Simultaneous Multilateral Search[*]

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Abstract

This paper studies simultaneous multilateral search (SMS) in over-the-counter markets: When searching, a customer simultaneously contacts several dealers and trades with the one offering the best quote. Higher search intensity (how often one can search) improves welfare, but higher search capacity (how many dealers one can contact) might be harmful. When the market is in distress, customers might inefficiently favor bilateral bargaining (BB) over SMS. Such preference for BB speaks to the sluggish adoption of SMS trading, like request-for-quote protocols, in over-the-counter markets. Furthermore, a market-wide shift to SMS may not be socially optimal. (*JEL* D40, D84, G12, G14)

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Search is a key feature in over-the-counter (OTC) markets. Duffie, Gârleanu, and Pedersen (2005, hereafter DGP) pioneered the theoretical study of OTC markets in a framework of random matching and *bilateral bargaining* (BB): Investors search for counterparties and are randomly matched over time. Upon successful matching, a buyer and a seller engage in Nash bargaining and split the trading gain according to their endowed bargaining power.

However, investors' interaction is not always bilateral. For example, in recent years, there is a rise of electronic trading in OTC markets, mainly in the form of Request-for-Quote (RFQ). In such marketplaces, where many corporate bonds and derivatives are traded, a customer contacts multiple dealers for quotes and then trades with the one offering the best price. Hendershott and Madhavan (2015) report that more than 10% of trades in the $8tn corporate bond market is completed via RFQ. O'Hara and Zhou (2021) document a continued growth of RFQ-based trading of corporate bonds, but only sluggishly, with the highest trading volume share below 14% in their sample.

This paper develops a theoretical model, tailored to the above one-to-many searching. Specifically, a customer is allowed to query *multiple* dealers *at the same time*, hence the name "Simultaneous Multilateral Search" (SMS). The objective is twofold. First, we examine how the SMS technology affects assets allocation and welfare. Second, we study how customers choose to search: Do they favor SMS over BB? Are their choices efficient? How do we understand the sluggish growth of SMS-type of electronic trading (O'Hara and Zhou, 2021)?

Section 1 sets up the model following Hugonnier, Lester, and Weill (2020, hereafter HLW), where a continuum of customers trade an asset through a continuum of homogeneous dealers.[1] All agents can hold either zero or one unit of the asset. The customers are subject to stochastic valuation shocks. Those who hold the asset but have low valuation want to sell, while those without the asset but with high valuation want to buy. They actively search for dealers according to independent Poisson processes with intensity ρ. We generalize the search process as follows to model SMS: (i) each searching customer can request quotes from up to n dealers; (ii) the best quote is determined via a first-price auction and (iii) the customer can potentially improve upon the best quote via bargaining: with probability q, the customer can make a take-it-or-leave-it offer (TIOLIO) to the

[1] In HLW the dealers are heterogeneous. We abstract from dealer heterogeneity to focus on SMS in a parsimonious way.

winning dealer after the auction. Notably, the search process nests BB as a special case when $n = 1$: The searching customer randomly contacts one dealer and sets the price with probability q. With probability $1 - q$, the dealer sets the price. In that special case the parameter q thus serves as the customer's Nash bargaining power parameter, as in DGP and HLW.

Section 2 characterizes the equilibrium and discusses novel findings. Notably, the two search parameters, the intensity ρ (how frequently one can search) and the capacity n (how many potential dealers one can reach), have contrasting implications for various equilibrium objects. For instance, a higher ρ always reduces the sizes of both the buyer- and the seller-customers, improving the asset allocation and also welfare. In contrast, a larger n can drive up the size of the short-side customers and possibly *hurt welfare.*

The key mechanism is a "dealer bottleneck," arising from the asymmetric effects of n on the matching of the two sides of the market. To see this, suppose the asset is in excess supply and 90% of the dealers have inventory while the other 10% do not. Let us examine what happens when the capacity increases from $n = 2$ to $n = 3$: For a customer-seller, the matching rate with a no-inventory dealer increases from $1 - 0.9^2 = 19\%$ to $1 - 0.9^3 = 27.1\%$. Such an improvement in matching significantly adds to the asset inflow to dealers from customer-sellers. However, the outflow rate—the matching between customer-buyers and dealer-sellers—only increases by 0.9%, from $1 - 0.1^2 = 99\%$ to $1 - 0.1^3 = 99.9\%$. The negligible increase of the outflow rate is not at all enough to balance the significant rise in the inflow rate. That is, the asset is "clogged" at the dealers, creating a bottleneck that leaves more customer-buyers unmatched.[2] This leads to a surge in unrealized trading gains and may reduce welfare. To emphasize, this bottleneck effect is unique to the search capacity n. In contrast, the search intensity ρ does not create asymmetry in matching and always improves welfare.

Such bottlenecks arise in our most general setup. In a specialized application, we allow customers to direct their searches to subsets of dealers of their choosing, based on noisy signals of dealer types. For example, a customer-buyer might have a rough idea of which dealers have in-

[2] It is the increase of the unmatched customer-buyers that eventually balances the asset inflow to and the outflow from dealers in the steady state equilibrium. Whereas the inflow increases with n via the higher matching *rate*, the outflow increases via the increment in the larger customer-buyer population *size*.

ventory, based on recently reported trades. She then optimally directs her searches only to those dealers for higher matching probabilities. A key parameter is the signal quality ψ, which can be interpreted as the transparency of dealer inventories. We show that a similar bottleneck can arise when ψ increases: As customers direct their searches more accurately, the matching on the short and on the long side is improved asymmetrically, hindering the efficient passing of the asset through dealers. Our model thus highlights a potential channel for how better inventory transparency might hurt welfare.

Another insight from the model is how SMS endogenizes the bargaining powers of customers and dealers. The key is the dual role of "dealer demographics"—how many dealers have the asset in their inventories and how many do not: As is standard, dealer demographics affect matching (e.g., how likely a customer can find a counterparty to trade). New in this model, dealer demographics also affect the split of trading gains between customers and dealers. For example, if there are many dealers with inventories, when contacted by a customer-buyer, they will quote more competitively, as they know that the customer has also contacted $n-1$ other dealers, who very likely might also have inventories to sell. Such fiercer competition cuts more trading gains to the searching customer and less to the dealers. Thus, SMS endogenizes the bargaining powers, which are by and large exogenous in existing search models. Further, in equilibrium, the dealers quote according to a mixed strategy, creating price dispersion despite dealer homogeneity. Notably, the distribution of such price dispersion is also endogenously determined via dealer demographics.

Section 3 studies the customers' choices between BB and SMS. We show that the choice ultimately boils down to the comparison between the two technologies' expected trading gain intensities, which are the respective products of (i) the search intensity—how frequent one can search, (ii) the matching rate—how likely it is to find at least one counterparty, and (iii) the expected trading gain share above—how much trading gain one can get given a match.

At first glance, one might conclude that SMS has advantages for customers over BB in all three aspects above: (i) it offers faster connection (via electronic platforms), (ii) it allows customers to contact multiple dealers, and (iii) it encourages the competition among the contacted dealers, hence giving larger trading gain shares to customers. The analysis, however, reveals a potential downside, especially when customers have , i.e., when q is low in SMS. In this case, the customer's expected trading gain is only determined by the endogenous competition among the contacted dealers. When such competition is insufficient, the customer expects very little, because any matched counterparty dealer will charge a monopoly price, knowing that she is likely the only counterparty that the customer can find (out of the n). In contrast, in BB, a customer always has some chances to secure some positive trading gains, given a positive q in BB.

It is worth emphasizing that the q in SMS (q^{SMS}) and that in BB (q^{BB}) are exogenous model parameters. For the customers to favor BB over SMS, the model effectively makes an assumption that the q^{SMS} is lower than q^{BB}, based on the real-world market structure described here. (A more general condition is given in Lemma 3.) Together with this, the novel equilibrium force of the n contacted dealers' competition (or the lack of it) makes it possible that SMS becomes less attractive than BB.

Indeed, the customers may favor BB over SMS, especially when the asset is in very imbalanced demand and supply. Consider the case of excess supply. The large number of customer-sellers flood the dealer sector with the asset, making most of the dealers full in inventory. Consequently, the remaining customer-sellers find it very difficult to find dealer-buyers. Even if they do, using SMS, any matched dealer-buyer will knowingly charge a very low monopoly price. Instead, resorting to BB, customer-sellers can still negotiate prices with dealers. This prediction echoes the empirical finding in O'Hara and Zhou (2021) that when corporate bonds are downgraded and under fire sell (i.e., in excess supply), the electronic trading volume share drops. Such an intrinsic tradeoff between SMS and BB could have hindered the adoption of electronic OTC trading of corporate bonds. This mechanism complements the existing explanation for customers' reluctance of using SMS, which largely relies on the concern of leaking private information to too many dealers (Hendershott and Madhavan, 2015). This information leakage argument, however, does not explain the downgrade-induced reduction of electronic volume shares, as downgrades are public information.

The customers' endogenous choices between BB and SMS also have welfare and market design implications. The analysis shows that when the asset trades very fast, i.e., for high search intensity ρ, a social planner strictly prefers SMS over BB, simply because SMS offers better matching, which creates large trading gains. Unlike the planner, who ignores the split of trading gains, the customers might shy away from SMS because the trading gain split there is inferior compared to BB. Such inefficiency in technology adoption can be reduced by policies and market designs that incentivizes customers to use SMS. In the model, this can be achieved by setting a large enough q in SMS, e.g., by allowing customers to further bargain in RFQ platforms, after running auctions among dealers.

However, such patches might not always work, depending on the characteristics of the asset traded. For example, when the asset is intrinsically slow, i.e., for sufficiently low search intensity ρ, having all investors using SMS is not efficient. The intuition goes back to the bottleneck: In the case of excess supply, for example, the planner would like customer-sellers to use BB and buyers SMS to reduce the asset inflow into the dealers, so as to mitigate the bottleneck. Such a distinction between fast- and slow-moving assets is realistic and important. While corporate bonds on SMS trade in a few minutes (Hendershott and Madhavan, 2015), auctions of collateralized loan obligations (CLOs) can take a day or two (Hendershott et al., 2020). Asset-specific design and policies should be considered, as opposed to market-wide, blanket recommendations.

Contribution and related literature

The paper contributes to four strands of the literature. First, adding to the search models of OTC markets, this paper introduces the possibility for investors to contact *multiple* potential counterparties *at the same time*. Previous search models largely focus on BB as in DGP, Weill (2007), Vayanos and Weill (2008), Lagos and Rocheteau (2009), Lagos, Rocheteau, and Weill (2011), Üslü (2019), and HLW. A noteworthy difference is that in SMS, the split of trading gain between customers and dealers (their respective bargaining powers) is endogenous of the equilibrium dealer demographics. This feature distinguishes our model from other works that also have multiple dealers competing simultaneously for a given transaction. For example, Hendershott et al. (2017) consider a stylized model of how customers choose to form dealer networks. There, a buyer simultaneously contacts all dealers in her network, who then compete to find the asset for the customer. Similarly, in Wang (2017), any agent may query quotes within her network simultaneously. In these models, the split of trading gains between dealers and customers is exogenous. In Zhu (2012) and An (2020), customers *sequentially* contact possibly multiple dealers and the resulting endogenous trading gain splits arise due to other frictions like information asymmetry.

Second, this paper contributes to the theory of electronic OTC markets. Vogel (2019) studies a hybrid OTC market where investors can trade in both the traditional voice market and the electronic RFQ platform. Liu, Vogel, and Zhang (2017) compare the the electronic RFQ protocol in an OTC market with a centralized exchange market. Both papers model the RFQ trading similarly to the current paper, with the key difference being that their RFQ matching rates are exogenous, whereas they are endogenous of dealer demographics in this paper. Riggs et al. (2019) study the RFQ trading in Swap Exchange Facilitites. Their analysis highlights order size as an important determinant of customers' choice of trading mechanism. Our analysis complements theirs and highlights another factor, dealer demographics. In a different line, Saar et al. (2020) compare dealers' market making (direct liquidity provision) and matchmaking (searching on the customers' behalf for counterparties) and study the effects of bank dealers' balance sheet costs.

Third, there is a growing body of literature comparing centralized versus decentralized trading (Pagano, 1989; Chowdhry and Nanda, 1991) in various aspects. Babus and Parlatore (2017) study the endogenous formation of fragmented markets due to investors' strategic behavior. Glode and Opp (2019) compare the efficiency of OTC and limit-order markets in a setting where investors endogenously acquire expertise. Lee and Wang (2019) study uninformed and informed investors' venue choice through an adverse selection channel. Dugast, Üslü, and Weill (2019) examine banks' choice among centralized trading, OTC trading, or both, in a setting where the banks differ in their risky asset endowment and in their capacity of OTC trading. This paper instead compares the conventional voice trading versus the relatively new electronic trading within the OTC setting.

Finally, this paper contributes to the auctions literature with uncertain number of bidders (see, e.g., the survey by Klemperer, 1999) and to the literature on pricing with heterogeneously informed consumers (e.g., Butters, 1977; Varian, 1980; and Burdett and Judd, 1983). Apart from the above literature on OTC markets, applications of such "random pricing" mechanisms are also seen recently in exchange trading, as in Jovanovic and Menkveld (2021). The main insight from this paper is that such uncertainty about the number of quoters (bidders) can arise endogenously from the search process.

1 Model setup

Time is continuous. The model concerns the trading of an asset in fixed supply s.

Customers and dealers. There is a continuum of *c*ustomers with mass m_c and a continuum of *d*ealers with mass m_d. Both groups are risk-neutral, discount future utility at the same rate r, and can each hold either zero or one unit of the asset. An asset owner will be denoted by o and a non-owner n.

The agents derive flow utility when holding the asset. A customer owner derives $y(t) \in \{y_h, y_l\}$ (*h*igh or *l*ow), which evolves stochastically according to a continuous time Markov chain: $\mathbb{P}[y(t+\mathrm{d}t) = y_h | \, y(t) = y_l] = \lambda_u \mathrm{d}t$ and $\mathbb{P}[y(t+\mathrm{d}t) = y_l | \, y(t) = y_h] = \lambda_d \mathrm{d}t$, where λ_d and λ_u are the respective switching intensities. A dealer-owner instead derives constant flow utility y_d.

In summary, there are four types of customers, $\{ho, hn, lo, ln\}$, and two types of dealers, $\{do, dn\}$. Their population size at any time t are denoted by $m_\sigma(t)$ for $\sigma \in \{ho, hn, lo, ln, do, dn\}$, with $m_{ho}(t) + m_{hn}(t) + m_{lo}(t) + m_{ln}(t) = m_c$ and $m_{do}(t) + m_{dn}(t) = m_d$.

Both customers and dealers experience independent exogenous exit shocks: at a Poisson intensity f_d (resp., f_c) a dealer (resp., a customer) leaves the market and gets zero utility flow going forward. Immediately after leaving, she is replaced by a trader of the same type, so that the total population sizes, m_c and m_d, do not change.

Search. The setup above follows HLW (with all dealers having the same preference). Notably, customers cannot contact each other and have to search for dealers to trade with. We generalize how customers interact with dealers by introducing a trading technology characterized by $\{\rho, n, q\}$. Using the technology, at a Poisson process with intensity ρ, each customer can contact up to n dealers.[3] Each contact by a customer-buyer (-seller) is a "match" if the contacted dealer is of type-*do*

[3] Customers can choose to contact fewer than n dealers. Since there is no cost of contacting more, in equilibrium, they will always choose to contact n dealers. With contact costs, investors in Riggs et al. (2019) choose an interior number of contacts. Such a cost does not bring novel insights in the current model and, hence, is set to zero.

(-dn). The probability that any given contact turns into a match is π_{do} for a buyer (π_{dn} for a seller). A customer-buyer's probability of finding *at least one* matching dealer is then $1-(1-\pi_{do})^n$; and, similarly, $1-(1-\pi_{dn})^n$ for a customer-seller. Both π_{do} and π_{dn} are functions of the "availability" of the target dealer type; that is, $\pi_{do} = \pi\left(\frac{m_{do}}{m_d}\right)$ and $\pi_{dn} = \pi\left(\frac{m_{dn}}{m_d}\right)$. We assume the function $\pi(x)$ has support $x \in [0,1]$ and is monotone increasing with $\pi(0) = 0$, $\pi(1) = 1$, $\pi'(0) < \infty$, and $\pi(x) \geq x$. Consider the following two examples:

- *Pure random matching:* Each dealer is selected from the whole dealer population at random. In this case, $\pi(x) = x \in [0,1]$, which is standard, as in DGP and HLW.
- *Random matching with signals:* Right before contacting the dealers, each customer can observe signals $\{\delta_i\}$, $i \in [0, m_d]$. Each signal $\delta_i \in \{1, 0\}$ reveals correctly the inventory of dealer i with probability $\psi \in (\frac{1}{2}, 1]$: $\psi = \mathbb{P}[\delta_i = 1 | \, \sigma_i = do] = \mathbb{P}[\delta_i = 0 | \, \sigma_i = dn]$. One can think of ψ as the signal quality and interpret it as the transparency of dealer inventories. (The signals are conditionally independent of each other.) Customers can *direct their search* to the subset of dealers with a particular realization of a signal. Within the chosen subset, the search is random. A customer-buyer (-seller) would then like to direct her search only to the subset of dealers whose signals equal one (zero). Define

$$\pi(x) := \frac{\psi x}{\psi x + (1-\psi)(1-x)}. \tag{1}$$

  Then by Bayes' rule, each contact by a customer-buyer (-seller) has success rate $\pi_{do} = \pi\left(\frac{m_{do}}{m_d}\right)$ ($\pi_{dn} = \pi\left(\frac{m_{dn}}{m_d}\right)$). Note that $\pi(x)$ degenerates to $\pi(x) = x$ if $\psi \to \frac{1}{2}$ (uninformative signals).

