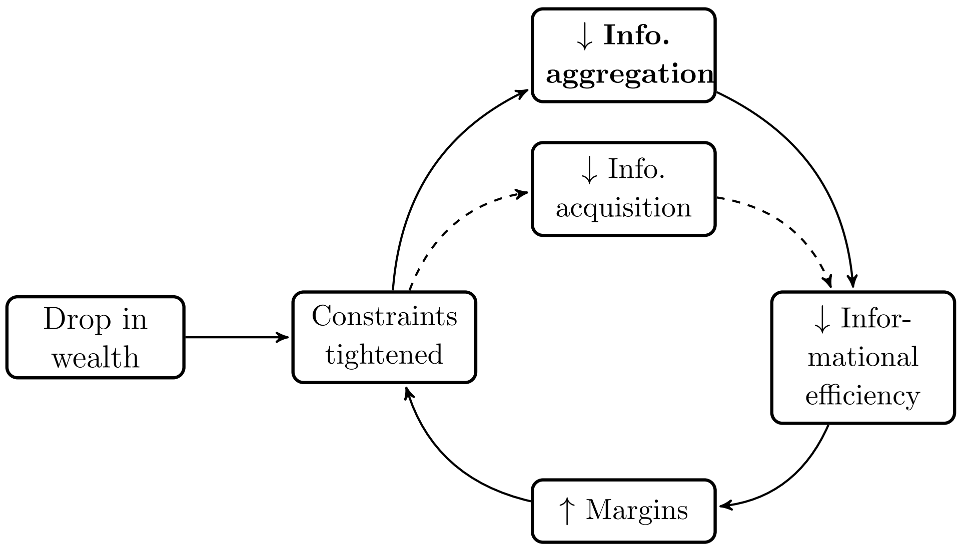
In the baseline model, because of irrelevance result, there was no information aggregation channel. In order to focus on the new mechanism, in this section we consider a setting with exogenous information (shutting down the information acquisition channel).[24] We show that an exogenous shock that tightens constraints of investors leads to lower informational efficiency. In response, financiers set higher margins, further tightening investors' constraints. This information spiral mechanism is similar to the one studied in the baseline model, with the difference that it acts through the information aggregation function of price. We illustrate this new channel in Figure 5.

Setup

Consider a two-date model with $t = \{1, 2\}$. Suppose the payoff of the risky asset at $t = 2$ is $f = v + \theta$ where v is the learnable component and θ is the unlearnable component. We assume that fundamental v is drawn from an improper uniform distribution, whereas $\theta \sim \mathcal{N}\left(0, \tau_\theta^{-1}\right)$. There are three classes of agents in the economy: investors, noise traders, and a market maker. There is a unit mass of investors with constant absolute risk aversion γ who have wealth W_0 and observe the same signal $s = v + \epsilon, \epsilon \sim \mathcal{N}\left(0, \tau_\epsilon^{-1}\right)$. Among these investors, fraction $\lambda \in (0, 1)$ are subject to margin constraints while the remaining fraction $(1 - \lambda)$ are unconstrained. As in the baseline model, we first study fixed margins m for both long and short positions and later study value-at-risk based margins. The noise traders submit exogenous liquidity demands $u \sim \mathcal{N}\left(0, \tau_u^{-1}\right)$ and finally, the uninformed market maker

[24] It is also worth noting that information acquisition is much less tractable in the setting we consider in this section.

Figure 5: Amplification mechanism



is unconstrained and risk-neutral, i.e., $\gamma_m = 0$.

Financial Market Equilibrium

Denote $\tau = \frac{\tau_\epsilon \tau_\theta}{\tau_\epsilon + \tau_\theta}$. The optimal demand for an unconstrained investor is given by

$$x_{i,u} = \frac{\tau}{\gamma}(s-p).$$

For a constrained investor, his demand is

$$x_{i,c} = \begin{cases} x_{i,u}, & \text{if } -\frac{W_0}{m} < \frac{\tau}{\gamma}(s-p) < \frac{W_0}{m}, \\ -\frac{W_0}{m}, & \text{if } \frac{\tau}{\gamma}(s-p) < -\frac{W_0}{m}, \\ +\frac{W_0}{m}, & \text{if } \frac{\tau}{\gamma}(s-p) > \frac{W_0}{m}. \end{cases}$$

Hence, the aggregate demand of investors is

$$X \equiv \lambda x_{i,c} + (1-\lambda) x_{i,u} + u = \begin{cases} \frac{\tau}{\gamma}(s-p) + u & \text{, if } s-p \in \left[-\frac{W_0\gamma}{m\tau}, \frac{W_0\gamma}{m\tau}\right]; \\ -\frac{W_0}{m}\lambda + \frac{\tau}{\gamma}(s-p)(1-\lambda) + u & \text{, if } s-p < -\frac{W_0\gamma}{m\tau}; \\ \frac{W_0}{m}\lambda + \frac{\tau}{\gamma}(s-p)(1-\lambda) + u & \text{, if } s-p > \frac{W_0\gamma}{m\tau}. \end{cases}$$

The market maker's inferred information from price s_p is an affine transformation of the intercept of the above aggregate demand:

$$s_p = \begin{cases} s + \frac{\gamma}{\tau} u & , \text{if } s - p \in \left[-\frac{W_0 \gamma}{m\tau}, \frac{W_0 \gamma}{m\tau} \right]; \\ s + \frac{\gamma}{(1-\lambda)\tau} u & , \text{if } s - p < -\frac{W_0 \gamma}{m\tau}; \\ s + \frac{\gamma}{(1-\lambda)\tau} u & , \text{if } s - p > \frac{W_0 \gamma}{m\tau}. \end{cases}$$

Given that market maker is risk-neutral, she sets the semi-strong efficient price $p = \mathbb{E}\left[v|s_p\right]$.

Proposition 13. *There exists a piecewise linear REE with price function given by*

$$p = \begin{cases} s + \frac{\gamma}{\tau} u, & \text{if } \ u \in \left[-\frac{W_0}{m}, +\frac{W_0}{m} \right]; \\ s + \frac{\gamma}{\tau(1-\lambda)} \left(u - \lambda \frac{W_0}{m} \right), & \text{if } \ u > \frac{W_0}{m}; \\ s + \frac{\gamma}{\tau(1-\lambda)} \left(u + \lambda \frac{W_0}{m} \right), & \text{if } \ u < -\frac{W_0}{m}. \end{cases}$$

In the piecewise linear REE, the price function takes different forms in different states of the world. Consider $u \in \left[-\frac{W_0}{m}, +\frac{W_0}{m} \right]$. In this case, prices and the price is same as in the economy without constraints. However, if $u < -\frac{W_0}{m}$, lower constraint binds for informed investors and their demand is information insensitive (a constant $-\frac{W_0}{m}$). In this case, the aggregate demand that the market maker observes is given by

$$X = \underbrace{-\frac{W_0}{m}\lambda}_{\text{constrained demand}} + \underbrace{\frac{\tau}{\gamma}(s-p)(1-\lambda)}_{\text{unconstrained demand}} + \underbrace{u}_{\text{noise}}.$$

This implies that the aggregate demand (and hence the market clearing price) is more sensitive to noise trading in this region. Hence, informational efficiency is lower in this region (compared to the economy without constraints). Similar logic follows in the case of $u > \frac{W_0}{m}$. As a result, the market maker's posterior is not normal and is characterized by the conditional probability density function:

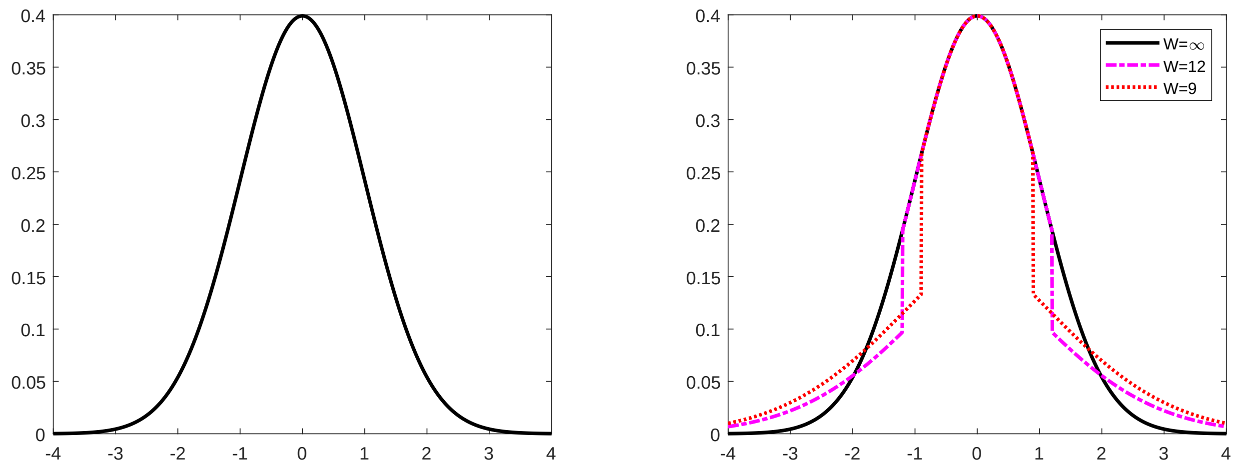
$$f_{s|p} = \begin{cases} \frac{\tau}{\gamma}\sqrt{\tau_u}\phi\left(\frac{\tau}{\gamma}\sqrt{\tau_u}(s-p)\right), & \text{if } s - p \in \left[-\frac{W_0 \gamma}{m\tau}, \frac{W_0 \gamma}{m\tau} \right]; \\ (1-\lambda)\frac{\tau}{\gamma}\sqrt{\tau_u}\phi\left((1-\lambda)\frac{\tau}{\gamma}\sqrt{\tau_u}(s-p) - \lambda\sqrt{\tau_u}\frac{W_0}{m}\right), & \text{if } s - p < -\frac{W_0 \gamma}{m\tau}; \\ (1-\lambda)\frac{\tau}{\gamma}\sqrt{\tau_u}\phi\left((1-\lambda)\frac{\tau}{\gamma}\sqrt{\tau_u}(s-p) + \lambda\sqrt{\tau_u}\frac{W_0}{m}\right), & \text{if } s - p > \frac{W_0 \gamma}{m\tau} \end{cases}$$

where $\phi(.)$ denotes the pdf of a standard normal distribution.

Figure 6 illustrates the distribution of s conditional on the price with and without the constraints. The left panel shows the distribution without constraints and it is Gaussian. The right panel shows the distribution with constraints and it is not Gaussian. The various lines show the distribution when the agents face different constraints. For the states of the world in which constraints do not bind (i.e., center region of the distribution), the posterior variance is the same as in the unconstrained case. For the states in which constraints bind for some agents (tails of the distribution), there

Figure 6: Posterior distribution of s conditional on p

The left panel shows the distribution in a model without constraints. The right panel shows the distribution in a model with constraints. Other parameters are: $\tau_u = 1; \lambda = 0.9; \gamma = 3$; $\alpha = 0.99$.



(a) No constraints

(b) Constraints tightening

is less informed trade in the market and hence the posterior variance is higher and leads to fatter tails. We therefore arrive at the following proposition about the interaction between constraints and informational efficiency.

Proposition 14. *If constraints become tighter for all investors, i.e. if* $\frac{W_0}{m}$ *decreases: (1) price informativeness (defined as the inverse of the conditional variance of the payoff) decreases (2) conditional distribution of losses on short and long positions becomes heavier-tailed, i.e. probability of a loss greater than* x *on a long position (given by* $Pr(p - f > x|p)$*) and probability of a loss greater than* x *on a short position (given by* $Pr(f - p > x|p)$*) increase for any* $x > 0$.

The proposition above demonstrates that the first part of our informational spiral holds in the setting with noise traders. As constraints tighten, informational efficiency falls. As a result, conditional distribution of losses becomes heavier-tailed. We show in the section below that these heavier tails feed back into higher margins, closing the loop in our spiral.

Value-at-risk based margins

Up to now, we assumed that margins are exogenously fixed. Next, as in the baseline model, we study how the price (and its informational efficiency) affects margins when they are set to control financiers' value-at-risk. VaR-based margins are described in (10) and we have the following result.

Proposition 15. *Suppose margins are value-at-risk based. Then* $m^+ = m^- = m$ *and margins solve*

$$1 - \alpha = E_\eta \left[Pr\left(p - v > m + \eta | p, \eta\right)\right]$$

where $\eta \equiv \theta - \epsilon$ *and* $\eta \sim N(0, \tau_\eta^{-1})$*. Both* τ_η *and* $E_\eta \left[Pr\left(p - v > m + \eta | p, \eta\right)\right]$ *are explicitly solved in the proof. Moreover, as the conditional distribution of losses becomes heavier-tailed, margins become higher.*

Proposition 15 shows that heavier tails of conditional loss distribution imply higher margins set by financiers. Combining the results of Propositions 15 and 14, we get that the following version of information spiral holds. As constraints tighten, informational efficiency drops and the distribution of losses becomes heavier-tailed. This implies higher margins, feeding back in to tighter constraints. This is represented in the Figure 5. One consequence of the information spiral highlighted in this section is the following complementarity.

Corollary 4. *Suppose that margins are increased for all investors except investor* i*. Then informational efficiency drops and the distribution of losses becomes heavier-tailed. As a result investor* i *will face higher margins as well.*

Figure 7 illustrates the above proposition. The various lines in panel (b) represent the function $f(m)$: the margin that an investor of interest faces, given that the margins faced by other investors is m. The fact that the function $f(m)$ is upward-sloping signifies the complementarity outlined in the proposition above. Different lines corresponds to the functions $f(m)$ for different levels of wealth. Note that for some wealth levels there could be multiple equilibria (since $f(m)$ crosses the 45-degree line in multiple points). We conclude Appendix B by arguing that the analysis provided here highlights that the economic mechanism of the informational spiral is present even in a setting with standard noise traders.