Note that it is natural to require $\pi(x) \geq x$: the least a customer can do is to randomly search among all dealers, in which case each contact by a customer-buyer (-seller) has probability $\pi_{do} = \frac{m_{do}}{m_d}$ ($\pi_{dn} = \frac{m_{dn}}{m_d}$) to be a match as in "pure random matching."

Price determination. When a searching customer is in contact with n dealers:

- With probability q, the customer makes a take-it-or-leave-it offer (TIOLIO) to all the contacted dealers, who then choose to accept the offer or walk away. If more than one dealer accepts, the customer randomly chooses one to trade with.
- With probability $1-q$, the n dealers simultaneously make independent TIOLIOs to the

customer, who then chooses the best quote or walks away.

A contacted dealer may be unable to accommodate the contacting customer due to the inventory constraint (i.e., not a match). Importantly, each dealer makes his decision independently, not knowing the types of the other $(n-1)$ contacted dealers. To note, this specific price determination mechanism does not affect the results about demographics and welfare (Sections 2.1–2.3), which we show hold much more generally.

Parameter values and supports. We normalize the customer mass to $m_c = 1$ and require the dealer mass $m_d > 0$. We also require $s \in (0, 1+m_d)$ so as to study asset allocation meaningfully. All Poisson processes are independent of one another. The customers' preference-switching intensities $\lambda_u > 0$ and $\lambda_d > 0$. The agents' exit rates $f_c \geq 0$ and $f_d \geq 0$. We set $y_h > y_l$ so that some customers are of "high" type and some "low." An additional constraint on y_d will be introduced in Proposition 1 to ensure positive trading gains. The technology parameters have supports $\rho \in (0, \infty)$, $n \in \mathbb{N}$ (the natural numbers), and $q \in [0, 1]$.

Remarks

Remark 1. The trading technology is general enough to encompass some of the most common protocols in OTC trading. For example, the case of $n = 1$ can be thought of as customers reaching dealers by phone and negotiating the terms of trade via bargaining (BB, as in DGP and in HLW). The case of $n > 1$ captures technologies that allow a customer to reach multiple dealers in one click, hence the name "simultaneous multilateral search" (SMS). For example, this is the case for the RFQ protocol on electronic platforms (like MarketAxess and Swap Execution Facilities, SEFs); for auctions like bid/offer-wanted-in-competition (B/OWIC); and in housing markets where a seller can be in touch with possibly many buyers at the same time.

Remark 2. In practice, customers can choose how to get in touch with dealers. They can always call dealers (BB) but they can also click buttons on electronic platforms like RFQs (SMS). After exploring the equilibrium properties of one general technology in Section 2, we study how customers choose between "call" and "click" in Section 3.

Remark 3. The general trading technology is governed by three parameters:

- The search intensity ρ, inherited from DGP and HLW, implies that the technology connects a customer with dealers in exponential waiting time with mean $1/\rho$. For example, auctions on MarketAxess vary in length, from 5 to 20 minutes (Hendershott and Madhavan, 2015). Trading of collateralized loan obligations (CLOs) is typically organized through B/OWIC by email (Hendershott et al., 2020) and can take a considerably longer time.
- The search capacity n, new in this paper, flexibly nests BB ($n = 1$) with SMS ($n > 1$). For example, on Bloomberg Swap Execution Facility (SEF), this upper bound is set to $n = 5$ (Riggs et al., 2019). On MarketAxess, a customer typically contacts 20 to 30 dealers (Hendershott and Madhavan, 2015; O'Hara and Zhou, 2021).
- The probability q reflects the customer's ability to extract rents above and beyond the direct competition among dealers. In BB ($n = 1$) such ability arises due to customers' opportunities to bargain with dealers, where q reflects the customers' Nash bargaining power as in DGP and HLW. In SMS ($n > 1$) such ability arises as the searching customer may further negotiate the price after soliciting dealers' (non-firm) quotes.[4] On typical RFQ platforms like MarketAxess, q is effectively zero, as customers can only solicit quotes from dealers but cannot further negotiate with them afterwards (O'Hara and Zhou, 2021). Instead, when trading is less formally organized, q can be large. The BWICs to sell CLOs are conducted by email, where dealers often report "soft" quotes, that customer can improve upon. In housing markets, sellers often post indicative, negotiable asks.

Remark 4. Dealers sometimes broadcast indicative bids and offers to customers on electronic platforms (called "dealer runs," Section III.B of Bessembinder, Spatt, and Venkataraman, 2020). This feature likens the RFQ interpretation of the current model to models of "directed search," where dealers first post quotes and then customers direct their queries to selected dealers (see, e.g., Wright et al., 2020). Supplementary Appendix S2 studies a directed search model and shows that it is the limiting case of $\psi \to 1$, i.e., when the signals of dealer inventories become perfect in our special case of "random matching with signals." Thus, similar to Shi (2019) but with a different approach,

[4] Supplementary Appendix S6 provides an alternative price-setting mechanism that allows customers to first run a first-price auction among n contacted dealers and then bargain bilaterally with the winning dealer. We show that such an alternative price-setting mechanism is equivalent to the one presented here, in that, in expectation, it results in exactly the same split of trading gains.

our setup bridges “random matching” and “directed search.”

Remark 5. This paper focuses on SMS technologies like RFQ trading, which, to our knowledge, only lets customers search for dealers, not the other way. The model thus shuts down dealer-to-customer searches. However, in reality, dealers probably do take initiatives to reach customers (though not via SMS) to, e.g., arrange riskless principal trades. Studying dealers’ search for customers will be an interesting and fruitful future research that goes beyond the scope of this paper. See, e.g., Saar et al. (2020) for an analysis of dealers’ matchmaking versus market making.

Remark 6. We generalise the standard framework of DGP and HLW by introducing the exit shocks: the model without such shocks is a special case with $f_d = f_c = 0$. In reality such shocks might come from the labor market, where an employee-trader might be fired or assigned to a different post, or might be due to changing investment opportunities, where traders exit one market in order to participate in the other. These exit shocks provide parameter flexibility that ensures strictly positive trading gains (see Proposition 1 for details).

2 Stationary equilibrium

There are three sets of equilibrium objects: i) the demographics $\{m_\sigma\}$ (Section 2.1); ii) the value functions $\{V_\sigma\}$ (Section 2.2); and iii) the split of trading gains (Section 2.4, together with the dealer’s pricing strategies). We look for a stationary Markov perfect equilibrium, under which these objects are time-invariant constants. We also focus on symmetric equilibrium; that is, the agents of the same type have the same value functions and receive the same fraction of the trading gain. We discuss in Section 2.3 how welfare is affected by search parameters.

2.1 Demographics

There are in total six demographic variables, $\{m_{ho}, m_{ln}, m_{hn}, m_{lo}, m_{do}, m_{dn}\}$, one for each agent type. The following three conditions must hold in equilibrium by definition:

$$\text{market clearing:} \qquad m_{ho} + m_{lo} + m_{do} = s; \tag{2}$$

$$\text{total customer mass:} \qquad m_{ho} + m_{ln} + m_{hn} + m_{lo} = 1; \text{ and} \tag{3}$$

$$\text{total dealer mass:} \qquad m_{do} + m_{dn} = m_d. \tag{4}$$

In a stationary equilibrium, the total measure of *h*igh type customers must be time-invariant; i.e., the net flow during any instance $\mathrm{d}t$ must be zero:

$$\text{net flow of high type customers:} \qquad (m_{lo} + m_{ln})\lambda_u - (m_{ho} + m_{hn})\lambda_d = 0, \tag{5}$$

which also ensures that the net flow of low type customers is zero.

Two more equations, from agents' trading, help pin down the six demographic variables. In equilibrium, only two types of customers want to trade with dealers: The *lo*-type wants to sell to *dn*-buyer, and the *hn*-type wants to buy from *do*-seller. The other two types, *ho* and *ln*, stand by and do not trade (which is a conjecture for now, and we will later verify it after Proposition 1).

Consider the inflows to and the outflows from the the *lo*-sellers. In a short period of $\mathrm{d}t$, a "fringe" of $m_{lo}\rho\mathrm{d}t$ of sellers will be searching, each having probability

$$\nu_{lo} = \nu(\pi_{dn}) := 1 - (1 - \pi_{dn})^n$$

to find at least one *dn*-buyer (out of n) to trade with.[5] Hence, there is an outflow of $\rho m_{lo}\nu_{lo}\mathrm{d}t$ due to the searching *lo*-sellers. In addition, due to preference shocks, there is an inflow of $\lambda_d m_{ho}\mathrm{d}t$ and an outflow of $\lambda_u m_{lo}\mathrm{d}t$. In a stationary equilibrium, the sum of the in/outflows above must be zero:

$$\text{net flow of } lo\text{-sellers:} \qquad -\rho m_{lo}\nu_{lo} - \lambda_u m_{lo} + \lambda_d m_{ho} = 0. \tag{6}$$

Analogously, define $\nu_{hn} = \nu(\pi_{do})$ as the probability for a searching *hn*-buyer to find at least one

[5] The exact law of large numbers in Duffie, Qiao, and Sun (2019) is applied so that the fractions of the populations of each type are their expected values. See also Sun (2006) and Duffie and Sun (2007, 2012).

do-seller. Then the zero net flow condition for *hn*-buyers becomes

$$\text{net flow of } hn\text{-buyers:} \qquad -\rho m_{hn}\nu_{hn} - \lambda_d m_{hn} + \lambda_u m_{ln} = 0, \tag{7}$$

which is the last equation needed to pin down the stationary demographics:

Lemma 1 (Stationary demographics). The demographics Equations (2)-(7) uniquely pin down the population sizes $\{m_{ho}, m_{ln}, m_{hn}, m_{lo}\} \in (0,1)^4$ and $\{m_{do}, m_{dn}\} \in (0, m_d)^2$.

There are other stationarity conditions. For example, the *hn*-buyer-initiated trading volume amounts to $\rho m_{hn}\nu_{hn}$, while the *lo*-seller-initiated volume is $\rho m_{lo}\nu_{lo}$. They are also, respectively, the asset flow out of and into the dealer sector. Therefore, the trading volume intensity t must satisfy

$$t := \rho m_{hn}\nu_{hn} = \rho m_{lo}\nu_{lo}, \tag{8}$$

for otherwise the dealer-owner mass, m_{do}, will not be stable. Indeed, Equation (8) is guaranteed by $(6) - (7) + (5)$. Supplementary Appendix S1.1 shows that Equations (2)-(7) are indeed sufficient for the stationarity of all other types of agents. Note also that the exit shock intensities do not enter the above conditions, because the exited are immediately replaced by newborns.

2.2 Value functions

Denote by V_σ a type-σ agent's value function. Then the reservation values for the asset are

$$R_l := V_{lo} - V_{ln}, \; R_h := V_{ho} - V_{hn}, \text{ and } R_d := V_{do} - V_{dn}$$

for the low-type customers, the high-type customers, and the dealers, respectively. For an *lo*-seller and a *dn*-dealer to trade, the price p must fall between $R_l \le p \le R_d$; and likewise, for an *hn*-buyer and a *do*-dealer to trade, the price must fall between $R_d \le p \le R_h$. Such prices split the trading gains, which are written as, respectively,

$$\Delta_{dl} := R_d - R_l \text{ and } \Delta_{hd} := R_h - R_d \tag{9}$$

for the two kinds of trades. For now, we make the conjecture that there are positive trading gains: $R_l \le R_d \le R_h$, which will be guaranteed by a condition on y_d (see Proposition 1 below).

2.2.1 The split of the trading gain

The split of the trading gain between an hn-buyer and a do-dealer can always be written as $\gamma_{hn}\Delta_{hd}$ and $(1-\gamma_{hn})\Delta_{hd}$, where $\gamma_{hn} \in [0,1]$ represents the hn-buyer's "expected trading gain share." Likewise, the split of trading gain between a dn-dealer and an lo-seller can be written as $(1-\gamma_{lo})\Delta_{dl}$ and $\gamma_{lo}\Delta_{dl}$ for some lo-seller's expected trading gain share $\gamma_{lo} \in [0,1]$.

The shares $\{\gamma_{hn}, \gamma_{lo}\}$ reflect how in expectation prices are set between trading pairs. For now we allow general $\{\gamma_{hn}, \gamma_{lo}\} \in [0,1]^2$, so that our results up to Section 2.4 hold for arbitrary price-setting mechanisms, up to two minimal assumptions: (i) a trade occurs whenever at least one of the n contacts is a match; and (ii) the shares $\{\gamma_{hn}, \gamma_{lo}\}$ do not depend on the value functions $\{V_\sigma\}$ (but they can depend on any other endogenous variables).[6] We complete the equilibrium characterization by deriving $\{\gamma_{hn}, \gamma_{lo}\}$ endogenously in Section 2.4, verifying also the two minimal assumptions.

2.2.2 Hamilton-Jacobi-Bellman equations

Consider first an ho-bystander. Over a short period $\mathrm{d}t$, the ho-bystander gets utility $y_h\mathrm{d}t$ from holding the asset; plus, with intensity $\lambda_d\mathrm{d}t$, she switches to lo-type and her value changes by $V_{lo} - V_{ho}$; minus $(r+f_c)V_{ho}\mathrm{d}t$ due to discounting and exit shocks. Hence, her HJB equation is

$$0 = y_h + \lambda_d \cdot (V_{lo} - V_{ho}) - (r+f_c)V_{ho}. \tag{10}$$

Similarly, an ln-bystander has HJB equation

$$0 = \lambda_u \cdot (V_{hn} - V_{ln}) - (r+f_c)V_{ln}. \tag{11}$$

Consider next an lo-seller. Over $\mathrm{d}t$, her value increases by $y_l\mathrm{d}t$ due to the asset holding. It may also change by $V_{ho} - V_{lo}$ with intensity $\lambda_u\mathrm{d}t$ due to a preference shock. The value also reduces by $rV_{lo}\mathrm{d}t$ due to discounting. Finally, from trading she expects an instantaneous gain of $\rho\nu_{lo}\gamma_{lo}\Delta_{dl}\mathrm{d}t$—she searches for dealers at intensity ρ, finds at least one match out of the n contacts with probability ν_{lo}, and expects a trading gain share of γ_{lo}. For notation simplicity, we write $\zeta_{lo} := \rho\nu_{lo}\gamma_{lo}$ as an lo-seller's "expected trading gain intensity." Therefore, the lo-seller's HJB

[6] If $\{\gamma_{hn}, \gamma_{lo}\}$ depend on the value functions $\{V_\sigma\}$, they will also enter the Bellman equation system below in Section 2.2.2, thus *nonlinearly* affecting the equilibrium value functions and, hence, also welfare. We show later in Section 2.4, however, that this is not a concern in our setup.

equation is

$$0 = y_l + \lambda_u \cdot (V_{ho} - V_{lo}) - (r + f_c)V_{lo} + \zeta_{lo}\Delta_{dl}. \tag{12}$$

Similarly, an *hn*-buyer's HJB equation is

$$0 = \lambda_d \cdot (V_{ln} - V_{hn}) - (r + f_c)V_{hn} + \zeta_{hn}\Delta_{hd}, \tag{13}$$

where the expected trading gain intensity is $\zeta_{hn} := \rho\nu_{hn}\gamma_{hn}$.