Proofs for Appendix B

Proof. (Propostion 13) Let $A = \frac{W_0}{m}$. Conjecture that a piecewise linear REE can be written with a price function of the form

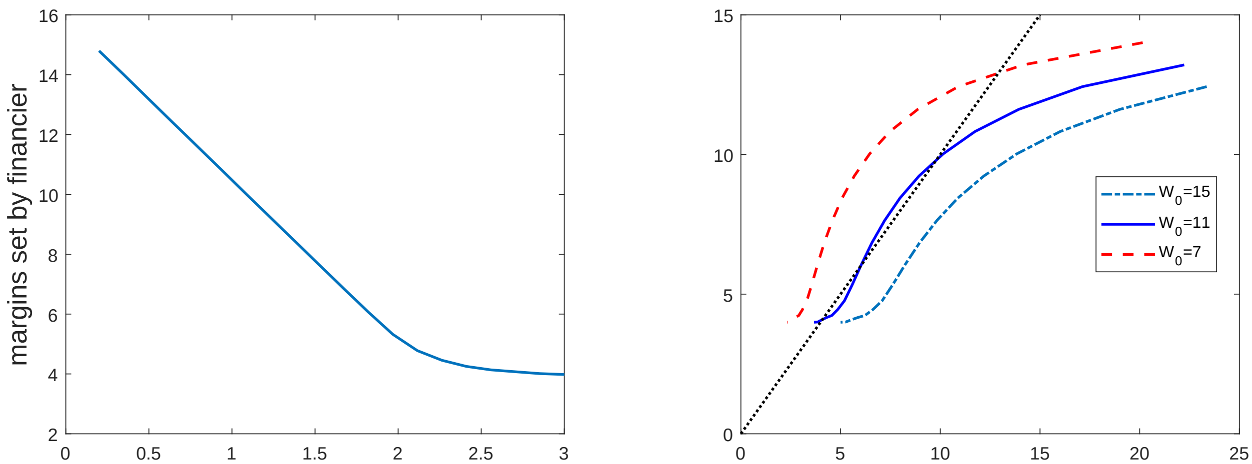
$$p = \begin{cases} s + \frac{\gamma}{\tau}\left(u - u_0\right), & \text{if } \; u \in [u_0 - A, u_0 + A]\,; \\ s + \frac{\gamma}{\tau(1-\lambda)}\left(u - u_0 - \lambda A\right), & \text{if } \; u > u_0 + A; \\ s + \frac{\gamma}{\tau(1-\lambda)}\left(u - u_0 + \lambda A\right), & \text{if } \; u < u_0 - A. \end{cases}$$

The constant u_0 is pinned down by the equilibrium condition $p = E[v|p]$ which implies

$$E[p] = E[v].$$

Figure 7: VaR-based margins set by a financier

The figure shows the VaR-based margins set by a financier. The left panel plots the margins as a function of investors' wealth W_0. The right panel shows the margins as a function of margins set by other financiers for different levels of wealth as indicated in the legend Other parameters are: $\tau_u = 1; \lambda = 0.9; \gamma = 3;\ \alpha = 0.99$.



(a) as a function of investors' initial wealth W_0 (b) as a function of other financiers' margins

The above condition gives a unique solution $u_0 = 0$. It can be then verified that the condition $p = E[v|p]$ holds with $u_0 = 0$. ■

Proof. (Proposition 14) *Part 1 (Tightening of constraints leads to increase in conditional variance).* Since the price is efficient, one can write

$$Var\,(v|p) = E\left[(v - E\,[v|p])^2\,|p\right] = E\left[(v-p)^2\,|p\right].$$

We can further expand the above as follows:

$$E\left[(v-p)^2\,|p\right] = E\left[(v-p)^2\,1\,[u \in [-A, A]]\,|p\right] + E\left[(v-p)^2\,1\,[u > A]\,|p\right] + E\left[(v-p)^2\,1\,[u < -A]\,|p\right].$$

Now we expand the terms above

$$\begin{aligned} E\left[(v-p)^2\,1\,[u \in [-A, A]]\,|p\right] &= E\left[\left(\epsilon + \frac{\gamma}{\tau}u\right)^2 1\,[u \in [-A, A]]\,|p = v + \epsilon + \frac{\gamma}{\tau}u\right] \\ &= E\left[\left(\epsilon + \frac{\gamma}{\tau}u\right)^2 1\,[u \in [-A, A]]\right] \end{aligned}$$

The last equality is true because $\tau_v = 0$, and hence price is infinitely noisy signal of $\epsilon + \frac{\gamma}{\tau}u$.

Proceeding similarly we get

$$E\left[(v-p)^2 1\left[u>A\right]|p\right]=E\left[\left(\epsilon+\frac{\gamma}{\tau(1-\lambda)}\left(u-\lambda A\right)\right)^2 1\left[u>A\right]\right]$$

$$E\left[(v-p)^2 1\left[u<-A\right]|p\right]=E\left[\left(\epsilon+\frac{\gamma}{\tau(1-\lambda)}\left(u+\lambda A\right)\right)^2 1\left[u<-A\right]\right].$$

To derive the sign of $\frac{\partial}{\partial A}E\left[(v-p)^2|p\right]$ we note that due to symmetry we can evaluate this sign conditional on $\epsilon=0$. This is true since the effect of a decrease in the upper constraint A on $E\left[(v-p)^2|p,\epsilon\right]$ will be the opposite of the effect of an increase in the lower constraint $-A$ on $E\left[(v-p)^2|p,-\epsilon\right]$, and they will therefore cancel out once we integrate with respect to ϵ.

Now it is easy to see that

$$\begin{aligned}E\left[(v-p)^2 1\left[u\in[-A,A]\right]|p,\epsilon=0\right]=&\ E\left[\left(\frac{\gamma}{\tau}u\right)^2 1\left[u\in[-A,A]\right]\right]\\&+E\left[\left(\frac{\gamma}{\tau(1-\lambda)}\left(u-\lambda A\right)\right)^2 1\left[u>A\right]\right]\\&+E\left[\left(\frac{\gamma}{\tau(1-\lambda)}\left(u+\lambda A\right)\right)^2 1\left[u<-A\right]\right]\end{aligned}$$

increases as A decreases.

Part 2 (Distribution of losses). Let $\eta=\theta-\epsilon$. Then $\eta\sim\mathcal{N}\left(0,\tau_\theta^{-1}+\tau_\epsilon^{-1}\right)$. We first derive the distribution of losses conditional on p and η. We split it into three parts:

$$\begin{aligned}Pr\left(p-s>x+\eta|p,\eta\right)=&Pr\left(p-s>x+\eta|u>A\right)\cdot Pr\left(u>A\right)\\&+Pr\left(p-s>x+\eta|-A<u<A\right)\cdot Pr\left(-A<u<A\right)\\&+Pr\left(p-s>x+\eta|u<-A\right)\cdot Pr\left(u<-A\right).\end{aligned}$$

For the first part we can write

$$\begin{aligned}Pr\left(p-s>x+\eta|u>A\right)\cdot Pr\left(u>A\right)=&Pr\left(p-s>x+\eta,u>A\right)\\=&Pr\left(u>\max\left\{\frac{(1-\lambda)(x+\eta)}{\gamma\sigma_\theta^2+\lambda A},A\right\}\right).\end{aligned}$$

Proceeding analogously with the other two term one can obtain

$$Pr\left(p - s > x + \eta | p, \eta\right) = \begin{cases} 1 - \Phi\left(\frac{1}{\sigma_u}\left(\frac{(1-\lambda)\tau(x+\eta)}{\gamma} + \lambda a\right)\right), & \text{if } x + \eta > \frac{\gamma}{\tau}a; \\ 1 - \Phi\left(\frac{1}{\sigma_u}\frac{(x+\eta)\tau}{\gamma}\right), & \text{if } -\frac{\gamma}{\tau}a \leq x + \eta \leq \frac{\gamma}{\tau}a \\ 1 - \Phi\left(\frac{1}{\sigma_u}\left(\frac{(1-\lambda)\tau(x+\eta)}{\gamma} - \lambda a\right)\right) & otherwise \end{cases}$$

Consider $\eta = y > 0$. It can be seen from above that

$$-\frac{\partial}{\partial A} Pr\left(p - s > x + \eta | p, \eta = y\right) > \frac{\partial}{\partial A} Pr\left(p - s > x + \eta | p, \eta = -y\right) > 0.$$