Finally, consider the dealers. A *do*-seller's HJB equation has the similar structure as before

$$0 = y_d - (r + f_d)V_{do} + \zeta_{do}\Delta_{hd}, \tag{14}$$

just without the type-switching term. To find a *do*-seller's expected trading gain intensity ζ_{do}, note that the total trading gain from all *hn*-buyer initiated trades amounts to $m_{hn}\rho\nu_{hn}\Delta_{hd}$. Since each *hn*-buyer expects $\zeta_{hn}\Delta_{hd}$, a *do*-seller gets the per capita remainder; that is,

$$\zeta_{do} := \frac{m_{hn}\rho\nu_{hn} - m_{hn}\zeta_{hn}}{m_{do}} = \frac{m_{hn}\rho\nu_{hn}}{m_{do}}(1 - \gamma_{hn}).$$

Similarly, a *dn*-buyer has

$$0 = -(r + f_d)V_{dn} + \zeta_{dn}\Delta_{dl} \tag{15}$$

with expected trading gain intensity

$$\zeta_{dn} := \frac{m_{lo}\rho\nu_{lo} - m_{lo}\zeta_{lo}}{m_{dn}} = \frac{m_{lo}\rho\nu_{lo}}{m_{dn}}(1 - \gamma_{lo}).$$

Recall from Equation (9) that both trading gains Δ_{hd} and Δ_{dl} are linear combinations of the value functions $\{V_\sigma\}$. Thus, Equations (10)-(15) constitute a linear system with six equations and six unknowns, solved by the proposition below.

Proposition 1 (Equilibrium value functions). Let $\xi := \frac{r+f_d}{r+f_c}$ and define $\overline{y}_d$ and $\underline{y}_d$ as

$$\overline{y}_d := y_h\xi - \frac{(y_h - y_l)\lambda_d\xi}{\lambda_d + \lambda_u + r + f_c} \quad \text{and} \quad \underline{y}_d := y_l\xi + \frac{(y_h - y_l)\lambda_u\xi}{\lambda_d + \lambda_u + r + f_c}.$$

When $\underline{y}_d \leq y_d \leq \overline{y}_d$, the reservation values satisfy $0 < R_l < R_d < R_h$ and the value functions are the solution to the linear equation systems (10)-(15). (Note that $\overline{y}_d > \underline{y}_d$ always holds.)

The parameter constraint of $y_d \in (\overline{y}_d, \underline{y}_d)$ ensures positive trading gains, i.e., $\Delta_{hd} = R_h - R_d > 0$

and $\Delta_{dl} = R_d - R_l > 0$. When $y_d \notin (\overline{y}_d, \underline{y}_d)$, intuitively, the dealers are no longer "intermediaries" between buyers and sellers and the economy might enter a steady state without trading. For example, suppose $y_d \notin (\overline{y}_d, \underline{y}_d)$ and $R_d > R_h$. Then *do*-dealers and *hn*-buyers do not trade, and by the stationarity condition (8), there must be no trade between *dn*-dealers and *lo*-sellers, either. Therefore, through the rest of the paper, we focus on the more interesting and empirically relevant case with trades by assuming that $y_d \in (\overline{y}_d, \underline{y}_d)$ always holds.

Finally, we can now verify the conjecture that *ho*- and *ln*-customers do stand by: If one did switch to trading, her expected trading price p would fall between the reservation values. For example, if an *ho*-customer sold, she would get a price between $R_l = V_{lo} - V_{ln} \leq p \leq V_{do} - V_{dn} = R_d$ and continue with V_{hn}. Given the positive trading gains, we have $R_d < R_h = V_{ho} - V_{hn}$, implying $V_{ho} > V_{hn} + p$, and the *ho*-customer never wants to sell. The same holds for an *lo*-customer. They are really bystanders.

2.3 Search technology and allocation efficiency

This subsection examines how allocation efficiency is affected by search technology. We are particularly interested in the contrast of the two search parameters, the intensity ρ and the capacity n—how fast customers can find dealers versus how many dealers can be reached in one "click."

We focus on the case in which the asset is in excess demand, formally defined below:

Lemma 2. The *lo*-sellers are on the short side of the market, i.e., $m_{lo} < m_{hn}$, if and only if

$$s < \eta + \frac{1}{2} m_d, \text{ where } \eta := \frac{\lambda_u}{\lambda_u + \lambda_d}.$$

Intuitively, the threshold $\eta + \frac{1}{2} m_d$ represents the asset's "intrinsic demand:" The fraction η is the size of the steady-state high-type customers, who are natural holders of the asset. In addition, since the dealers are homogeneous, half of them are also natural asset holders. When such intrinsic demand exceeds the supply s, the *hn*-buyers (*lo*-sellers) are on the short (long) side of the market. The calibration in Appendix A finds that $s < \eta + \frac{1}{2} m_d$, suggesting that the RFQ trading of corporate bonds is, on average, in excess demand. Therefore, below we mainly focus on the case of excess demand. (The analysis of the excess supply case is symmetric.) The only exception is Section 3.2, where we consider excess supply due to the fire selling of corporate bonds.

2.3.1 Trading volume and customer sizes

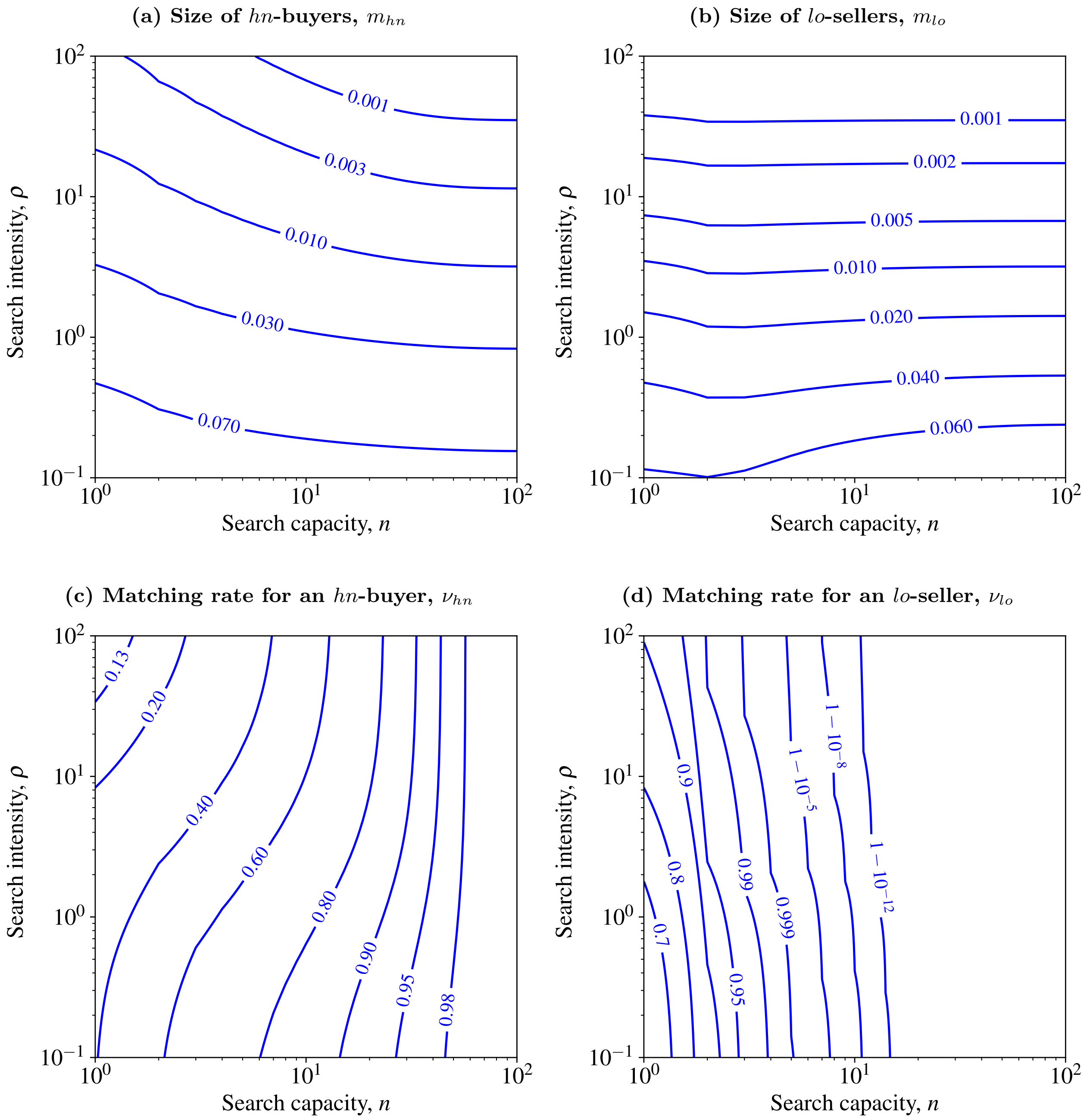
The contrasting effects between ρ and n can be seen most clearly from Figure 1(a) and 1(b), which are based on parameters calibrated against the real-world RFQ trading of corporate bonds (Appendix A). Specifically, while the intensity ρ always reduces the customer sizes, the effect of the capacity n is more nuanced: It monotonically reduces m_{hn} (the long side) but might increase m_{lo} (the short side). The proposition below sums up the patterns formally.

Proposition 2 (Search technology, customer sizes, and trading volume). The search intensity ρ reduces both m_{hn} and m_{lo}. The search capacity n reduces the long-side customer mass but has ambiguous effect on the short-side customer mass. In particular, when ρ is sufficiently small, the short-side customer mass increases with n. The trading volume intensity t increases in both n and ρ.

The proposition also states that both search technologies monotonically improve trading volume. This is rather intuitive as the matching between customers and dealers become more efficient with either higher ρ or larger n. Such increased trading volume, however, does not always translate to allocation efficiency. Notably, a larger search capacity n might exacerbate inefficient allocation: more low-type customers end up holding the asset. Indeed, the increase of m_{lo} with n can be seen along every horizontal cut in Figure 1(b), thought more saliently for lower than higher ρs. This is "inefficient" as such holdings could have been better appreciated by high-type customers (as the asset is in excess demand). As explained below, such inefficiency has a novel dealer "bottleneck" effect to blame.

2.3.2 The bottleneck effect

Note that an increase in the search capacity n does help matching: Both the probabilities ν_{lo} and ν_{hn} of finding at least one dealer counterparty increase with n, as can be seen from Figure 1(c) and (d) (formally shown in Lemma S1.2 in the supplementary appendix). However, the magnitudes of the increases are far from equal. The increment in ν_{hn} is much more substantial than that in


(a) Size of hn-buyers, m_{hn}

(b) Size of lo-sellers, m_{lo}

(c) Matching rate for an hn-buyer, ν_{hn}

(d) Matching rate for an lo-seller, ν_{lo}

Figure 1: Customer sizes and matching rates. This figure plots how the search intensity ρ and the search capacity n affect the customer sizes in (a) and (b) and the matching rates in (c) and (d). Apart from ρ and n, the other parameters are set at $s = 0.12$, $m_d = 0.10$, $\lambda_u = 0.04$, and $\lambda_d = 0.31$, based on the calibration exercise detailed in Appendix A. The other model parameters are irrelevant here as the equilibrium demographics do not depend on them; see Equations (2)-(7) and Lemma 1.

ν_{lo}. This is because the *lo*-sellers are on the short side of the market and there are many more *dn*-dealers to find (than *do*-dealers for the long side *hn*-buyers). Correspondingly, ν_{lo} is much closer to its upper bound of 100% and cannot be increased by as much as ν_{hn}. Put differently, the increase in n matches many more *hn*-*do* pairs than *lo*-*dn* pairs.

The *lo*-*dn* trades let the asset flow into the dealer sector, while the *hn*-*do* trades let the asset flow out of the dealers. The above asymmetric effects of n—the substantially smaller inflow compared to the outflow—imply that the asset flow is clogged when passing through the dealer sector, hence the "bottleneck."[7] As the dealers give out a lot of the asset to *hn*-buyers but only take in little from *lo*-sellers, the *lo*-seller size m_{lo} increases and the *hn*-buyer size m_{hn} reduces.

Summing up, there are two pairs of asymmetric effects: In terms of matching probability, ν_{hn} increases much more than ν_{lo}. In terms of population sizes, m_{hn} shrinks, whereas m_{lo} balloons. These effects ensure the stationarity of dealers in equilibrium, with $\rho\nu_{lo}m_{lo} = \rho\nu_{hn}m_{hn}$ (Equation 8). The above discussion is for the case of excess demand. When the asset is in excess supply, the dealer bottleneck also arises as n increases: A substantially larger asset inflow than outflow from the dealers raises both m_{do} and m_{hn}, as the matching probability ν_{lo} increases much more than ν_{hn}.

It is worth emphasizing that the bottleneck arises only with the search capacity n but *not* with the intensity ρ. This is because ρ scales up both the inflow $\rho\nu_{lo}m_{lo}$ and the outflow $\rho\nu_{hn}m_{hn}$ and there is no asymmetry. This is a novel finding, thanks to the flexibility of n.[8] For example, the bottleneck does not manifest in HLW, as their customers and dealers only meet bilaterally ($n = 1$).

Proposition 2 emphasizes that the dealer bottleneck arises only when the search intensity ρ is "low." How "low" is low enough? To provide some perspective, Appendix A calibrates the model parameters against the real-world RFQ trading of corporate bonds and finds robust presence of

[7] The terminology of "bottleneck" also emphasizes that the inefficiency hinges on the existence of a sector of dealers. Absent of such intermediaries, for example, the matching between the high-type and low-type customers always results in the maximum trading gains, hence no inefficiency.

[8] Supplementary Appendix S4 studies an extension of the model, where customers are allowed to endogenously choose their search intensity (subject to a flow cost). In equilibrium, *lo*-sellers search with ρ_{lo}, while *hn*-buyers with a possibly different ρ_{hn}. Numerically, such endogenously asymmetric ρs still create *no* dealer bottleneck. Our analysis suggests that this is because the search intensities still enter the stationarity condition proportionally on both sides: $\rho_{lo}m_{lo}\nu_{lo} = \rho_{hn}m_{hn}\nu_{hn}$. In contrast, the search capacity n asymmetrically affects the matching rates ν_{lo} and ν_{hn} (by exponentiating the intrinsically different matching probabilities π_{dn} and π_{do}).

dealer bottlenecks, suggesting room for improving the efficiency of RFQ trading in practice.

2.3.3 Welfare

When does the dealer bottleneck translate to welfare losses? We examine next welfare as the present value of all asset-owners' utility flows: $w := \frac{1}{r}(y_h m_{ho} + y_d m_{do} + y_l m_{lo})$. Note that, unsurprisingly, welfare is endogenously determined only by population sizes, because the pricing strategies (Section 2.4) only affect the cut but not the size of the "pie." Also, only the discount rate r, but not the exit shock intensities $\{f_c, f_d\}$, enter the welfare expression, because an exited agent is immediately replaced by a newborn, who inherits the same utility flow.

Let the customer owners' average utility flow be $\hat{y} := \frac{m_{ho}y_h + m_{lo}y_l}{m_{ho}+m_{lo}}$. Welfare can then be rewritten as

$$w = \frac{y_d}{r} m_{do} + \frac{\hat{y}}{r}(m_{ho} + m_{lo}) = \frac{y_d}{r} m_{do} + \frac{\hat{y}}{r}(s - m_{do}), \tag{16}$$

where the second equality follows the market clearing condition (2). That is, the m_{do} mass of dealers get a flow utility y_d, while the $(s - m_{do})$ mass of customer-owners get an average flow utility $\hat{y}$. When the search intensity ρ is low, $\hat{y}$ becomes approximately exogenous:

$$\hat{y} = \frac{m_{ho}y_h + m_{lo}y_l}{m_{ho} + m_{lo}} \approx \eta y_h + (1 - \eta) y_l \tag{17}$$

because the stationarity condition (6) implies that $m_{lo}\lambda_u \approx m_{ho}\lambda_d$ and hence $\frac{m_{ho}}{m_{lo}} \approx \frac{\lambda_u}{\lambda_d} = \frac{\eta}{1-\eta}$.

Expression (16) highlights that for low ρ, welfare simply depends on the split of the asset between dealers (m_{do}) and customers ($s - m_{do}$). In particular, welfare increases (decreases) with m_{do} when $y_d > \hat{y}$ ($< \hat{y}$), as dealers are the better (worse) users than an *average* customer. When n increases, a "swelling" bottleneck of m_{dn} clogs the asset flow, reducing m_{do}. Welfare losses then occur with larger n, if $y_d > \hat{y}$, as seen in the low-ρ range of Figure 2.

Proposition 3 (Search technology and welfare). A higher search intensity ρ always improves welfare. A larger search capacity n improves welfare when ρ is sufficiently high. If ρ is instead low enough and if $y_d >$ ($<$) $\hat{y}$, a larger n reduces welfare, if the asset is in excess demand (supply).

Consistent with this prediction, Appendix A shows that welfare losses due to the bottleneck can

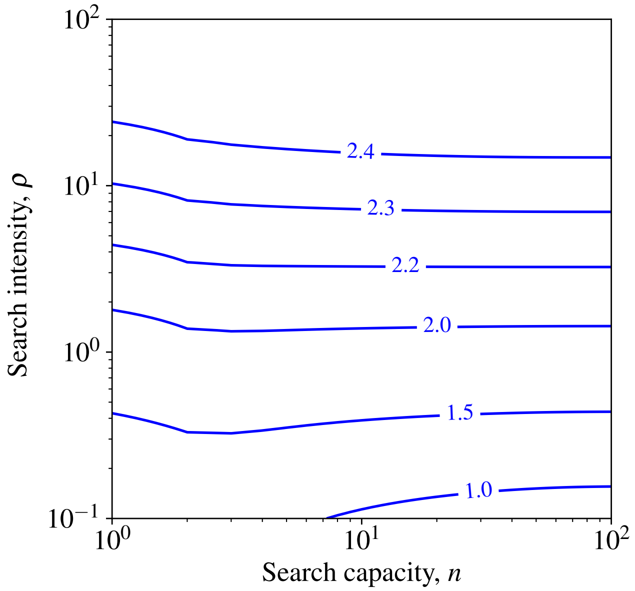


Figure 2: Search technology and welfare. This figure plots how the search intensity ρ and the search capacity n affect welfare, $w = \frac{1}{r}(y_h m_{ho} + y_d m_{do} + y_l m_{lo})$ Apart from ρ and n, the other four demographic parameters are set at $s = 0.12$, $m_d = 0.10$, $\lambda_u = 0.04$, and $\lambda_d = 0.31$, based on the calibration exercise detailed in Appendix A. The uncalibrated valuation parameters are set at $r = 0.05$, $y_h = 1.0$, $y_d = 0.9$, and $y_l = 0.0$. (The exit intensities $\{f_c, f_d\}$ and the customers' intrinsic bargaining power q are irrelevant here.)

indeed arise in the RFQ trading of corporate bonds, precisely when the dealer valuation for the bond is high.

It should be noted that Proposition 3 only compares welfare between the two steady states, before and after a shock in the search technology ρ or n. This neglects the transition dynamics induced by the shock, or more precisely, the welfare flows enjoyed by the traders in between the two steady states. In this sense, the proposition implicitly focuses on the *long-run* effects of search technology shocks.

Naturally, one might ask: if such transition welfare flows are accounted for, does the welfare ranking stated in Proposition 3 still hold? Supplementary Appendix S5 analyzes this question and finds an intuitive answer: When the discount rate r is relatively small, Proposition 3 remains robust. This is because with a lower r, welfare—the present value of all future utility flows—puts

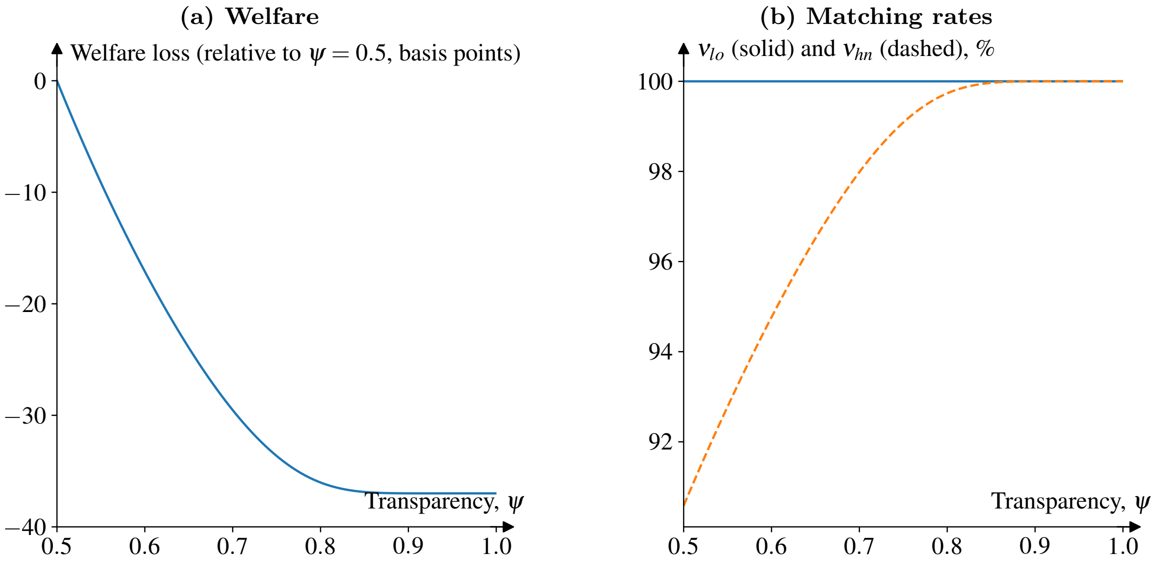


Figure 3: Effects of transparency, ψ. This figure plots how transparency ψ affects welfare in Panel (a) and the matching rates in Panel (b). The illustration is for a low level of search intensity $\rho = 1.0$. The other demographic parameters are set following the calibration in Appendix A: $n = 27$, $s = 0.12$, $m_d = 0.1$, $\lambda_d = 0.31$, and $\lambda_u = 0.04$, which are sufficient for Panel (b). For Panel (a), the additional valuation parameters are set to $y_h = 1.0$, $y_d = 0.9$, and $y_l = 0.0$. (The exit intensities $\{f_c, f_d\}$ and the customers' intrinsic bargaining power q are irrelevant here.)

a higher weight on future flows.

2.3.4 Inventory transparency and allocation efficiency

Our analysis so far admits generic forms of $\pi(\cdot)$, the probability for a searching customer's contact to be a match. Below we turn to the specific parameterization of "random matching with signals" (p. 8), with $\pi(\cdot)$ given by Equation (1). We interpret customers' signal quality ψ as dealer *inventory transparency* and examine how it affects welfare.

Proposition 4 (Transparency and welfare). Better inventory transparency ψ improves welfare only when ρ is sufficiently high. If ρ is low enough and if $y_d > (<)\ \hat{y}$, a higher ψ reduces welfare when the asset is in excess demand (supply).

Figure 3(a) shows an example where improved transparency hurts welfare. The key intuition is that the change in transparency ψ asymmetrically affects customers on the short and the long sides of the market, similar to the asymmetric effects of n in Section 2.3. A higher ψ improves matching by increasing both ν_{hn} and ν_{lo}, helping customers direct more accurately their searches to dealers with the right inventory capacity. Yet, the short side matching rate (ν_{lo}) increases much less than the long side (ν_{hn}), since it is already close to the upper bound of 100%, as illustrated in Figure 3(b). (In fact, ν_{lo} is too close to 100% to begin with, making it almost flat in the illustration.) The bottleneck again emerges and possibly hurts welfare, mirroring Proposition 3.

The dissemination of post-trade information of corporate bonds via TRACE (Transaction Reporting and Compliance Engine), starting in 2002, was perhaps the most significant transparency shock in the corporate bonds market. A large volume of the literature has documented its impact on market quality, applauding the improved liquidity and the reduced trading costs (e.g., Bessembinder, Maxwell, and Venkataraman, 2006, Edwards, Harris, and Piwowar, 2007, and, Goldstein, Hotchkiss, and Sirri, 2007). The extant theory models also seem to agree that welfare always improves with inventory transparency (e.g., Cujean and Praz, 2015, who study *bilateral* searches by customers, without going through a dealer sector). To the extent that post-trade transparency from TRACE also improves customers' inference about dealer inventories, our model cautions that the resulting better matching—the improved "liquidity"—not necessarily always translates to better welfare in terms of allocation (Figure 3a vs. 3b). In particular, our model highlights the importance of empirically examining how dealer inventories respond to such transparency shocks.

2.4 The endogenous split of trading gain

The analysis so far only requires the general form of expected trading gain shares, $\{\gamma_{hn}, \gamma_{lo}\}$. Under the assumed trading mechanism ("Price determination" on p. 8), such expected trading gain shares can be *endogenously* determined:

Proposition 5 (The split of trading gains). Define $\gamma(\pi, n) := q + (1-q)\Big(1 - \frac{n\pi\cdot(1-\pi)^{n-1}}{1-(1-\pi)^n}\Big)$ for $\pi \in (0,1)$ and $n \in \mathbb{N}$. Then, $\gamma_{hn} = \gamma(\pi_{do}, n)$ and $\gamma_{lo} = \gamma(\pi_{dn}, n)$.

This proposition thus completes the equilibrium characterization. Several features are worth discussing. First, as long as $n \geq 2$, $\gamma(\cdot)$ is strictly increasing in π, from $\gamma(0, n) = q$ to $\gamma(1, n) = 1$.

Take, for example, an *hn*-buyer searching for *do*-dealers. With a higher π_{do}, each contacted *do*-dealer knows that she is more likely competing with some other *do*-dealers among the other $(n-1)$ contacts. Such fiercer competition gives more trading gains to the *hn*-buyer. Indeed, when $\pi_{do} \to 1$, *hn*-buyers extract full surplus with $\gamma_{hn} \to 1$ from the dealers' perfect competition. On the other extreme, if $\pi_{do} \to 0$, each *do*-dealer knows that she is likely the monopolist among all n contacted and therefore quotes a monopolistic price. Indeed, as $\pi_{do} \to 0$, $\gamma_{hn} \to q$, which is the baseline probability that the customer can make TIOLIOs to the contacted dealers.

Second, when $n = 1$, $\gamma_{hn} = \gamma_{lo} = q$, as if the searching customer engages in a Nash bargaining with one matched dealer with respective bargaining power parameters q and $1 - q$. Our setup thus nests such exogenous splits of trading gains, commonly assumed in the literature (see, e.g., DGP and HLW). When $n \geq 2$, our model highlights that under SMS, the expected trading gain shares $\{\gamma_{hn}, \gamma_{lo}\}$ are endogenous, in particular, of the dealer composition m_{do} and m_{dn}. Such an endogenous split of trading gains is a distinguishing feature of our model.

Finally, whenever $n \geq 2$, the contacted dealers compete against an *unknown number of others*, as some of the n contacted dealers might not be of the matching type. That is, every contacted matching dealer knows that there is a non-zero probability that she actually is the only match. As is known in the literature (e.g., Burdett and Judd, 1983), in this case, dealers follow mixed strategies in setting their prices. This suggests that dealers' strategic behavior can be a source of price dispersion. Even though dealers are homogeneous in our model, it still features price dispersion, a robust empirical feature of OTC markets. For example, Hendershott and Madhavan (2015) document a significant dispersion in dealers' responding quotes in corporate bond market. Hau et al. (2017) find evidence for price dispersion in foreign exchange derivatives.

Corollary 5 only characterizes the split of trading gains. The implied $\gamma(\pi, n)$ also feed back to the equilibrium price formation in terms of dealers' quotes, the average price level in the economy, and the price dispersion. Supplementary Appendix S3 detail these results regarding the price.

3 SMS versus BB: How to search

In real-world trading, investors can choose their trading technologies. For example, while bilateral bargaining is still the dominant form of trading in corporate bonds, electronic platforms with RFQ protocols have been on the rise (O'Hara and Zhou, 2021). We consider investors' choice of "Click or Call" (Hendershott and Madhavan, 2015) in this section.

Specifically, we introduce two technologies, BB and SMS. They differ in parameters $\{n^k, \rho^k, q^k\}$, $k \in \{\mathrm{BB}, \mathrm{SMS}\}$ (some realistic parameter restrictions are imposed below). Each customer can choose, at any point in time, which technology to use to contact dealers, if she wants to trade. All dealers can be reached either by BB or by SMS. The other model ingredients remain the same as in Section 1.

Section 3.1 analyzes how customers choose between the two technologies in a steady state equilibrium. We then examine whether SMS-like electronic trading (e.g., RFQ) can completely replace traditional bilateral bargaining. The answer is no, as Section 3.2 shows that in stress periods (e.g., after a fire sale), BB is used more often than SMS. Finally, Section 3.3 draws implications on welfare, policy, and market design.

Parameter constraints. Motivated by "calls" (BB) and "clicks" (SMS), we assume

$$n^{\mathrm{BB}} = 1, \quad n^{\mathrm{SMS}} > 2, \text{ and } \rho^{\mathrm{BB}} \leq \rho^{\mathrm{SMS}}. \tag{18}$$

In a bilateral call, a customer bargains with one dealer, hence $n^{\mathrm{BB}} = 1$. By clicking, a typical real-world RFQ protocol connects the customer to multiple dealers, at least three in most of the applications (see Remark 3), hence $n^{\mathrm{SMS}} > 2$.[9] Earlier research finds that electronic platforms can "provide considerable time savings relative to ... bilateral negotiations" (Hendershott and Madhavan, 2015); and can "improve the speed of execution" (O'Hara and Zhou, 2021), motivating $\rho^{\mathrm{BB}} \leq \rho^{\mathrm{SMS}}$.

The probabilities to set prices in respective technology, q^{BB} and q^{SMS}, also play an important role. In most of the applications (e.g., MarketAxess), a customer using RFQ is always on the

[9] Excluding the special case of $n^{\mathrm{SMS}} = 2$ reduces the cases to consider when characterizing the equilibrium, streamlining the exposition. The full characterization for $n^{\mathrm{SMS}} \geq 2$ is provided in the proof of Proposition 6.

receiving end of dealers' TIOLIOs, suggesting that $q^{\mathrm{SMS}} = 0$. On the other hand, in bilateral calls, there is always room for negotiation and it is natural to expect that $q^{\mathrm{BB}} > 0$. We impose no such constraints here and proceed to examine how q^{SMS} and q^{BB} affect the customers' technology choices.

3.1 Choosing between SMS and BB

As before, we only focus on steady states, characterized by three sets of equilibrium objects: (i) customers' optimal technology choices, (ii) demographics, and (iii) value functions. Compared to Section 2, the novel part is the analysis of (i), detailed below. The analyses of (ii) and (iii) are analogous to those in Section 2 and, hence, collated in Supplementary Appendix S1.2-S1.3.

Recall from Section 1 that there are four types of customers, $\sigma \in \{ho, ln, hn, lo\}$. Now the σ-type customers can be further split into subtypes σ-BB and σ-SMS, which we distinguish by superscripting the relevant variables with the chosen technology $k \in \{\mathrm{BB}, \mathrm{SMS}\}$. For example, their masses satisfy $m_\sigma^{\mathrm{BB}} + m_\sigma^{\mathrm{SMS}} = m_\sigma$ and they have (possibly different) value functions V_σ^{BB} and V_σ^{SMS}.

The analysis can be simplified in two ways. First, note that in a stationary equilibrium, the value functions are time-invariant. That is, if a type-σ customer prefers one technology over the other at some point of time, her technology choice will persist until her type changes (due either to a preference shock or to trading). Hence, without loss of generality, we can focus on a type-σ customer's technology choice at the moment she becomes type-σ. Second, both ho and ln customers will be bystanders in equilibrium, just like in the case of one trading technology before. Therefore, there is no need to distinguish ln^{SMS} versus ln^{BB} or ho^{SMS} versus ho^{BB}. Only the technology choices of the trading customers, hn and lo, need to be studied below.

Denote by $\theta_\sigma \in [0, 1]$ the probability of a customer, who just received a preference shock and becme type-σ, to choose SMS (hence choosing BB with probability $1 - \theta_\sigma$), where $\sigma \in \{hn, lo\}$.

Then

$$\theta_\sigma \begin{cases} = \mathbb{1}_{\{V_\sigma^{\mathrm{SMS}} > V_\sigma^{\mathrm{BB}}\}}, & \text{if } V_\sigma^{\mathrm{SMS}} \neq V_\sigma^{\mathrm{BB}}; \\ \in [0,1], & \text{if } V_\sigma^{\mathrm{SMS}} = V_\sigma^{\mathrm{BB}}. \end{cases} \tag{19}$$

We shall focus on *symmetric* equilibria, where all customers of type σ choose the same θ_σ.

To sustain an equilibrium, the technology choices $\{\theta_{hn}, \theta_{lo}\}$ must agree with the value functions $\{V_\sigma\}$ according to Equation (19). The value functions are, in turn, chained to $\{\theta_{hn}, \theta_{lo}\}$ via many layers of endogenous variables (see Supplementary Appendix S1.3): the trading gain intensities $\{\zeta_\sigma\}$, the dealers' pricing, and the many demographic variables $\{m_\sigma\}$— a big fixed-point problem. It turns out that the equilibrium $\{\theta_{hn}, \theta_{lo}\}$ ultimately boil down to comparing the probabilities of finding a match, i.e., $\pi_{do} = \pi\left(\frac{m_{do}}{m_d}\right)$ and $\pi_{dn} = \pi\left(\frac{m_{dn}}{m_d}\right)$, with some threshold π^*:

Lemma 3. If the technologies satisfy

$$\rho^{\mathrm{SMS}} q^{\mathrm{SMS}} n^{\mathrm{SMS}} < \rho^{\mathrm{BB}} q^{\mathrm{BB}} n^{\mathrm{BB}}, \tag{20}$$

then Equation (19) can be equivalently written as

$$\theta_{hn} \begin{cases} = \mathbb{1}_{\{\pi_{do} > \pi^*\}}, & \text{if } \pi_{do} \neq \pi^* \\ \in [0,1], & \text{if } \pi_{do} = \pi^* \end{cases} \text{ and } \theta_{lo} \begin{cases} = \mathbb{1}_{\{\pi_{dn} > \pi^*\}}, & \text{if } \pi_{dn} \neq \pi^* \\ \in [0,1], & \text{if } \pi_{dn} = \pi^* \end{cases}, \tag{21}$$

where π^* uniquely solves $z^{\mathrm{SMS}}(\pi) = z^{\mathrm{BB}}(\pi)$, with $z^k(\cdot)$ defined in Equation (23) below for $k \in \{\mathrm{SMS}, \mathrm{BB}\}$. If, instead, $\rho^{\mathrm{SMS}} q^{\mathrm{SMS}} n^{\mathrm{SMS}} \geq \rho^{\mathrm{BB}} q^{\mathrm{BB}} n^{\mathrm{BB}}$, then $\theta_{hn} = \theta_{lo} = 1$.