Given the symmetry of the distribution of η this implies

$$\frac{\partial}{\partial A} Pr(p - s > x + \eta | p) = E_\eta\left[\frac{\partial}{\partial A} Pr\left(p - s > x + \eta | p, \eta = y\right)\right] < 0.$$

Thus if A decreases, conditional distribution of losses becomes heavier-tailed. ■

Proof. (Proposition 15) Conjecture that $m^+ = m^- = m$. For long positions, financier sets margins such that $Pr\left(p - f > m | p\right) = 1 - \alpha$. Note that

$$\begin{aligned} \Pr\left(p - f > m | p\right) &= \Pr\left(p - s + \epsilon - \theta > m | p\right) \\ &= E_\eta\left[\Pr\left(p - s > m + \eta | p, \eta\right)\right] \end{aligned}$$

The expression for $\Pr\left(p - s > m + \eta | p, \eta\right)$ was derived in the proof of the Proposition 14. The margins are given by $1 - \alpha$ quantile of the conditional distribution of losses derived in Proposition 14. Therefore, as this distribution become heavier-tailed, the margins increase. ■

C Investors with initial endowments of risky asset

In the baseline model, we assume that investors have cash as initial endowment. In this section we assume that investors are endowed with y_0 units of risky asset. As we show below, the functional form of the constraints $a(p)$ and $b(p)$ will change in that case. In particular, the upper constraint $b(p)$ will resemble the borrowing constraint in Yuan (2005). The main result of this section is that the informational spiral will still be present in this economy with different form of constraints.

Setup

The model is identical to the model in Section 2.1, except that the investors are initially endowed with $y_0 > 0$ units of risky asset. Moreover, we assume that they sell these assets to relax their constraints.[25] Investors wealth at date 2 is thus given by

$$W_i = y_0 p + x_i(v + \theta - p) + e_i \theta.$$

In this section we assume that the market maker is risk-neutral. We now derive the functional form of constraints $a(p)$ and $b(p)$. As before, we assume that to build a long position in the risky asset, an investor can borrow from a financier at the risk-free rate, but he has to pledge a cash margin of $m^+ \geq 0$ per unit of asset to the financier as collateral. The investor can similarly establish a short position by providing, as collateral, a cash margin of m^- per unit of asset. The maximum positions they can take are constrained by the amount of cash C they have:

$$m^-[x_i]^- + m^+[x_i]^+ \leq C.$$

The difference compared to baseline seeting is that the amount of cash now depends on p:

$$C = y_0 p.$$

We can derive portfolio constraints as follows:

$$b(p) = \left[\frac{y_0 p}{m^+}\right]^+, \;\; a(p) = -\left[\frac{y_0 p}{m^-}\right]^+, \tag{32}$$

[25] Strictly speaking, selling the assets relaxes constraints and hence is always optimal only if price is greater than both margins. This is always true in practice because otherwise the agents could take larger positions without the financiers. To have this reasonable property in the model, one can choose a high enough mean payoff of the asset to ensure that the the price is almost always greater than margins. This is because changing the mean payoff does not affect the margins but changes the level of price.

Where in the above we accounted for the fact that by definition the maximum long position $b(p)$ has to be positive, whereas maximum short position $a(p)$ has to be negative. We note the similarity between the upper constraint $b(p)$ and the linear borrowing constraint in Yuan (2005).

Equilibrium and the informational spiral

The proposition below characterizes the equilibrium in the financial market.

Proposition 16. *Suppose that investors have identical signal precisions τ_ϵ and face constraints as described above, then there exists a unique linear equilibrium in which informational efficiency $\beta = \beta^u$ and the function $g(p) = \frac{\beta^2\tau_z+\tau_v}{\beta^2\tau_z} \cdot p$.*

Proof. Follows from Proposition 2 by taking the limit $\gamma_m \to 0$. ■

We now establish the two parts of the informational spiral are present in the alternative setup considered in this Appendix. First, we establish that as constraints tighten, investors' incentives to acquire information decrease. in the Proposition below.

Proposition 17. *If y_0 drops, investors constraints become tighter and their marginal value of information decreases.*

Proof. The fact that constraints tighten as y_0 drops follows directly from (32). The rest of the proof follows from Proposition 4. ■

As in the section 3.2 we assume that each financier sets her margin in order to control her value-at-risk. As we show in the proposition below, the second part of the informational spiral continues to hold: as informational efficiency drops, margins increase and hence constraints tighten.

Proposition 18. *If portfolio constraints are of the form of margin requirements, and if margins are value-at-risk based, then there exists a unique generalized linear equilibrium in which the function $g(p)$ is as characterized by Proposition 16 and the equilibrium margins are given by $m^+ = m^- = \Phi^{-1}(\alpha)\sqrt{Var[f-p|p]} = \Phi^{-1}(\alpha)\sqrt{(\tau_v+\beta^2\tau_z)^{-1}+\tau_\theta^{-1}}$. Consequently, for a given investors' wealth y_0, if informational efficiency (β) decreases then the margins (m^+ and m^-) both increase. This implies that the lower constraint (i.e., $a(p)$) increases and the upper constraint (i.e., $b(p)$) decreases. In other words, as informational efficiency declines, constraints become tighter.*

Proof. Follows from Propositions 6 and 16 ■

Combining the results of Propositions 17 and 18 we get the a version of the informational spiral with risky asset endowment y_0 as wealth.

D Risk-neutral measure and microfoundation for VaR- and VaR^P- margins

Investors in our model borrow from financiers, who impose a risk-based margin per unit of risky asset invested (long or short). While fully endogenizing the risk-based margin as an optimal contract is beyond the scope of our paper, in this Appendix we attempt to describe the problem and frictions faced by the financier to rationalize the use of risk-based margin. We start by defining the risk-neutral measure, which is used to compute the value-at-risk in our baseline specification.

Risk-neutral measure

Consider an unconstrained investor who solves

$$\max_x E[-\exp(-\gamma x(f-p))|p].$$

The first-order condition implies

$$p = E\left[\frac{\exp(-\gamma x^*(f-p))}{E[\exp(-\gamma x^*(f-p))|p]} f|p\right] \tag{33}$$

where x^* denotes the investor's optimal holding. Define a random variable $Z = \frac{\exp(-\gamma x^*(f-p))}{E[\exp(-\gamma x^*(f-p))|p]}$ which is a Radon-Nikodym derivative that defines the risk-neutral measure. Substituting the optimal demand of an uninformed investor $x^* = \frac{E[f|p]-p}{\gamma Var[f|p]}$, the Radon-Nikodym derivative can be written as

$$Z = \frac{\exp\left(-\frac{E[f|p]-p}{Var[f|p]}(f-p)\right)}{E\left[\exp\left(-\frac{E[f|p]-p}{Var[f|p]}(f-p)\right)|p\right]} \tag{34}$$

Definition 2. *The risk-neutral meausre is defined by the Radon-Nikodym derivative Z given by (34). That is, for any event A measurable with respect to information in prices, the risk-neutral conditional probability of that event $Pr^Q(A|p) = E[Z \cdot I(A)|p]$. The unconditional probability is defined as $Pr^Q(A) = E[Z \cdot I(A)]$.*

The first order condition (33) can be rewritten as

$$p = E^Q[f|p],$$

which justifies the name of the new measure. Our definition implies that the risk-neutral distribution of $f|p$ is characterised by the probability density function

$$g_{f|p}(f|p) = \phi\left(\frac{f - E[f|p]}{\sqrt{Var(f|p)}}\right) \cdot Z,$$

where $\phi(\cdot)$ is a density of a standard normal random variable. Direct calculation leads to the following result.