Below we discuss the key steps behind this lemma. First, the value functions are pinned down by the HJB equations (S11)-(S14) in Supplementary Appendix S1.3. The proof of Lemma 3 shows that V_σ^k is a monotone function in ζ_σ^k, for $\sigma \in \{lo, hn\}$. This intuitive result says that when a searching customer chooses between SMS vs. BB, she is essentially comparing the trading gain intensities $\zeta_\sigma^{\mathrm{SMS}}$ vs. $\zeta_\sigma^{\mathrm{BB}}$. Hence, the technology choices (19) can be equivalently written as:

$$\theta_\sigma \begin{cases} = \mathbb{1}_{\{\zeta_\sigma^{\mathrm{SMS}} > \zeta_\sigma^{\mathrm{BB}}\}}, & \text{if } \zeta_\sigma^{\mathrm{SMS}} \neq \zeta_\sigma^{\mathrm{BB}}; \\ \in [0,1], & \text{if } \zeta_\sigma^{\mathrm{SMS}} = \zeta_\sigma^{\mathrm{BB}}. \end{cases} \tag{22}$$

Second, analogous to $\{\zeta_{lo}, \zeta_{hn}\}$ in Section 2.2, we write $\zeta_{hn}^k = z^k(\pi_{do})$ and $\zeta_{lo}^k = z^k(\pi_{dn})$, where

$z^k(\cdot)$ is defined for $\pi \in (0,1)$ as

$$z^k(\pi) := \rho^k \nu^k(\pi)\gamma^k(\pi) = \rho^k \cdot \Big(1-(1-\pi)^{n^k}\Big)\left(q^k + (1-q^k)\left(1 - \frac{n^k\pi \cdot (1-\pi)^{n^k-1}}{1-(1-\pi)^{n^k}}\right)\right). \quad (23)$$

The superscript k is not exponent but indicates the technology $k \in \{\text{BB}, \text{SMS}\}$. That is, customers essentially choose $\{\theta_\sigma\}$ by examining whether and how $z^{\text{SMS}}(\pi)$ and $z^{\text{BB}}(\pi)$ cross each other.

Lemma 3 essentially characterizes such crossing. Under the condition (20), $z^{\text{SMS}}(\pi)$ crosses $z^{\text{BB}}(\pi)$ from below once at $\pi^* \in \left(0, \frac{1}{2}\right)$. That is, a *hn*-buyer (*lo*-seller) prefers BB over SMS when $\pi_{do} < \pi^*$ ($\pi_{dn} < \pi^*$). This might come as a surprise, given that the condition (18) has guaranteed that SMS not only helps reach dealers faster but also induces more competitive quotes. Why would a customer still prefer BB?

To see the potential advantage of BB, consider for example an *hn*-buyer looking for *do*-sellers. Suppose m_{do} is very low and, hence, so is $\pi_{do} = \pi\left(\frac{m_{do}}{m_d}\right)$. Then the *hn*-buyer customer finds one counterparty dealer with probability approximately $n^k\pi_{do}$—one and only one success from n^k Bernoulli draws at rate π_{do}. (For small π_{do}, finding multiple dealers is negligibly unlikely.) It follows that a successfully contacted dealer in this case knows that she is almost surely a monopolist and will quote a very expensive ask, leaving no trading gain to the *hn*-buyer. The customer only gets non-zero trading gain if she can make a TIOLIO, i.e., with probability q^k. Taken together, for small π, the customers' trading gain intensity is $z^k(\pi) \approx \rho^k \cdot (n^k\pi) \cdot q^k$. Comparing BB with SMS in this case yields:

$$\lim_{\pi \downarrow 0} \frac{z^{\text{BB}}(\pi)}{z^{\text{SMS}}(\pi)} = \frac{\rho^{\text{BB}} n^{\text{BB}} q^{\text{BB}}}{\rho^{\text{SMS}} n^{\text{SMS}} q^{\text{SMS}}}.$$

The condition (20), therefore, ensures that for sufficiently small π, i.e., for relatively few counterparty dealers, BB has an advantage over SMS. In real-world trading, the condition (20) seems to hold because customers using SMS, like RFQ protocols, do not have many opportunities, if at all, to further bargain with dealers. We, therefore, argue that q^{SMS} is close to zero in reality.[10]

[10] Complementing the condition (20), the condition (18) in turn ensures that SMS is preferred when there are sufficiently many dealer counterparties.That is, $\lim_{\pi\uparrow 1}\big(z^{\text{BB}}(\pi)/z^{\text{SMS}}(\pi)\big) = \rho^{\text{BB}} q^{\text{BB}}/\rho^{\text{SMS}} \leq 1$. It is interesting to note that only q^{BB} appears but not q^{SMS} in the limit of $\pi \uparrow 1$. With $n^{\text{SMS}} > 1$ and $\pi \uparrow 1$, the multiple counterparty dealers in SMS almost always engage in Bertrand competition, and the customer always gets the full trading gain, regardless of q^{SMS}. On the contrary, with $n^{\text{BB}} = 1$, a customer using BB meets only one counterparty dealer, who will always set the monopolist price, leaving surplus to the customer only with probability q^{BB}.

We are now ready to state the equilibrium.

Proposition 6 (Steady state equilibrium with technology choices). A unique stationary equilibrium exists depending on the asset supply s: There exist thresholds $0 < s_{hn,0} < s_{hn,1} \leq s_{lo,1} < s_{lo,0} < 1 + m_d$ so that

	(a) hn-buyers' probability to use SMS, θ_{hn}	(b) lo-sellers' probability to use SMS, θ_{lo}	(c) asset holding by dealers, m_{do}
(1) $0 < s \leq s_{hn,0}$	0	1	$g(0, 1, m_{do}) = s$
(2) $s_{hn,0} \leq s \leq s_{hn,1}$	$g(\theta_{hn}, 1, m_d^*) = s$	1	m_d^*
(3) $s_{hn,1} < s < s_{lo,1}$	1	1	$g(1, 1, m_{do}) = s$
(4) $s_{lo,1} \leq s \leq s_{lo,0}$	1	$g(1, \theta_{lo}, m_d - m_d^*) = s$	$m_d - m_d^*$
(5) $s_{lo,0} < s < 1 + m_d$	1	0	$g(1, 0, m_{do}) = s$

where $g(x_1, x_2, x_3) = s$ uniquely solves θ_{hn}, θ_{lo}, and m_{do} in columns (a), (b), and (c), respectively. The constant π^* is given in Lemma 3 and $m_d^* := \pi^{-1}(\pi^*)m_d$. The function $g(\cdot)$ and the thresholds $\{s_{hn,0}, s_{hn,1}, s_{lo,1}, s_{lo,0}\}$ are given in the proof. As a special case, when $\rho^{\text{SMS}} q^{\text{SMS}} n^{\text{SMS}} \geq \rho^{\text{BB}} q^{\text{BB}} n^{\text{BB}}$, the thresholds collapse to $s_{hn,0} = s_{hn,1} = 0$ and $s_{lo,0} = s_{lo,1} = 1 + m_d$, and the equilibrium is described by (3) of the above table, consistent with Lemma 3.

Figure 4 illustrates the equilibrium by plotting the technology choices θ_{hn} (solid) and θ_{lo} (dashed) on the left axis and the dealer-owner population size m_{do} (dot-dashed) on the right axis. The four thresholds of $\{s_{hn,0}, s_{hn,1}, s_{lo,1}, s_{lo,0}\}$ cut the support of $s \in (0, 1 + m_d)$ into five regions on the horizontal axis. Consider the solid line, i.e., θ_{hn}, for example. When the asset supply s is extremely low, SMS is very unattractive for the hn-buyers, because they know it is very difficult to find a counterparty do-dealer (the dot-dashed line), and even if they do, they are going to be charged with a monopoly price using SMS. When s is sufficiently high, there are sufficiently many do-dealers, whose price competition makes SMS sufficiently attractive with high trading gain intensity ζ_{hn}^{SMS} for hn-buyers. As such, the solid line flattens at $\theta_{hn} = 1$ for $s > s_{hn,1}$. In between, we see θ_{hn} monotonically increases for $s_{hn,0} \leq s \leq s_{hn,1}$. Such a mixed strategy is supported by the constant $m_{do} = \pi^* m_d$ in the region—the hn-buyers are indifferent to BB and SMS. The pattern for the dashed line, i.e., θ_{lo}, is exactly the opposite, as lo-sellers seek dn-dealers, whose mass is $m_{dn} = m_d - m_{do}$.

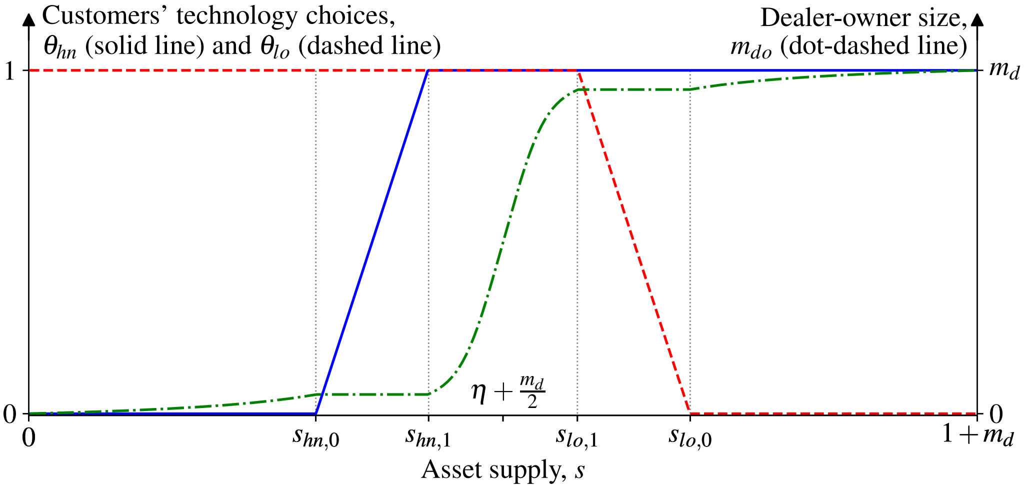
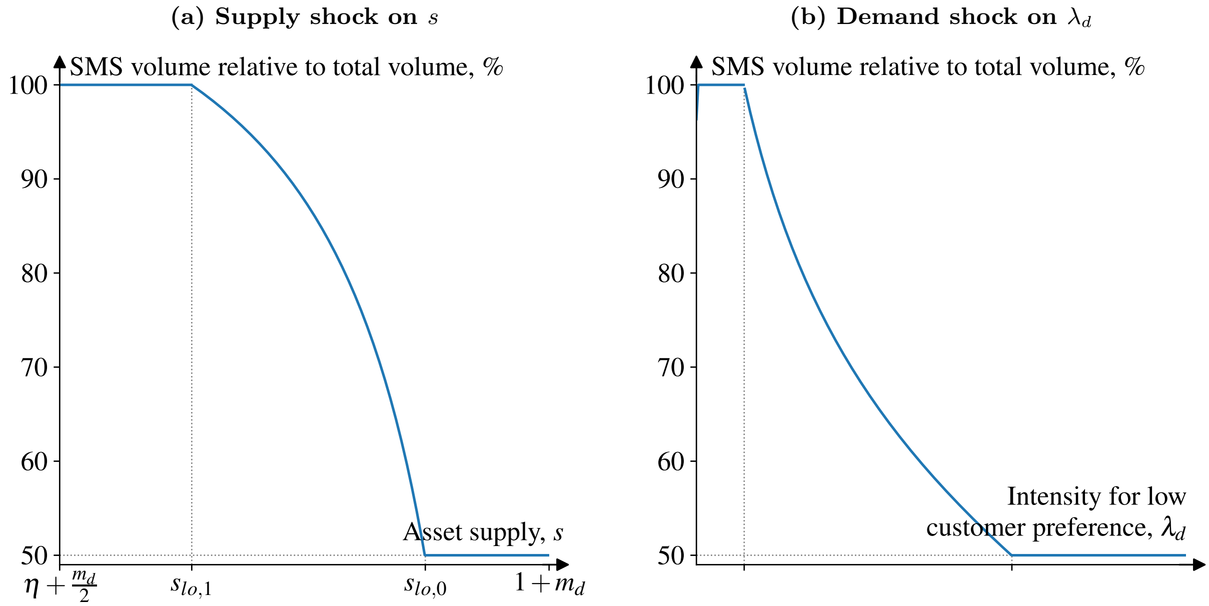


Figure 4: Equilibrium technology choice plotted against asset supply. This figure sketches customers' technology choices against the asset supply s in equilibrium. The hn-buyers' choice θ_{hn} is plotted in the solid line, while the lo-sellers' choice θ_{lo} is plotted in the dashed line (the left axis). The dot-dashed line plots the population size of do-seller dealers (the right axis).

3.2 Stress periods

O'Hara and Zhou (2021) show that after downgrade, a corporate bond's electronic (SMS) volume share falls relative to voice trading (BB). The analysis developed above provides a theoretical framework to study investors' endogenous technology choice when under such stress. We emphasize that the results below pertain only to steady states, e.g., before and after corporate bond downgrades, following the steady state equilibrium characterized above.

One consequence of a corporate bond downgrade is that many previously buy-and-hold long-term investors now no longer wish to hold such bonds. Ambrose, Cai, and Helwege (2008) and Ellul, Jotikasthria, and Lundblad (2011) document such fire sales by insurance companies. In the context of our model, we interpret such fire selling in two different ways, (i) an exogenous increase in the total supply s of the asset—a supply shock; and / or (ii) an exogenous increase in the customers' intensity of drawing low preference λ_d—a demand shock. (A third alternative, reducing λ_u, is equivalent to increasing λ_d and is omitted for brevity.) To fit the fire-selling interpretation, we also assume that the asset is in excess supply, as defined in Lemma 2.


(a) **Supply shock on s**

(b) **Demand shock on λ_d**

Figure 5: Usage of SMS in a stationary equilibrium after surges in supply. This figure plots the usage of SMS (in a stationary equilibrium)—SMS volume relative to total volume—when the asset supply s surges in Panel (a) and when the customers' low-valuation preference shock intensity λ_d increases in Panel (b).

The SMS volume share is defined as the share of the total trading volume executed using SMS:

$$\frac{\rho^{\mathrm{SMS}} m_{lo}^{\mathrm{SMS}} \nu_{lo}^{\mathrm{SMS}} + \rho^{\mathrm{SMS}} m_{hn}^{\mathrm{SMS}} \nu_{hn}^{\mathrm{SMS}}}{\left(\rho^{\mathrm{SMS}} m_{lo}^{\mathrm{SMS}} \nu_{lo}^{\mathrm{SMS}} + \rho^{\mathrm{SMS}} m_{hn}^{\mathrm{SMS}} \nu_{hn}^{\mathrm{SMS}}\right) + \left(\rho^{\mathrm{BB}} m_{lo}^{\mathrm{BB}} \nu_{lo}^{\mathrm{BB}} + \rho^{\mathrm{BB}} m_{hn}^{\mathrm{BB}} \nu_{hn}^{\mathrm{BB}}\right)}. \tag{24}$$

Figure 5(a) and (b) below illustrate how this SMS volume share responds to shocks in s and λ_d, respectively. In Panel (a), the volume ratio is initially flat at 100% because both *lo*-sellers and *hn*-buyers always use SMS ($\theta_{hn} = \theta_{lo} = 1.0$). As the supply s rises higher (between $s_{lo,1}$ and $s_{lo,0}$), *lo*-sellers start to use less SMS, resulting in the decreasing segment. As s increases further, there are no more *lo*-sellers using SMS—all of them use BB, while all *hn*-buyers use SMS. That is, $m_{lo}^{\mathrm{SMS}} = m_{hn}^{\mathrm{BB}} = 0$. In this case, the SMS volume ratio above reduces to

$$\frac{\rho^{\mathrm{SMS}} m_{hn}^{\mathrm{SMS}} \nu_{hn}^{\mathrm{SMS}}}{\rho^{\mathrm{SMS}} m_{hn}^{\mathrm{SMS}} \nu_{hn}^{\mathrm{SMS}} + \rho^{\mathrm{BB}} m_{lo}^{\mathrm{BB}} \nu_{lo}^{\mathrm{BB}}} = \frac{t}{2t} = 50\%,$$

where the equality follows because the trading volume in this case can be written as $t = \rho^{\mathrm{SMS}} m_{hn}^{\mathrm{SMS}} \nu_{hn}^{\mathrm{SMS}} =$

$\rho^{\mathrm{BB}} m_{lo}^{\mathrm{BB}} \nu_{lo}^{\mathrm{BB}}$. Overall, the SMS volume ratio drops with the decline of the SMS usage θ_{lo}, as seen before in Figure 4. The same pattern is observed from Panel (b), where we increase the customers' negative preference shock intensity λ_d, effectively reducing the demand for the asset.