Lemma 15. *Under the risk-neutral measure, the distribution of $f|p$ is Normal with mean p and variance $Var(f|p)$.*

Proof. *(Lemma 15).* We proceed by direct calculation

$$g_{f|p}(f|p) = \phi\left(\frac{f - E[f|p]}{\sqrt{Var(f|p)}}\right) \cdot Z$$

Substitute Z above and collect the terms that depend on f, we get

$$\begin{aligned} g_{f|p}(f|p) &= \psi(p) \cdot \exp\left(-\frac{1}{2Var(f|p)}\left(f^2 - 2fE[f|p]\right) - \frac{E[f|p] - p}{Var[f|p]} f\right) \\ &= \psi(p) \cdot \exp\left(-\frac{1}{2Var(f|p)}\left(f^2 - 2f \cdot p\right)\right). \end{aligned} \tag{35}$$

In the above, $\psi(p)$ combines all the terms that do not depend on f. Since the density has to integrate to 1 we can express

$$\psi(p) = \left(\int \exp\left(-\frac{1}{2Var(f|p)}\left(f^2 - 2f \cdot p\right)\right) df\right)^{-1}. \tag{36}$$

Since the normal density with mean p and variance $Var(f|p)$ can be represented by (35) and (36) the Lemma follows. ■

Financiers' problem and VaR-margins.

We first consider the case in which investors take long position in the asset, and therefore, microfound the expression for m^+.

When investors take a long position of the asset

We assume that the financier can borrow at a rate $1 - \epsilon$ and lend to investors at a rate of 1 i.e., there are gains from trade between financiers and investors.[26] We also assume that the investors' date-2 wealth is not pledgeable and the financier has to pay a proportional cost to enforce the investor to

[26]One way to rationalize this is to assume that investors valuation of the risk free asset is different from financiers, e.g., due to relative tax disadvantage as in Duffie, Gârleanu, and Pedersen (2005).

repay with date-2 wealth. Therefore, for every unit of asset that the investor has purchased, he can transfer the asset and some cash m^+ to the financier's account as a collateral. Effectively, the financier is lending an amount $(p - m^+)$ per unit to the investor while holding the asset as collateral.

At $t = 2$, the investor has to repay $(p - m^+)$ to get the asset dividend f back. If the dividend is more than the promised repayment, i.e., $f > p - m^+$, costly enforcement is not needed because the financier can just take the repayment from the dividend of the asset which is at his custody. If instead the dividend from the asset is less than the promised repayment, we assume the financier has to pay an enforcement cost $k = \frac{\epsilon}{1-\alpha}$ per dollar lent to force the investors to pay with his date-2 wealth.[27] Thus, the financier earns a return "spread" ϵ on lending to investors but has to pay an enforcement cost $\frac{\epsilon}{1-\alpha}$ per dollar lent in the states where $f < p - m^+$.

Assume financiers have CARA utility over terminal wealth with risk aversion parameter γ_F . Financier's time-2 wealth consists of two parts: the first one comes from his investment in the assets, W_A and the second comes from lending to investors W_L. For the second part we write:

$$\begin{aligned} W_L &= x_i \left(p - m^+\right) \left(\epsilon - \frac{\epsilon}{1-\alpha} \times 1\left(f < p - m^+\right)\right) \\ &= \epsilon x_i \left(p - m^+\right) \left(1 - \frac{1}{1-\alpha} \times 1\left(f < p - m^+\right)\right) \end{aligned}$$

We will later assume that ϵ is small and calculate our expressions in the limit as $\epsilon \to 0$. Given that the financier is uninformed and unconstrained, we can write

$$W_A = \frac{E\left[f|p\right] - p\left(1-\epsilon\right)}{\gamma_F Var\left[f|p\right]} \left(f - p\right).$$

Note that financier's information set is $\mathcal{I}_F = \{p, x_i\}$. However, it is easy to see that information content in x_i is subsumed in prices and hence financier only conditions on prices. Financier's utility can then be written as

$$E\left[U_F\left(W_A + W_L\right)|p\right] = U_F\left(W_A\right) + E\left[U_F^{'}\left(W_A\right) W_L|p\right] + o(\epsilon) \tag{37}$$

$$= U_F\left(W_A\right) + E\left[U_F^{'}\left(W_A\right) \epsilon x_i \left(p - m^+\right) \left(1 - \frac{1}{1-\alpha} \times 1\left(f < p - m^+\right)\right)|p\right] + o(\epsilon) \tag{38}$$

$$= U_F\left(W_A\right) + \frac{\epsilon x_i \left(p - m^+\right)}{1-\alpha} E\left[U_F^{'}\left(W_A\right)|p\right] \cdot \tag{39}$$

$$\left(1 - \alpha - E\left[\frac{U_F^{'}\left(W_A\right)}{E\left[U_F^{'}\left(W_A\right)|p\right]} 1\left(f < p - m^+\right)|p\right]\right) + o(\epsilon) \tag{40}$$

We assume that there is perfect competition between financiers, so that each of them should be indifferent between lending to investors and getting (40) or not lending and getting the outside

[27]Here α is just a normalisation constant, but later on we will derive that it will be equal to the VaR confidence level α.

option of $U_F(W_A)$. Equalising the above expression to $U_F(W_A)$, and taking the limit as $\epsilon \to 0$ we get

$$E\left[\frac{U_F'(W_A)}{E\left[U_F'(W_A)|p\right]} 1\left(f < p - m^+\right)|p\right] = 1 - \alpha,$$

Noting that $\lim_{\epsilon\to 0} \frac{U_F'(W_A)}{E\left[U_F'(W_A)\right]} = Z$, where Z is the Radon-Nikodym derivative associated with the rsik-neutral measure it then follows that m^+ should be such that

$$Pr^Q\left[f < p - m^+|p\right] = 1 - \alpha,$$

which coincides with expression for VaR-based margins.

When investors take a short position of the asset

Financiers are also endowed with a large but finite amount of assets. We call them security lenders here because they act as such. Their outside option is to lend the security to some unmodelled agents at a return $1-\epsilon$. Investors can borrow the asset from the security lenders by pledging $(p+m^-)$ cash collateral per unit at $t=1$. As the investors sell the asset immediately for p, they have to put up m^- from their own wealth. At $t=2$, the investors buy back the asset at f, return it and retrieve $(p+m^-)$ from the security lenders. We assume that if the value of the asset is higher than the cash collateral, i.e., $f > p + m^-$, the security lenders have to expend k per dollar of asset lent in order to force the investors to buy back and return the asset. Using the same steps as in the above subsection, the margin m^- should satisfy the following break-even condition

$$\epsilon x_i\left(p+m^-\right) E\left[U_F'(W_A)|p\right]\left(1 - \frac{1}{1-\alpha} \times E\left[\frac{U_F'(W_A)}{E\left[U_F'(W_A)|p\right]} 1\left(f > p + m^-\right)|p\right]\right) = 0$$

which after simplification reduces to

$$Pr^Q\left[f - p > m^-|p\right] = 1 - \alpha,$$

which coincides with expression for VaR-margins.

VaR^P-margin

In the previous subsection, we rationalized the use of risk-based margin where risk is evaluated under in risk-neutral measure. Here, we provide a microfoundation for situations where risk should be evaluated under physical measure.

We assume that financiers do not participate in financial markets and are exposed to only idiosyncratic shocks. Given this, the wealth from financing activity is not correlated with their other wealth and the financier is effectively risk-neutral with respect to income from financing. The financier is endowed with large but finite amount of cash. We assume that the the financier gets a gross return of 1 when lending to the investors but only $(1-\epsilon)$ when investing in the risk-free asset.