Proposition 7 (SMS usage under stress). The usage of SMS decreases with either the asset's excess supply or with its excess demand. That is, all else equal, for $s > s_{hn,1}$ ($< s_{lo,0}$), the ratio defined in (24) weakly decreases when s increases (decreases) or when λ_d increases (decreases).

The proposition also gives the mirroring result: SMS usage also drops when the asset's excess demand exacerbates ($s < \eta + \frac{m_d}{2}$).

The key intuition for the decrease of the SMS volume share can be understood from the worsening pricing for the *lo*-sellers. As the asset supply s increases after the fire sell, there are more and more *do*-dealers, as shown in the dot-dashed line in Figure 4. This is also evidenced empirically by Anand, Jotikasthira, and Venkataraman (2021), who show that the majority of dealers enter a positive inventory cycle upon a corporate bond's downgrade (e.g., their Figure 2C). The remaining *dn*-dealers, facing less competition, therefore, will charge worse and worse prices to the *lo*-sellers in SMS. Expecting such worsening prices from SMS, the *lo*-sellers then avoid using SMS and switch to BB. In particular, our model yields an additional prediction regarding prices in SMS and in BB under a fire sell:

Proposition 8 (Prices in SMS versus in BB under fire sell). When there is excess supply, an *lo*-seller's expected trading price using SMS worsens relative to using BB.

Therefore, one way to empirically test our theory is to compare the trading prices in BB and in SMS when the asset is under fire sell and examine if the price in SMS is worse than that in BB.

To compare, Hendershott and Madhavan (2015) also shed light on customers' choice between "call" and "click." There the key disadvantage of SMS (click) is the leakage of one's private information to the multiple contacted dealers, as opposed to the only one in BB (call). Our mechanism complements theirs by explaining the volume shift to BB after shocks *not* affecting information asymmetry, such as corporate bond downgrades.

3.3 Efficiency and welfare

Are the market's equilibrium technology choices socially optimal? Given the technologies $\{n^k, \rho^k, q^k\}$, how would a social planner choose $\{\theta_{lo}, \theta_{hn}\}$ for the customers? When, if at all, do the market's equilibrium choices $\{\theta_{lo}, \theta_{hn}\}$ coincide with the planner's $\{\theta^*_{lo}, \theta^*_{hn}\}$?

The answers critically depend on the characteristics of the asset. Among others, how quickly customers can find dealers, i.e., $\{\rho^{\mathrm{BB}}, \rho^{\mathrm{SMS}}\}$, matters a lot. Recall from the technology assumption (18) that $\rho^{\mathrm{BB}} \leq \rho^{\mathrm{SMS}}$. Therefore, it suffices to consider the cases of high-ρ^{BB} and the low-ρ^{SMS} below.

3.3.1 The case of high search intensity

Proposition 9 (A social planner's technology choices, I). When the search intensity ρ^{BB} ($\leq \rho^{\mathrm{SMS}}$) is sufficiently high, welfare w is monotone increasing in SMS usages by both types of customers and the social planner chooses $\theta^*_{lo} = \theta^*_{hn} = 1$.

The intuition largely follows Proposition 3. When the search intensity is high, Proposition 3 shows that welfare is monotone increasing in n. As such, by assigning both $\theta^*_{lo} = \theta^*_{hn} = 1$, the planner chooses n^{SMS} over n^{BB} to maximize welfare.

However, the market's technology choices do not always coincide with the planner's. This is because a customer cares not only about the probability of finding a counterparty dealer but also about the endogenous split of the trading gain. Figure 6 sketches such possible discrepancies. The solid line and the dashed line plot, respectively, the market's choices of θ_{hn} and θ_{lo} against the asset supply s. (Note that the patterns are qualitatively the same as in Figure 4.) The shaded areas indicate that there is inefficiency in the market's technology choices. For example, when the excess supply s is relatively extreme, $s > s_{lo,1}$, as in fire sell (Section 3.2), the dealer sector becomes overloaded (m_{do} too large), giving lo-sellers a hard time finding dn-dealers. Then lo-sellers become less willing to use SMS (θ_{lo} decreases with s) because in SMS their trading gains are too low. The same holds when $s < s_{hn,1}$ (extreme excess demand).

Since the planner wants to encourage SMS usage, a simple, welfare-improving market design mandate readily follows: Let customers indicate their reservation values when searching dealers via

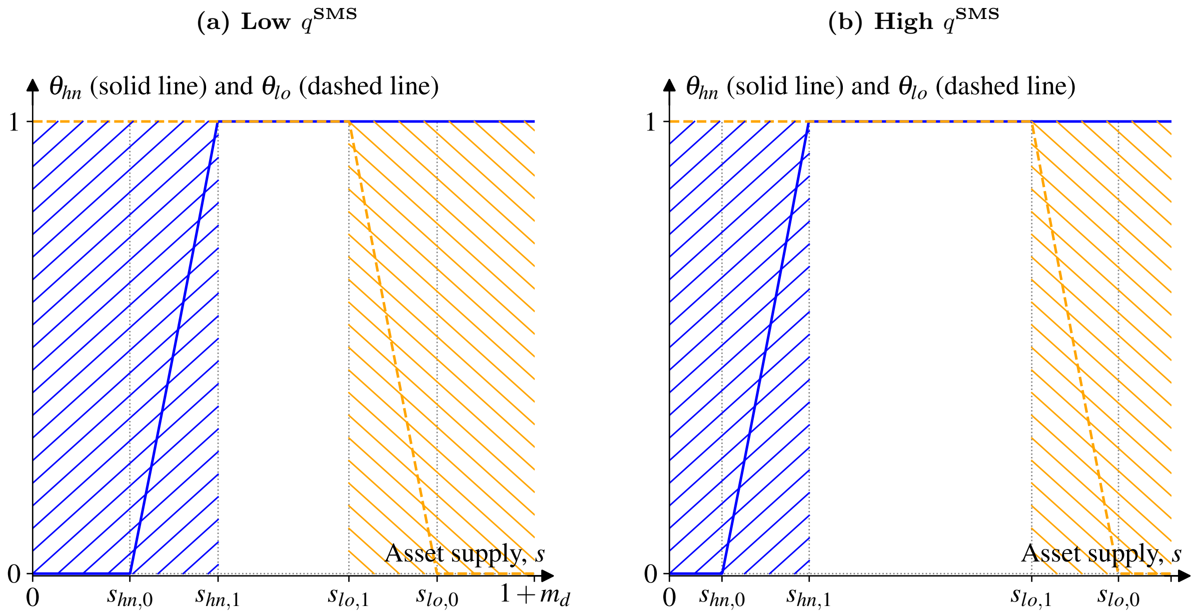


Figure 6: Market's technology choices versus a social planner's under high search intensity. This figure sketches the inefficiency due to the difference between the market's equilibrium technology choices and a social planner's when the search intensity $\rho := \min[\rho^{\text{BB}}, \rho^{\text{SMS}}]$ is high. The solid (blue) line and the dashed (orange) line are θ_{hn} and θ_{lo}, respectively, the hn-buyers' and the lo-sellers' equilibrium probabilities of using SMS. The "//"(blue) and "\\" (orange) shaded areas indicate, respectively, where $\theta_{hn} \neq \theta^*_{hn}$ and $\theta_{lo} \neq \theta^*_{lo}$. Panel (a) shows the patterns for low q^{SMS}, while Panel (b) shows for a higher q^{SMS}.

SMS. In the model, such a design translates to an increase in q^{SMS}. By Lemma 3, when q^{SMS} is large enough, such that the inequality (20) flips, the customers endogenously choose SMS efficiently: $\theta^{\text{SMS}}_{hn} = 1 = \theta^*_{hn}$ and $\theta^{\text{SMS}}_{lo} = 1 = \theta^*_{lo}$. The improvement can be seen by contrasting Figure 6(a) and (b): The shaded area of the inefficient technology adoption is reduced from (a) to (b).

In practice, however, customers are almost always on the receiving end of TIOLIOs on electronic platforms; i.e., $q^{\text{SMS}} = 0$. We argue that one reason behind such an inefficient design is the dealers' incentive to participate. For example, when q^{SMS} becomes large, close to one, the dealers get a vanishing share of trading gains. Therefore, to the extent that the dealers have certain influence on the design of trading protocols on the electronic platforms, they would avoid a high q^{SMS}, or perhaps none at all, to let customers make TIOLIOs. Even if the dealers are independent of the trading protocol design, the platform operator will have to set a low q^{SMS} to incentivize dealers' participation.

3.3.2 The case of low search intensity

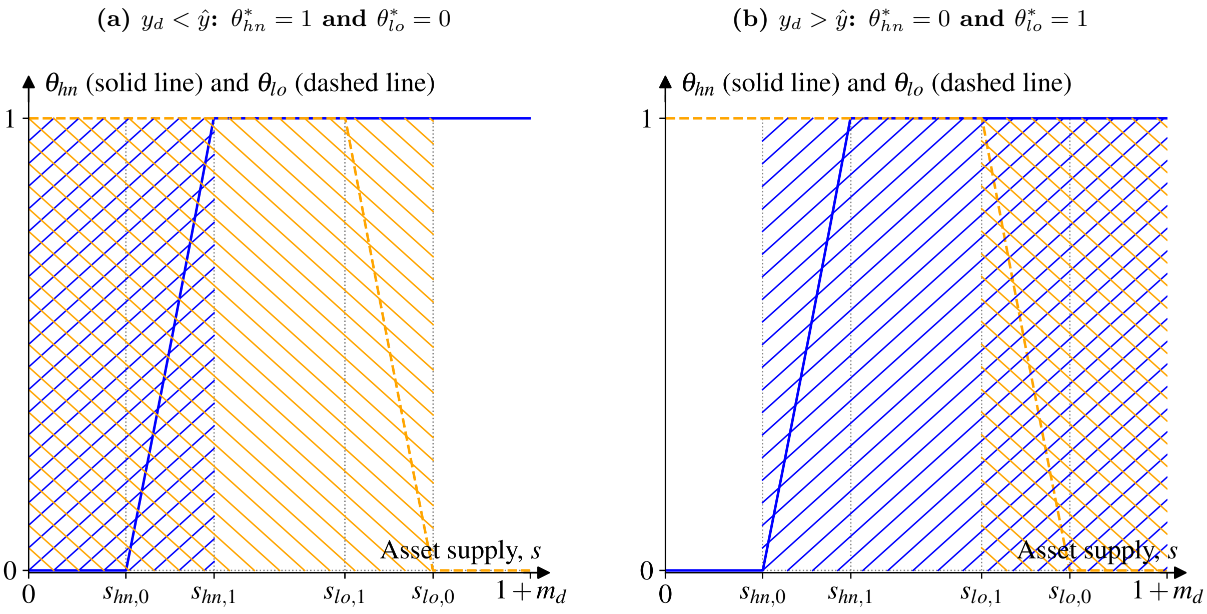
The case of low search intensity is more nuanced. The planner's choices in addition depend on the comparison between dealers' instantaneous utility y_d and an average customer's $\hat{y}$:

Proposition 10 (A social planner's technology choices, II). When the search intensity ρ^{SMS} ($\geq \rho^{\text{BB}}$) is sufficiently low, the social planner chooses $\theta^*_{lo} = 1 - \theta^*_{hn} = \mathbb{1}_{\{y_d > \hat{y}\}}$ to maximize welfare.

To see why, recall from Equation (16): $w = \frac{\hat{y}}{r}(s - m_{do}) + \frac{y_d}{r} m_{do}$. Thus, the planner wants to maximize (minimize) m_{do}, i.e., to shift all asset holding to dealers (customers), if and only if $y_d > \hat{y}$ ($y_d < \hat{y}$). To do so, the planner will polarize $\{\theta^*_{lo}, \theta^*_{hn}\}$ because they affect m_{do} in opposite directions: If more *lo*-sellers use SMS, *dn*-dealers get to buy more often, increasing m_{do}; but if more *hn*-buyers use SMS, more *do*-dealers get to sell their assets, decreasing m_{do}. (This is formally shown in Lemma S1.1 in the supplementary appendix.) As a result, the planner sets $\theta^*_{lo} = \mathbb{1}_{\{y_d > \hat{y}\}}$ and $\theta^*_{hn} = 1 - \theta^*_{lo}$.

Figure 7(a) sketches the proposition for the case of $y_d < \hat{y}$, in which the planner wants to allocate the asset to the customers as much as possible, thus assigning $\theta^*_{hn} = 1$ and $\theta^*_{lo} = 0$. This is against the *lo*-sellers' wish, as they want to sell the asset to the dealers. As a result, the market's technology choices are efficient (coinciding with the planner's) only when the asset is in extreme supply, i.e., when $s > s_{lo,0}$. Panel (b), flipping Panel (a), sketches the case of $y_d > \hat{y}$.

The patterns shown in Figure 7 caveat that the intuition regarding welfare and market design obtained from the high-ρ case does not carry through when the matching of the asset is intrinsically slow. For example, compared to corporate bonds, whose matching on MarketAxess take only a few minutes (Hendershott and Madhavan, 2015), collateralized loan obligations (CLOs) trade much more slowly, taking days as the B/OWIC run through emails require considerably longer time to organize (Hendershott et al., 2020). For such slow-moving assets, the planner always wants some customers to use BB to prevent the asset from being held inefficiently in the wrong hands.


(a) $y_d < \hat{y}$**:** $\theta^*_{hn} = 1$ **and** $\theta^*_{lo} = 0$

(b) $y_d > \hat{y}$**:** $\theta^*_{hn} = 0$ **and** $\theta^*_{lo} = 1$

Figure 7: Market's technology choices versus a social planner's under low search intensity. This figure sketches the inefficiency due to the difference between the market's equilibrium technology choices and a social planner's when the search intensity $\rho := \min[\rho^{\text{BB}}, \rho^{\text{SMS}}]$ is low. The solid (blue) line and the dashed (orange) line are θ_{hn} and θ_{lo}, respectively, hn-buyers' and lo-sellers' equilibrium probabilities of using SMS. The "//"(blue) and "\\" (orange) shaded areas indicate, respectively, where $\theta_{hn} \neq \theta^*_{hn}$ and $\theta_{lo} \neq \theta^*_{lo}$. Panel (a) shows the pattern for the case of $y_d < \hat{y}$, in which case $\theta^*_{hn} = 1$ and $\theta^*_{lo} = 0$, and Panel (b) the opposite, in which case $\theta^*_{hn} = 0$ and $\theta^*_{lo} = 1$.

4 Conclusion

This paper studies "simultaneous multilateral searching" (SMS), which has been popularized in practice recently through trading protocols like "Request-for-Quote" (RFQ) in OTC markets. The idea is that a searching customer can reach out to multiple dealers simultaneously, solicit quotes from them, and then trade with the one offering the best quote. This search mechanism differs from the conventional "bilateral bargaining" (BB), in which a searching customer meets a single dealer and negotiates the terms of trade.

A steady state equilibrium is characterized in an extension of the standard search framework. The key insight revealed is that the split of the trading gain between a searching and a quoting investor is an endogenous equilibrium outcome, as opposed to the exogenous split (à la Nash) in the literature assuming BB. In addition, two search parameters, the intensity and the capacity, are analyzed in terms of their contrasting welfare implications. A novel bottleneck effect, arising from (and only from) the search capacity, is shown to hinder the efficient asset allocation and might possibly hurt welfare. Such a bottleneck might arise also from transparency of dealer inventories.

Allowing customers to endogenously choose between SMS and BB, the model finds an intrinsic hindrance in the adoption of SMS and further suggests potential inefficiency in terms of asset allocation. The model underscores channels through which both regulation and market design can affect customers' search preferences and, ultimately, the allocation efficiency. Notably, the adoption of SMS-like trading protocols, like RFQ platforms, might significantly improve if customers are given more room to set their reservation prices. For example, RFQ platforms can provide channels for customers to further bargain and negotiate with their selected dealers after the initial auction (see Supplementary Appendix S6). Such increased usage of SMS protocols can improve welfare.