We also assume that the investors' date-2 wealth is not pledgeable and the financier has to pay a cost to enforce the investor to repay with date-2 wealth. Therefore, for every unit of asset that the investor has invested, he can transfer the asset and some cash m^+ to the financier's account as a collateral. Effectively, the financier is lending an amount $(p-m^+)$ to the investor while holding the asset as collateral.

At $t=2$, the investor has to repay $(p-m^+)$ to get the asset dividend f back. If the dividend is more than the promised repayment, i.e., $f > p-m^+$, costly enforcement is not needed because the financier can just take the repayment from the dividend of the asset which is at his custody. If instead the dividend from the asset is less than the promised repayment, we assume the financier has to pay an enforcement cost k per dollar lent to force the investors to pay with his date-2 wealth. In sum, upon observing p, a competitive financier chooses a cash margin m^+ so that he is indifferent between lending to the investors and investing in the risk-free asset

$$x_i(p-m^+) - Pr(x_i f < x_i(p-m^+)|p)kx_i(p-m^+) \geq x_i(p-m^+)(1-\epsilon) \tag{41}$$

After simplification, we have

$$Pr(p-f > m^+|p) \leq \frac{\epsilon}{k} \tag{42}$$

which coincides with the VaR margin constraint in (10) when $\frac{\epsilon}{k}$ is replaced with $1-\alpha$.

E Equilibrium characterization with VaR^P-margins

Assume that financiers set the margins under physical measure. With risk-averse market maker, the price can be written as $p = \mathbb{E}[v|p] - rp(p)$. As in Brunnermeier and Pedersen (2009), we assume that the financiers use information from prices to set margin in order to control their value-at-risk:

$$m^+(p) = \inf\left\{m^+(p) \geq 0 : Pr(p - f > m^+(p)|p) \leq 1 - \alpha\right\},$$

$$m^-(p) = \inf\left\{m^-(p) \geq 0 : Pr(f - p > m^-(p)|p) \leq 1 - \alpha\right\}$$

where $m^+(p)$ and $m^-(p)$ are the margins on long and short positions (per unit of asset) respectively. We now derive the expressions for these margins. To compute $m^+(p)$, we first determine the function $m_n^+(p)$ that satisfies

$$\begin{aligned}
1 - \alpha &= Pr\left(E[f|p] - rp(p) - f > m_n^+(p)|p\right) \\
&= Pr\left(\sqrt{\tau_m}(E[f|p] - f) > \sqrt{\tau_m}(m_n^+(p) + rp(p))|p\right) \\
&= 1 - \Phi\left(\sqrt{\tau_m}(m_n^+(p) + rp(p))\right).
\end{aligned}$$

Thus, we can write

$$m^+(p) = [m_n^+(p)]^+ = \left[\frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}} - rp(p)\right]^+. \tag{43}$$

Similarly, one can define $m_n^-(p)$ which satisfies $Pr(f - p > m_n^-(p)|p) = 1 - \alpha$ and can write

$$m^-(p) = [m_n^-(p)]^+ = \left[\frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}} + rp(p)\right]^+. \tag{44}$$

The endogenous VaR margins $m^+(\cdot)$ and $m^-(\cdot)$ are determined by three variables. Both margins on long and short positions increase in the exogenous level of confidence α and decrease in the endogenous informational efficiency of price β (through $\tau_m^{-1} = \left(\tau_v + \beta^2\tau_z\right)^{-1} + \tau_\theta^{-1}$). In addition, the margin on long (short) position decreases (increases) in the endogenous risk premium $rp(p)$. We would like to emphasize the fact that informational efficiency of price always affect the tightness of margin constraint.

Financial market equilibrium with a risk-averse market maker

Formally, our financial market equilibrium with endogenous margin constraints is defined as follows: (1) financiers and investors determine demands and margins anticipating a particular price function (2) in equilibrium demands and margins are consistent with anticipated price function. We hold the precision of investors' signals fixed.

Proposition 19. *(Equilibrium with endogenous margin requirements) When the portfolio constraints* *are of the form of margin as in equation (9) and margins are endogenously determined by VaR, there exists a unique generalized linear equilibrium. Moreover, in this unique equilibrium the function $g(p)$, i.e. the sufficient statistic ϕ, is increasing in price.*

Proof. (Proposition 19) One can prove that for every p there exists unique $\phi = g(p)$ such that the market clears (similar to Proposition 2). We now prove that $g(p)$ is invertible. We plug expression for our endogenous margins into ODE (6) assuming that both m_n^+ and m_n^- are positive. We get

$$\frac{\partial g(p)}{\partial p} = \frac{c_p^m + \pi_2 c_p - \left(\frac{\pi_1 W_0}{m^-(p)^2} + \frac{\pi_3 W_0}{m^+(p)^2}\right)\frac{\partial rp(p)}{\partial p}}{\pi_2 c_\phi + c_\phi^m}. \tag{45}$$

Moreover using the fact that $rp(p) = \frac{\gamma_m}{\tau_m}\left(c_0^m + c_\phi^m g(p) - c_p^m p\right)$, we get

$$\frac{\partial rp(p)}{\partial p} = \frac{\gamma_m}{\tau_m}\left(c_\phi^m \frac{\partial g(p)}{\partial p} - c_p^m\right). \tag{46}$$

Substituting (46) into (45), we get

$$\frac{\partial g(p)}{\partial p} = \frac{c_p^m + \pi_2 c_p + \frac{\gamma_m}{\tau_m}\left(\frac{\pi_1 W_0}{m^-(p)^2} + \frac{\pi_3 W_0}{m^+(p)^2}\right) c_p^m}{\pi_2 c_\phi + c_\phi^m + \frac{\gamma_m}{\tau_m}\left(\frac{\pi_1 W_0}{m^-(p)^2} + \frac{\pi_3 W_0}{m^+(p)^2}\right) c_\phi^m}.$$

Clearly, the derivative above is always positive, which means that the equilibrium function $g(p)$ is invertible. Thus, for each fundamental ϕ there exists a unique p clearing the market. The initial condition for the ODE above can be found by clearing the market for a particular price, e.g. $p = 0$. ■

Value of information with a risk-averse market maker

Here, we study how the incentives of investors to acquire information at $t = 0$ are affected by general portfolio constraints in the case of risk-averse market maker. The goal of this section is to generalize Proposition 4. When market maker is risk-averse, tightening constraints for all investors is complicated because the equilibrium price distribution will change, which affects price-dependent constraints.

Proposition 20. *The following results hold:*

1. *Suppose all investors are unconstrained. Once finite constraints $a(p)$ and $b(p)$ are introduced for all investors, the marginal value of information decreases for all of them.*

2. *Suppose that all investors face portfolio constraints $a(p)$ and $b(p)$. Once all investors are constrained to hold 0 positions in the asset the marginal value of information decreases for all of them.*

3. *Suppose there exists an equilibrium with monotone function g(p). There exist $\overline{\gamma_m}$ such that for all $\gamma_m < \overline{\gamma_m}$ as constraints tighten, investors acquire less information.*

Proof. Parts 1. and 2. follow directly from Proposition 3 and the fact that $0 < \frac{U_0^u}{U_0} < 1$. Part 3 follows from Proposition 4. ■

The part 1 of the Proposition above demonstrates that when constraints $a(p)$ and $|b(p)|$ are tightened from infinity to some finite positive numbers, marginal value of information for all investors decreases. Similarly, part 2 demonstrates that when constraints $a(p)$ and $|b(p)|$ are tightened from some finite positive numbers to zero, the marginal value of information decreases as well. Part 3 of the Proposition considers general changes in the constraints. It shows that if risk aversion of market maker is below some threshold, investors acquire less information as their constraints tighten. All these results yield confidence that the Proposition 4 holds with risk-averse market maker. Even though we cannot prove the statement of Proposition 4 beyond the case of γ_m small enough, we verify it numerically.