Appendix

A A quantitative exercise

This appendix details the calibration of the model parameters (as in Section 1) against the RFQ trading of corporate bonds. Our focus lies in the novel bottleneck effect (Section 2.3)—is it a relevant concern in the real-world trading of corporate bonds? To provide a tentative answer, we match the model-generated moments of an average corporate bond with real-world statistics, mainly drawn from Hendershott and Madhavan (2015), whose data (from 2010 to April, 2011) are provided by MarketAxess, the dominant RFQ platform of corporate bonds trading. In addition, we rely also on statistics reported by MarketAxess; on O'Hara and Zhou (2021), which contain longer-horizon trends of electronic trading of corporate bonds; on Bessembinder et al. (2018) for over-all market statistics of corporate bonds trading; and on the survey of Bessembinder, Spatt, and Venkataraman (2020) for the market structure details. The calibration methodology follows HLW.

A.1 Calibrating the demographic parameters

The endogenous masses of six agent types, $\{m_{ho}, m_{hn}, m_{lo}, m_{ln}, m_{do}, m_{dn}\}$, are determined by the six model parameters listed in Table 1. Note that in Section 1, the customers' preference shocks are characterized by $\{\lambda_u, \lambda_d\}$, which are equivalent to $\{\lambda, \eta\}$ here, with $\lambda := \lambda_d + \lambda_u$ and $\eta := \lambda_u/\lambda$. Given our focus on the dealer bottleneck, we follow Proposition 2 to examine a large, reasonable

Parameter	Calibrated value	Source
ρ	$1 \sim 1,000$	Free ranging on the whole support, not calibrated
n	27	Average from Table VI of Hendershott and Madhavan (2015)
m_d	0.10	MarketAxess press release[*]
$\lambda \; (= \lambda_d + \lambda_u)$	0.35	Jointly calibrated against turnover (excluding inter-dealer trades)[§], per capita trading volume[†], and customer queries' no-response rate[‡], fixing $\rho = 100$
$\eta \; (= \lambda_u / \lambda)$	0.11	
s	0.12	

Table 1: Calibration of demographic parameters. This table summarizes the demographic parameters used in the quantitative exercise: the search intensity ρ, the search capacity n, the dealer sector size m_d (relative to customer size), the customers' type-switching shock intensity $\lambda \; (= \lambda_d + \lambda_u)$, and the probability η of switching to high type upon a shock. The table notes are explained below (see Section A.1 for details):

[*] See "MarketAxess Expands Liquidity in European Credit With Addition of 4 Dealers" from MarketAxess website; retrieved on August 25, 2021.

[§] Estimated from Table I and II of Bessembinder et al. (2018).

[†] Estimated based on Table I of Hendershott and Madhavan (2015).

[‡] Reported in Table VI of Hendershott and Madhavan (2015).

support of $\rho \in [1, 1000]$ for the search intensity.[11] The other five parameters are calibrated against the data as follows and the results are summarized in Table 1.

The dealer sector size, m_d (relative to the customer size). MarketAxess in a press release reports that there are "[o]ver 1,000 institutional investor and broker-dealer firms" who are "active users of the MarketAxess trading platform," and "over 90 global dealers are [...] providing liquidity [...]." This translates to

$$m_d = \frac{90}{1000 - 90} \approx 0.1$$

dealers per customer. See "MarketAxess Expands Liquidity in European Credit With Addition of 4 Dealers" from MarketAxess website; retrieved on August 25, 2021. We do note that the press release was made in 2015. Therefore, by applying the same ratio of $m_d \approx 0.1$ to the 2010-2011 statistics reported by Hendershott and Madhavan (2015), we effectively assume that the composition had not changed dramatically. We also acknowledge the concern that real-world agents are not bound to hold one unit of the asset as assumed in the model. As such, a more realistic calibration of m_d would be to compute the asset holding ratio, as is done by HLW. Constrained by data availability, we instead resort to the above approximation.

The search capacity n. Hendershott and Madhavan (2015) report in their Table VI that across "all bonds," the number of dealers queried in their sample (2010 to April 2011) is between 25.1

[11] Another reason to examine a large support of ρ is that it seems to be the most "free" parameter in existing the calibrations. For example, Duffie, Gârleanu, and Pedersen (2007) set a rather high search intensity of 625 per year; Lester, Rocheteau, and Weill (2015) set it to be 85 per year; Pagnotta and Philippon (2018) find it to be around 1 and around 39 (daily frequency), respectively, for slow and fast duopoly market makers; HLW require an average trading time of about five days, or a customer-dealer search intensity of 77. See a similar remark in Footnote 23 of HLW. We therefore examine the range of $\rho \in [1, 1000]$.

to 27.7, depending on the trade size. A similar number is reported by O'Hara and Zhou (2021) ("[t]he number of dealers contacted this way [...] is typically around 30," p. 370), whose data cover a much longer horizon from 2010 to 2017. To be precise, we take

$$n = 27,$$

closest to the 27.2 reported for "odd" size trades by Hendershott and Madhavan (2015), because Figure 2 of O'Hara and Zhou (2021) shows that this size category constitutes the lion's share of all electronic trading volume.

The asset supply s, the type-switching shock intensity λ, and the probability of becoming high type η. These three parameters are jointly solved from a three-equation system described below. First, Table II of Bessembinder et al. (2018) reports a turnover ratio (trading volume relative to amount outstanding) of 0.78 for all corporate bonds in 2010. Depending on the definition of dealers, about 71%-76% of the total trading volume is customer-based (as opposed to inter-dealer trading), according to their Table I. We take that roughly three quarters of the turnover ratio above is contributed by customer trading. We assume that these ratios apply to RFQ trading, equating the model implied turnover ratio with the above numbers:

$$\frac{2t}{s} = \frac{1}{s}(\rho m_{hn}\nu_{hn} + \rho m_{lo}\nu_{lo}) = 0.78 \times \frac{3}{4} = 58.5\%. \tag{A.1}$$

Recall from Equation (8) that $t = \rho m_{hn}\nu_{hn} = \rho m_{lo}\nu_{lo}$ is the trading volume from customer-buyers and sellers, respectively.

Second, Table I of Hendershott and Madhavan (2015) reports that there were in total 467,614 electronic trades across 5,528 bonds in their sample of $1\frac{1}{3}$ years (2010 to April 2011). Per our earlier estimate (see the calibration of m_d above), there were about 900 ($= 1000 \times (1 - 10\%)$) customer institutions. Therefore, the trading intensity (per customer, per year) for an average bond is

$$\frac{2t}{m_c} = \frac{2t}{1} = \frac{467,614/\frac{4}{3}}{5,528 \times 900} \approx 0.0705. \tag{A.2}$$

Third, Table VI of Hendershott and Madhavan (2015) reports the "no response" rate of the customers' queries. In particular, across all bonds, 8.6% of the queries of "odd size" trades had no responses. (We focus on "odd size" trades, again, because most of the electronic trades are in this size category, as shown in Figure 2 of O'Hara and Zhou, 2021.) In the model, a searching customer's query sees no reply only if all n contacts are unfortunately directed to the "wrong" type of dealers. For example, an hn-buyer will find no response from the dealers with probability $1 - \nu_{hn} = (1 - \pi_{do})^n = \pi_{dn}^n$, where we adopt the most parsimonious form of the matching rate $\pi(\cdot)$ and obtain $\pi_{dn} = m_{dn}/m_d$.[12] Since every instance there are ρm_{hn} queries from hn-buyers and ρm_{lo}

[12] More elaborate parametrization for $\pi(\cdot)$, e.g., like Equation (1), will require calibration of additional parameters, like the inventory transparency ψ.

from *lo*-sellers, the model implies a weighted average no-response rate of

$$\frac{m_{hn}}{m_{hn}+m_{ho}}\left(\frac{m_{dn}}{m_d}\right)^n+\frac{m_{lo}}{m_{hn}+m_{lo}}\left(\frac{m_{do}}{m_d}\right)^n=8.6\%. \tag{A.3}$$

The three-equation system, (A.1)-(A.3), jointly solves the last three demographic parameters, $\{s, \lambda, \eta\}$. In particular, note that $s = 0.1205$ is uniquely pinned down by the ratio of (A.2) over (A.1). For the other two parameters, λ and η, the nonlinear equation system always delivers stable and unique numerical solutions, based on the extensive trials.

The calibration of λ and η does depend on the exact choice of ρ, which varies from $\rho = 1$ to $1,000$. Fortunately, the results are not very sensitive: When varying ρ in the relevant support with 0.1 units increments, we find that the 0.5% and the 99.5% percentiles for the calibrated η are 0.1139 and 0.1150, respectively; and those for the calibrated λ are 0.3493 and 0.3713, respectively. That is, varying $\rho \in [1, 1000]$, 99% of the calibrated values of η and λ concentrate in the above very narrow bands. For concreteness, Table 1 reports the calibrated values of λ and η based on $\rho = 100$.

A.2 The demographic bottleneck

The calibrated parameter values in Table 1 produce the demographic graphs in Figure 1. In particular, the bottleneck effect can be seen from horizontal cuts in Figure 1(b), where the size of low-type customer-owners, m_{lo}, increases with the search capacity n. Here, Figure 8 below reproduces such an effect by zooming in on the change of m_{lo} due to an increase from $n = 27$ to $n = 28$, roughly a vertical strip at around $n = 27$ in Figure 1(b).[13]

It can be seen that regardless of the value of ρ, letting customers contact one more dealer—raising n from the empirical average of 27 to 28—*always increases* the size of m_{lo}. That is, we find that such a bottleneck, which hinders the efficient passing of corporate bonds through the dealer sector, is in place in the real-world RFQ trading of corporate bonds.

Figure 8 shows that the demographic bottleneck is likely in place in the RFQ trading of corporate bonds. Whether such a bottleneck translates to welfare losses depends on the exact values of the agents' flow utility $\{y_h, y_d, y_l\}$ (c.f. Proposition 2 and 3). The next subsection attempts such calibrations.

A.3 When does the bottleneck translate to welfare losses?

Recall that welfare is defined as $w = \frac{1}{r}(m_{ho}y_h + m_{do}y_d + m_{lo}y_l)$. Since the demographics have been successfully calibrated (up to ρ), so are the agent sizes $\{m_{ho}, m_{do}, m_{lo}\}$. Still, the other four valuation parameters $\{r, y_h, y_d, y_l\}$ are needed to pin down an exact number for welfare. Finding four more statistics from real-world RFQ trading to determine these parameters, however, is difficult

[13] A subtle difference is that in Figure 1(b) only ρ varies, all other demographic parameters fixed according to Table 1, while in Figure 8, as ρ changes, all other demographic parameters—in particular λ and η—are re-calibrated. The magnitude of the difference is negligibly small, as the variation in λ and η with ρ are tiny.

Figure 8

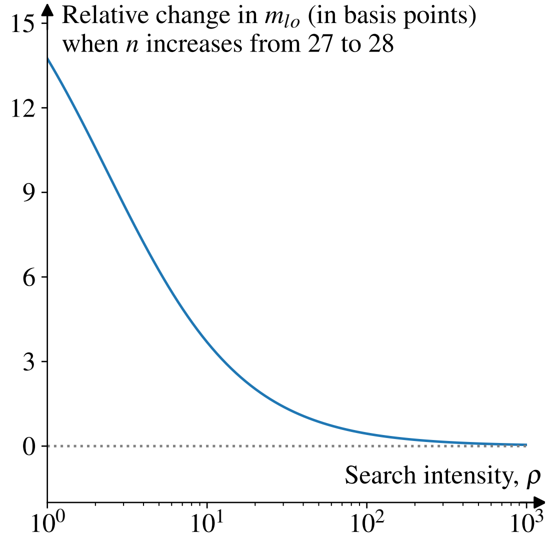


Figure 9

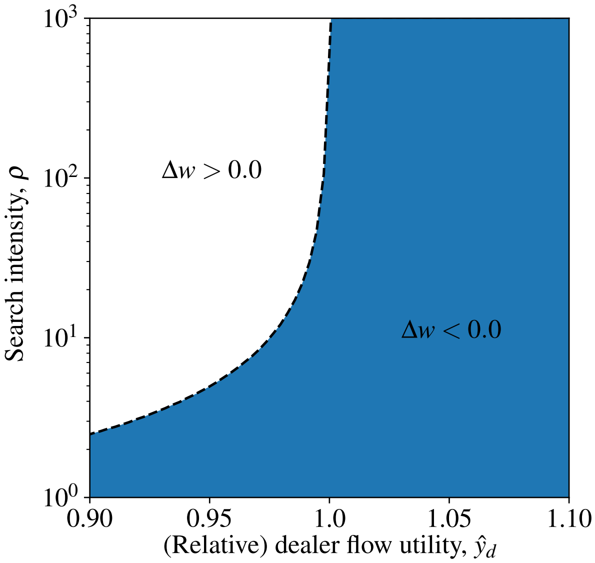


Figure 8: The bottleneck effect under the calibrated parameter values. This figure illustrates the bottleneck effect of an increase in the search capacity n (from 27 to 28) in terms of the increase in the low-type customer sizes, i.e., m_{lo}. Other than n, the relevant model parameters, $\{m_d, s, \eta, \lambda\}$, are calibrated according to Appendix A.1, as functions of the exogenously varying search intensity ρ.
Figure 9: Welfare change due to an increase in the search capacity n. This figure illustrates the effect of an increase in the search capacity n (from 27 to 28) on welfare w. Both the search intensity ρ and the dealers' (relative) flow utility $\hat{y}_d$ are varied exogenously. The demographics $\{m_{ho}, m_{do}, m_{lo}\}$ are solved based on the calibrated demographic parameters as in Appendix A.1, with $n = 27$ and ρ varying. The other valuation parameters are normalized at $y_h = 1.0$ and $y_l = 0.0$ (thus the varying y_d is relative to the normalization of y_h and y_l). The dashed line indicates the isoquant where the welfare change is exactly zero. The white and the blue areas indicate, respectively, positive and negative welfare changes.

(we detail the challenges at the end of this subsection). Instead, we ask a simpler question: When the bottleneck translate to welfare losses? That is, if the search capacity n increases, e.g., from 27 to 28, we want to know when the implied change in welfare is negative, i.e., $\Delta w = w(n = 28) - w(n = 27) < 0$.

To do so, use the market clearing condition $m_{lo} = s - m_{do} - m_{ho}$ to rewrite welfare as:

$$w = \frac{1}{r}(sy_l + m_{do}(y_d - y_l) + m_{ho}(y_h - y_l)) = \frac{y_h - y_l}{r}\left(\frac{y_l s}{y_h - y_l} + m_{ho} + \frac{y_d - y_l}{y_h - y_l} m_{do}\right).$$

It then follows that the sign of the welfare change is determined by

$$\text{sign}[\Delta w] = \text{sign}\left[\Delta m_{ho} + \frac{y_d - y_l}{y_h - y_l}\Delta m_{do}\right] = \text{sign}[\Delta m_{ho} + \hat{y}_d \Delta m_{do}]$$

where the difference notation Δ is understood as the difference between $n = 28$ and $n = 27$, under the demographic parameters calibrated in Section A.1. That is, instead of calibrating all four valuation parameters, it suffices to know the relative dealer valuation $\hat{y}_d := \frac{y_d - y_l}{y_h - y_l}$.

That is, we now have two "free" parameters, the search intensity ρ and the dealer relative flow utility $\hat{y}_d$. Figure 9 shows the result. It can be seen that the relative welfare change is negative whenever $\hat{y}_d$ is sufficiently high, for all levels of ρ. In particular, $\hat{y}_d \approx 1$ or, equivalently, $y_d \approx y_h$ suffices for the bottleneck to translate to welfare losses.[14]

To conclude, this quantitative exercise has carried the theoretical bottleneck prediction to the real-world RFQ trading of corporate bonds, finding two results. First, for a wide, realistic range of search intensity ρ, the bottleneck always exists (Figure 8). Second, the bottleneck indeed translates to welfare losses, when dealers and customers value the bonds' coupons similarly (i.e., $y_d \approx y_h$; Figure 9). Taken together, we find that restricting the search capacity n may have positive welfare impacts.

Difficulty in identifying the valuation parameters

Under additional assumptions, a number of studies have calibrated the exact welfare values, like HLW, Pagnotta and Philippon (2018), Lester, Rocheteau, and Weill (2015). Doing so requires the identification of four other model parameters, $\{r, y_h, y_d, y_l\}$, and below we detail the challenges of doing so in our setting. To begin with, the literature typically normalizes r and one of the flow utility ys. For example, HLW set $r = y_h = 0.05$. This leaves y_l and y_d to be pinned down by matching with statistics about the asset price, like trading costs and yield spreads.