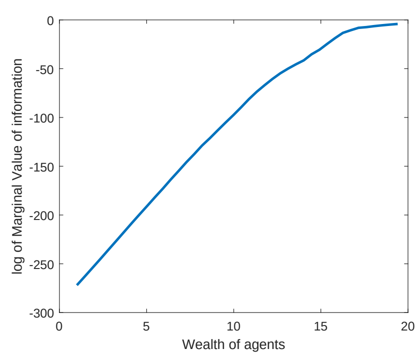
In Figure 8 we plot the marginal value of information, against the constraints investors face. This figure is typical: similar numerical results hold over the entire range of parameters. We record the following observation for future reference.

Observation *Suppose that all investors face portfolio constraints $a(p)$ and $b(p)$ and that the market maker is risk-averse. If constraints become tighter for all investors, i.e. $a(p)$ increases and $b(p)$ decreases , the marginal value of information decreases for all investors.*

In the rest of this Appendix, we focus on how margin requirements change with informational efficiency.

Figure 8: Value of information

The plot shows the marginal value of information as a function of investors' wealth. Other parameters are: $\tau_v = 1; \tau_z = 1; \gamma = 3; \alpha = 0.99$. All other paramters are chosen to be 1.



Approximation of the price function $p(\phi)$

In general, the equilibrium price function is non-linear (as a function of fundamental ϕ) in the economy with margin constraints. This is because, the fractions of investors who are constrained or not vary with ϕ. In order to facilitate additional analysis and interpretation of the results, we approximate the equilibrium price function with a piece-wise linear price function with three linear parts. The idea is that as ϕ increases from low to high value regions, demands of *almost all* investors change from being constrained by the margin on short position $m^{-}(\cdot)$, to being unconstrained, and to being constrained by the margin on long position $m^{+}(\cdot)$.[28]

Consider the case with some $\phi < \phi^{-}$ such that most investors' demand are constrained by the margin on short position, i.e. $X^{u} \approx -\frac{W_0}{m^{-}(p)} = -\frac{W_0}{\left[\frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}} + rp(p)\right]^{+}}$. With the market maker's demand $x_m = \frac{\tau_m}{\gamma_m} rp(p)$, the market clearing condition becomes

$$-\frac{W_0}{\left[\frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}} + rp(p)\right]} + \frac{\tau_m}{\gamma_m} rp(p) = 1 \quad \text{for } rp\,(p) > -\frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}}$$

Denote rp^{-} as the risk premium that satisfies the above equation. It is easy to check that the risk premium rp^{-} in this case is unique and does not depend on price. Finally, since the market maker's demand is a proportional to risk premium and thus also a constant in price this case, the equilibrium price must adjust with respect to the fundamental ϕ such that the market maker's demand stays constant. That is, from $rp^{-} = \frac{\gamma_m}{\tau_m} x_m = \frac{\gamma_m}{\tau_m}(c_0^m + c_\phi^m \phi - c_p^m p)$, we find the linear price function $p(\phi) = -\frac{\tau_m}{\gamma_m c_p^m} rp^{-} + \frac{c_0^m}{c_p^m} + \frac{c_\phi^m}{c_p^m}\phi$ for $\phi < \phi^{-}$.

One can characterize linear pricing functions with similar procedures for the case of unconstrained investors in the intermediate range of fundamental $\phi \in (\phi^{-}, \phi^{+})$ and the case of investors' demand being constrained by the margin on long margin for the high range of $\phi > \phi^{+}$. The boundary values of ϕ^{-} and ϕ^{+} are pinned down by imposing continuity on the approximated price functions. We summarize the result in the following lemma.

Lemma 16. *(An approximated price function). The equilibrium price function $p\,(\phi)$ can be approximated by three linear price functions for three different scenarios about investors' demand: i) constrained by the margin on short position $m^{-}(p)$; ii) unconstrained; iii) constrained by the margin on*

[28]For any fundamental ϕ, there are always constrained investors and unconstrained investors, thanks to dispersed idiosyncratic shocks on investors' endowment and signal. Even at extreme ϕ, there are a measure non-zero of investors who are unconstrained, which makes the aggregate demand vary with fundamentals. Importantly, such arbitrarily small variations in aggregate demand allows the market maker to learn about the fundamental and clear the market with a market-clearing price.

long position $m^+(p)$. The approximated price function $\hat{p}(\phi)$is

$$\hat{p}(\phi) = \begin{cases} \frac{1}{c_p^m}(-\frac{\tau_m}{\gamma_m} rp^- + c_0^m + c_\phi^m \phi) & \text{for } \phi < \phi^- \\ \frac{c_0 + c_0^m - 1}{c_p + c_p^m} + \frac{c_\phi + c_\phi^m}{c_p + c_p^m}\phi & \text{for } \phi \in [\phi^-, \phi^+] \\ \frac{1}{c_p^m}(-\frac{\tau_m}{\gamma_m} rp^+ + c_0^m + c_\phi^m \phi) & \text{for } \phi > \phi^+ \end{cases}$$

where $rp^- = \frac{\gamma_m - k\tau_m + \sqrt{\gamma_m^2 + k^2\tau_m^2 + 2k\gamma_m\tau_m + 4\gamma_m\tau_m W_0}}{2\tau_m}$, $rp^+ = \frac{\gamma_m + k\tau_m - \sqrt{\gamma_m^2 + k^2\tau_m^2 - 2k\gamma_m\tau_m + 4\gamma_m\tau_m W_0}}{2\tau_m}$ *with* $k = \frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}}$, *and* $\phi^j = \frac{-\frac{\tau_m}{\gamma_m c_p^m} rp^j + \frac{c_0^m}{c_p^m} - \frac{c_0 + c_0^m - 1}{c_p + c_p^m}}{\frac{c_\phi + c_\phi^m}{c_p + c_p^m} - \frac{c_\phi^m}{c_p^m}}$ *for* $j = \{-, +\}$. $rp^- > rp^+$ *and* $\phi^- < \phi^+$.

Note that the price function is more sensitive to fundamental at the intermediate values of ϕ when most investors' demands are not restricted by the margin constraints. Intuitively, this is because the investors can adjust their demand thus impound more information about fundamentals into price.

Margins with a risk-averse market maker

In this section, we provide conditions for Proposition 6 and Remark 4 from the main text to still hold with a risk-averse market maker. Assume that each financier sets her margin in order to control her value-at-risk, as in Brunnermeier and Pedersen (2009). Given fundamentals v and z, the margins are given by

$$m^+(p) = [m_n^+(p)]^+ = \left[\frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}} - rp(p)\right]^+ \quad \text{and} \quad m^-(p) = \left[\frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}} + rp(p)\right]^-.$$

Proposition 21. *For a given investors' wealth W_0, when informational efficiency (β) decreases, margins increase for all price realizations if*

$$\gamma_m < \frac{\sqrt{\tau_m}\Phi^{-1}(\alpha)}{2}$$

and W_0 and σ_ξ are small. This implies that, as informational efficiency drops, constraints become tighter.

Proof. Using the approximated price function, which is valid given that σ_ξ is small, we can write the risk premium as

$$rp(p) = \begin{cases} rp^- & \text{for } g(p) < \phi^- \\ \frac{\gamma_m}{\tau_m}\left(\frac{c_p^m}{c_p + c_p^m} + \frac{c_p c_\phi^m - c_\phi c_p^m}{c_p + c_p^m} g(p)\right) & \text{for } g(p) \in [\phi^-, \phi^+] \\ rp^+ & \text{for } g(p) > \phi^+. \end{cases} \tag{47}$$

Let $k = \frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}}$ and $l = \frac{\gamma_m}{\tau_m}$. Taking the derivative of margins with respect to τ_m, we get

$$\frac{\partial m^+(p)}{\partial \tau_m} = \begin{cases} \frac{\partial m_n^+(p)}{\partial \tau_m} = -\frac{\Phi^{-1}(\alpha)}{2\sqrt{\tau_m^3}} - \frac{\partial rp(p)}{\partial \tau_m} & \text{if } \frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}} > rp(p) \\ 0 & \text{otherwise} \end{cases} \tag{48}$$

$$\frac{\partial m^-(p)}{\partial \tau_m} = \begin{cases} \frac{\partial m_n^-(p)}{\partial \tau_m} = -\frac{\Phi^{-1}(\alpha)}{2\sqrt{\tau_m^3}} + \frac{\partial rp(p)}{\partial \tau_m} & \text{if } \frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}} > -rp(p) \\ 0 & \text{otherwise} \end{cases} \tag{49}$$

where $\frac{\partial k}{\partial \tau_m} = -\frac{k}{2\tau_m}$, $\frac{\partial l}{\partial \tau_m} = -\frac{l}{\tau_m}$ and

$$\begin{aligned}
\frac{\partial m_n^+(p)}{\partial \tau_m} &= \frac{\partial}{\partial \tau_m}[k - rp(p)] \\
&= \begin{cases} \frac{\partial}{\partial \tau_m}[k - rp^-] & \text{for } g(p) < \phi^- \\ \frac{\partial}{\partial \tau_m}\left[k - \frac{\gamma_m}{\tau_m}\left(\frac{c_p^m}{c_p + c_p^m} + \frac{c_p c_\phi^m - c_\phi c_p^m}{c_p + c_p^m} g(p)\right)\right] & \text{for } g(p) \in [\phi^-, \phi^+] \\ \frac{\partial}{\partial \tau_m}[k - rp^+] & \text{for } g(p) > \phi^+ \end{cases} \\
&= \begin{cases} \frac{2l - 3k}{4\tau_m} + \frac{(2l+k)(l+k) + 4W_0 l}{4\tau_m\sqrt{l^2 + k^2 + 2kl + 4lW_0}} & \text{for } g(p) < \phi^- \\ -\frac{k}{2\tau_m} + \frac{\frac{1}{\gamma}\frac{\partial \tau}{\partial \tau_m} + \frac{1}{l\tau_m}}{\left(\frac{\tau}{\gamma} + \frac{1}{l}\right)^2} & \text{for } g(p) \in [\phi^-, \phi^+] \\ \frac{2l - k}{4\tau_m} - \frac{(2l-k)(l-k) + 4lW_0}{4\tau_m\sqrt{l^2 + k^2 - 2kl + 4lW_0}} & \text{for } g(p) > \phi^+ \end{cases}
\end{aligned}$$

The third term in the above expression is negative if $2l - k < 0$. For W_0 small enough, even the first term in the above expression is negative. Similarly,

$$\frac{\partial m_n^-(p)}{\partial \tau_m} = \frac{\partial}{\partial \tau_m}[k + rp(p)] = \begin{cases} \frac{\partial}{\partial \tau_m}[k + rp^-] & \text{for } g(p) < \phi^- \\ \frac{\partial}{\partial \tau_m}\left[k + \frac{\gamma_m}{\tau_m}\left(\frac{c_p^m}{c_p + c_p^m} + \frac{c_p c_\phi^m - c_\phi c_p^m}{c_p + c_p^m} g(p)\right)\right] & \text{for } g(p) \in [\phi^-, \phi^+] \\ \frac{\partial}{\partial \tau_m}[k + rp^+] & \text{for } g(p) > \phi^+ \end{cases}$$

$$= \begin{cases} -\frac{3k}{4\tau_m} - \frac{l}{2\tau_m} - \frac{2l^2 + k^2 + 3kl + 4W_0 l}{4\tau_m\sqrt{l^2 + k^2 + 2kl + 4lW_0}} & \text{for } g(p) < \phi^- \\ -\frac{k}{2\tau_m} - \frac{\frac{1}{\gamma}\frac{\partial \tau}{\partial \tau_m} + \frac{1}{l\tau_m}}{\left(\frac{\tau}{\gamma} + \frac{1}{l}\right)^2} & \text{for } g(p) \in [\phi^-, \phi^+] \\ -\frac{k}{4\tau_m} - \frac{l}{2\tau_m} + \frac{2l^2 + k^2 - 3kl + 4lW_0}{4\tau_m\sqrt{l^2 + k^2 - 2kl + 4lW_0}} & \text{for } g(p) > \phi^+ \end{cases}$$

Note that the first term and second terms in the above expression are always negative. If W_0 is small enough, even the third term is negative. This implies that $m^-(p)$ always decreases with informational efficiency. So, the sufficient conditions for the margins to decrease with informational efficiency is $2l < k$ and small enough wealth. ■

Below we examine the conditions of the Proposition above, for which Remark 4 from the main text still holds.

Proposition 22. *Suppose that the conditions of Proposition 21 hold. Then margins increase when informational efficiency drops, even when financier does not learn from prices.*

Proof. We prove the proposition for m^+, the margin on a long position. The proof for m^- is analogous and is omitted for brevity.

When financiers cannot condition margins on prices, the margins are determined by the following equation:

$$\alpha = E\left[\Phi\left(\sqrt{\tau_m}(m^+ + rp(p))\right)\right]. \tag{50}$$

We prove this proposition by contradiction. Denote $m_n^+(p) = \frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}} - rp(p)$ (the margins the financier would set if he would be able to condition on prices). Suppose that m_n^+ decreases as informational efficiency drops. One can write

$$\begin{aligned}\sqrt{\tau_m}\left(m^+ + rp(p)\right) &= \sqrt{\tau_m}\left(m^+ - m_n^+(p) + m_n^+(p) - rp(p)\right)\\ &= \sqrt{\tau_m}\left(m^+ - m_n^+(p) + \frac{\Phi^{-1}(\alpha)}{\sqrt{\tau_m}}\right)\\ &= \sqrt{\tau_m}\left(m^+ - m_n^+(p)\right) + \Phi^{-1}(\alpha).\end{aligned}$$

We assume that the conditions of Proposition 21 hold, therefore, $m_n^+(p)$ increases for all p as informational efficiency drops. At the same time, $\sqrt{\tau_m}$ decreases and we assumed that m^+ drops as well. This implies that $E\left[\Phi\left(\sqrt{\tau_m}(m^+ + rp(p))\right)\right]$ drops when informational efficiency drops. A contradiciton with (50). ■

The following corollary to Propositions 20 and 21 generalizes Corollary 3.

Corollary 5. *Suppose that the conditions of Proposition 21 hold and γ_m is small enough. Then a decrease in investor wealth W_0 decreases informational efficiency β and increases VaR-based margins m^+, m^-.*

Proof. Follows directly form Propositions 20 and 21. ■

Intuitively, information efficiency decreases with wealth (Proposition 20); when information efficiency drops, margins rise (Proposition 21).

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