While welfare per se only depends on the above four parameters $\{r, y_h, y_d, y_l\}$, calibrating the model to real-world price statistics, however, also requires knowledge about additional parameters in our model, in particular, the exit rates $\{f_c, f_d\}$ and the trading gain splits γ_{lo} and γ_{hn}, which all enter the agents' value functions and, hence, also the prices (Section 2.2 and 2.4). [15] Thus, after normalizing r and y_h (as in HLW), we are still left with six valuation parameters, $\{y_l, y_d, f_c, f_d, \gamma_{lo}, \gamma_{hn}\}$, to calibrate. To the best of our knowledge, however, only two statistics about asset prices, applicable to our model, have been used in the literature for such calibration: the trading cost (bid-ask spread or dealer markups) and the yield (or yield / dividend spread, as in, e.g., Pagnotta and Philippon, 2018 and HLW).

The literature reduces the number of the unidentified valuation parameters by making reason-

[14] Absent of an exact calibration, we note that a relatively high $\hat{y}_d$ is economically plausible. For example, one can interpret the consumption good produced by the asset as the coupons paid out by the bond. Absent any shocks, naturally, both customers and dealers should value such dollar payments exactly the same, suggesting $y_d = y_h =$ "common value". When negative shocks, e.g., liquidity needs, strike, the valuation for such coupon flows will drop, resulting in $y_l < y_d = y_h$.

[15] Note that the customers' bargaining power q (above and beyond the dealers' competition) enters the trading gain splits $\{\gamma_{hn}, \gamma_{lo}\}$ through the specific form of price determination assumed in Section 1. Calibrating γ_{lo} and γ_{hn} is therefore more general and flexible, as no particular assumption on price formation is imposed.

able approximations. For example, most of the papers have $f_c = f_d = 0$, which would be reasonable in our model if the exits of both customers and dealers are sufficiently rare. Further, in the context of BB, DGP and in HLW, assume symmetry between customer buys and sells and argue for $\gamma_{lo} = \gamma_{hn} = \gamma$ (the same Nash bargaining across all customer-dealer meetings). Assuming so in the context of SMS, however, is less realistic, as customers on the short and long side of the market face different competition among the dealers and so should expect to extract different fractions of trading gains. Thus, we have at least 4 parameters $\{y_l, y_d, \gamma_{lo}, \gamma_{hn}\}$, facing only two price statistics.

In summary, we lack the moment conditions needed to pin down the valuation parameters and to quantify welfare implications of the bottleneck. Instead, we let the relative utility flow $\hat{y}_d$ to be "free" parameter. This allows this subsection to comprehensively examine when the bottleneck found in Section A.1 translates to welfare losses, as opposed to relying on a single point estimation of welfare.

B Collection of proofs

The proofs of all the lemmas are deferred to Supplementary Appendix S1, where we also provide additional useful results to facilitate equilibrium characterization.

Proposition 1

Proof. Note that the trading gain is $\Delta = R_{hn} - R_{lo} = (V_{ho} - V_{hn}) - (V_{lo} - V_{ln})$, a linear combination of the four unknown value functions. The four equations (10)-(13), therefore, is a linear equation system that uniquely pins down the four unknowns.

It only remains to prove that the trading gains are strictly positive when $\underline{y}_d \leq y_d \leq \overline{y}_d$. Difference Equation (10) and (13) to get $0 = y_h - (r + f_c)R_h - \zeta_{hn}\Delta_{hd} - \lambda_d \cdot (R_h - R_l)$. Similarly, difference Equation (12) and (11) to get $0 = y_l - (r + f_c)R_l + \zeta_{lo}\Delta_{dl} + \lambda_u \cdot (R_h - R_l)$. Finally, difference the two dealers' HJB equations, (14) and (15), to get $y_d - (r + f_d)R_d + \zeta_{do}\Delta_{hd} - \zeta_{dn}\Delta_{dl}$. Note that $\Delta_{hd} = R_h - R_d$ and $\Delta_{dl} = R_d - R_l$. Therefore, taking the ζs as given, the above form a 3-equation-3-unknown linear system, from which the reservation values $\{R_h, R_d, R_l\}$ can be uniquely solved. The resulting expressions are complicated and omitted here, but it is straightforward to verify that $R_h - R_d$ (resp., $R_d - R_l$) is monotone decreasing (resp., increasing) in y_d. (Note that the trading gain intensities ζs are independent of y_d). Therefore, one can find the upper and the lower thresholds by solving $\overline{y}_d^{'}$ explicitly from $R_h = R_d$ and $\underline{y}_d^{'}$ from $R_d = R_l$:

$$\overline{y}_d^{'} := \frac{\xi y_h(\zeta_{lo} + \lambda_u + r + f_c) + \xi\lambda_d y_l + \zeta_{dn}(y_h - y_l)}{\zeta_{lo} + \lambda_d + \lambda_u + r_c}, \quad \text{and}$$

$$\underline{y}_d^{'} := \frac{\xi y_l(\zeta_{hn} + \lambda_d + r + f_c) + \xi\lambda_u y_h + \zeta_{do}(y_l - y_h)}{\zeta_{hn} + \lambda_d + \lambda_u + r + f_c},$$

where ξ is defined in the proposition. The above thresholds are still endogenous of the ζs. To obtain the thresholds composed of exogenous parameters, note that

$$\overline{y}_d' \geq y_h\xi - \frac{(y_h - y_l)\xi\lambda_d}{\lambda_d + \lambda_u + r + f_c} =: \overline{y}_d \quad \text{and} \quad \underline{y}_d' \leq y_l\xi + \frac{(y_h - y_l)\xi\lambda_u}{\lambda_d + \lambda_u + r + f_c} =: \underline{y}_d.$$

Clearly, $\overline{y}_d > \underline{y}_d$. As such, $\underline{y}_d \leq y_d \leq \overline{y}_d$ is sufficient to ensure $R_l < R_d < R_h$. □

Proposition 2

Proof. The effects of ρ and n are proved separately below. For concreteness, assume that the asset is in excess supply. (The case of excess demand is symmetric and omitted.)

A higher search intensity ρ: The trading volume can be written as $t = \rho m_{hn}\nu_{hn}$ (Equation 8). Equation (S3) gives another link between t and m_{hn}. Combining the two gives

$$t = \frac{(1 + m_{do} - s)\lambda_u\rho}{(\lambda_d + \lambda_u)\nu_{hn}^{-1} + \rho}, \tag{B.1}$$

which is increasing in ρ and in m_{do} (note that ν_{hn} is also increasing in m_{do}). Lemma S1.2 has shown that a higher ρ increases m_{do} (given excess supply). Therefore, the volume increases with ρ. It is then also clear from (S2) that m_{lo} decreases. Finally, $m_{hn} = \frac{\nu_{lo}}{\nu_{hn}}m_{lo}$ by (8). The ratio $\frac{\nu_{lo}}{\nu_{hn}} = \frac{1-\pi_{do}^n}{1-(1-\pi_{do})^n}$. Simply computing the derivative with respect to π_{do} can show that the ratio decreases with π_{do}. That is, a higher ρ, increasing m_{do} and π_{do}, results in a lower m_{hn} as well.

A larger search capacity n: Lemma S1.2 has shown that a larger n also increases m_{do} (given excess supply). Note that since $\nu_{hn} = 1 - (1 - \pi(m_{do}/m_d))^n$, $\frac{\partial \nu_{hn}}{\partial m_{do}} > 0$ and $\frac{\partial \nu_{hn}}{\partial n} > 0$. From the same expression of t above, therefore, n also increases trading volume. Again, from Equation (S2), it is clear that m_{lo}, the long-side, then decreases with n.

The effect on $m_{hn} = \frac{\nu_{lo}}{\nu_{hn}}m_{lo}$, the short-side, is more complicated, because now n also affects the ratio $\frac{\nu_{lo}}{\nu_{hn}}$. To prove the statement, instead, it is easier to turn to the following equivalent expression:

$$m_{hn}(m_{do}, n) := \frac{t}{\rho\nu_{hn}} = \frac{(1 + m_{do} - s)\lambda_u}{\lambda_d + \lambda_u + \rho(1 - (1 - \pi(m_{do}/m_d))^n)}, \tag{B.2}$$

where the second equality follows Equation (S3). It is straightforward to find that $\lim_{\rho\to 0} \frac{\partial m_{hn}}{\partial n} = 0$; and $\lim_{\rho\to 0} \frac{\partial m_{hn}}{\partial m_{do}} = \frac{\lambda_u}{\lambda_d + \lambda_u} > 0$. Directly computation using (S23) implies $\lim_{\rho\to 0} \frac{\mathrm{d}m_{do}}{\mathrm{d}n} > 0$. Therefore, $\lim_{n\to\infty} \frac{\mathrm{d}m_{hn}}{\mathrm{d}n} = \lim_{n\to\infty}\left(\frac{\partial m_{hn}}{\partial n} + \frac{\partial m_{hn}}{\partial m_{do}}\frac{\mathrm{d}m_{do}}{\mathrm{d}n}\right) > 0$. □

Proposition 3

Proof. The proof considers the changes in ρ and in n separately. Only the case of excess supply, i.e., $s > \eta + m_d/2$, is analyzed (and the case of excess demand is analogous and is omitted).

When ρ increases: Recall welfare is $w = (y_h m_{ho} + y_d m_{do} + y_l m_{lo})/r$. By market clearing (2), substitute $m_{lo} = s - m_{ho} - m_{do}$ in the above welfare expression to get $w = (y_l s + (y_h - y_l)m_{ho} +$

$(y_d - y_l)m_{do})/r$. By Lemma S1.2, m_{do} increases with ρ. By Proposition 2, m_{hn} and m_{lo} decrease with ρ. That is, $m_{ho} = \eta - m_{hn}$ increases with ρ. Note that $y_d \in [\underline{y}_d, \overline{y}_d]$ is assumed to ensure positive trading gains (Proposition 1) and we also have that $y_l < \underline{y}_d < \overline{y}_d < y_h$. It then follows that $y_d \in (y_l, y_h)$. Therefore, welfare is increasing with ρ.

When n increases: Welfare can be written as $w = (y_l s + (y_d - y_l)m_{do} + (y_h - y_l)(\eta - m_{hn}))/r$. The effect of n goes through m_{do} and m_{hn}, which are linked through the trading volume definition of $t = \rho m_{hn}\nu_{hn}$. In the proof of Proposition 2, it has been shown that m_{hn} can be written as a function of m_{do} and n; see Equation (B.2). Applying the chain rule yields

$$\frac{\mathrm{d}m_{hn}}{\mathrm{d}n} = \frac{\partial m_{hn}}{\partial n} + \frac{\partial m_{hn}}{\partial m_{do}}\frac{\mathrm{d}m_{do}}{\mathrm{d}n}. \tag{B.3}$$

Combining the above, one can see that

$$\frac{\mathrm{d}w}{\mathrm{d}n} = \frac{1}{r}\left((y_d - y_l)\frac{\mathrm{d}m_{do}}{\mathrm{d}n} - (y_h - y_l)\frac{\mathrm{d}m_{hn}}{\mathrm{d}n}\right) = \frac{1}{r}\left(\left((y_d - y_l) - (y_h - y_l)\frac{\partial m_{hn}}{\partial m_{do}}\right)\frac{\mathrm{d}m_{do}}{\mathrm{d}n} - (y_h - y_l)\frac{\partial m_{hn}}{\partial n}\right).$$

Therefore, three derivatives of $\frac{\partial m_{hn}}{\partial n}$, $\frac{\partial m_{hn}}{\partial m_{do}}$, and $\frac{\mathrm{d}m_{do}}{\mathrm{d}n}$ need to be evaluated under $\rho \to 0$ and under $\rho \to \infty$.

Consider first the case of $\rho \to 0$. Directly computing the first partial derivative yields

$$\frac{\partial m_{hn}}{\partial n} = \frac{\lambda_u(1 + m_{do} - s)\left(1 - \frac{m_{do}}{m_d}\right)^n \rho \log\left(1 - \frac{m_{do}}{m_d}\right)}{\left(\lambda_d + \lambda_u + \rho\left(1 - \left(1 - \frac{m_{do}}{m_d}\right)^n\right)\right)^2}, \tag{B.4}$$

from which it follows that $\lim_{\rho\to 0}\frac{\partial m_{hn}}{\partial n} = 0$. Also, $\lim_{\rho\to 0}\frac{\partial m_{hn}}{\partial m_{do}} = \frac{\lambda_u}{\lambda_d+\lambda_u} = \eta$. Hence, $\lim_{\rho\to 0}\frac{\mathrm{d}m_{hn}}{\mathrm{d}n} = \eta\lim_{\rho\to 0}\frac{\mathrm{d}m_{do}}{\mathrm{d}n}$. Therefore, $\lim_{\rho\to 0}\frac{\mathrm{d}w}{\mathrm{d}n} = \frac{1}{r}\left((y_d - y_l)\lim_{\rho\to 0}\frac{\mathrm{d}m_{do}}{\mathrm{d}n} - (y_h - y_l)\eta\lim_{\rho\to 0}\frac{\mathrm{d}m_{do}}{\mathrm{d}n}\right) = \frac{1}{r}(y_d - \hat{y})\lim_{\rho\to 0}\frac{\mathrm{d}m_{do}}{\mathrm{d}n}/r$, where $\hat{y} := \eta y_h + (1-\eta)y_l$. Note that $\lim_{\rho\to 0}\frac{\mathrm{d}m_{do}}{\mathrm{d}n} > 0$ because (i) from (S18), $\lim_{\rho\to 0} m_{do} \in (0, m_d)$; and (ii) given the excess supply, m_{do} increases in n (Lemma S1.2). Therefore, $\mathrm{sign}\left[\lim_{\rho\to 0}\frac{\mathrm{d}w}{\mathrm{d}n}\right] = \mathrm{sign}[y_d - \hat{y}]$, proving the statement.

Next, consider the case of $\rho \to \infty$. Note that $\frac{\mathrm{d}m_{do}}{\mathrm{d}n} > 0$ (Lemma S1.2). Then signing $\frac{\mathrm{d}w}{\mathrm{d}n}$ in this case is equivalent to

$$\mathrm{sign}\left[\lim_{\rho\to\infty}\frac{\mathrm{d}w}{\mathrm{d}n}\right] = \mathrm{sign}\left[\frac{y_d - y_l}{y_h - y_l} - \lim_{\rho\to\infty}\left(\frac{\frac{dm_{hn}}{dn}}{\frac{dm_{do}}{dn}}\right)\right].$$

Next, we use (B.4) to calculate $\frac{\partial m_{hn}}{\partial n} \leq 0$. Equation (B.4) then implies that $\lim_{\rho\to\infty}\frac{\frac{dm_{hn}}{dn}}{\frac{dm_{do}}{dn}} \leq \lim_{\rho\to\infty}\frac{\partial m_{hn}}{\partial m_{do}} = 0$. This implies that $\lim_{\rho\to\infty}\frac{\mathrm{d}w}{\mathrm{d}n} > 0$. □

Proposition 4

Proof. Similar to the proof of Proposition 3 one can see that

$$\frac{\mathrm{d}w}{\mathrm{d}\psi} = \frac{1}{r}\left((y_d - y_l)\frac{\mathrm{d}m_{do}}{\mathrm{d}\psi} - (y_h - y_l)\frac{\mathrm{d}m_{hn}}{\mathrm{d}\psi}\right) = \frac{1}{r}\left(\left((y_d - y_l) - (y_h - y_l)\frac{\partial m_{hn}}{\partial m_{do}}\right)\frac{\mathrm{d}m_{do}}{\mathrm{d}\psi} - (y_h - y_l)\frac{\partial m_{hn}}{\partial \psi}\right).$$

Therefore, three derivatives of $\frac{\partial m_{hn}}{\partial \psi}$, $\frac{\partial m_{hn}}{\partial m_{do}}$, and $\frac{\mathrm{d}m_{do}}{\mathrm{d}\psi}$ need to be evaluated under $\rho \to 0$ and under $\rho \to \infty$.

Consider first the case of $\rho \to 0$. Directly computing the first partial derivative yields we get that $\lim_{\rho\to 0} \frac{\partial m_{hn}}{\partial \psi} = 0$. Also, $\lim_{\rho\to 0} \frac{\partial m_{hn}}{\partial m_{do}} = \frac{\lambda_u}{\lambda_d+\lambda_u} = \eta$. Hence, $\lim_{\rho\to 0} \frac{\mathrm{d}m_{hn}}{\mathrm{d}\psi} = \eta \lim_{\rho\to 0} \frac{\mathrm{d}m_{do}}{\mathrm{d}\psi}$. Therefore, $\lim_{\rho\to 0} \frac{\mathrm{d}w}{\mathrm{d}\psi} = \frac{1}{r}\Big((y_d - y_l)\lim_{\rho\to 0} \frac{\mathrm{d}m_{do}}{\mathrm{d}\psi} - (y_h - y_l)\eta \lim_{\rho\to 0} \frac{\mathrm{d}m_{do}}{\mathrm{d}\psi}\Big) = \frac{1}{r}(y_d - \hat{y})\lim_{\rho\to 0} \frac{\mathrm{d}m_{do}}{\mathrm{d}\psi}/r$, where $\hat{y} := \eta y_h + (1-\eta)y_l$. Note that $\lim_{\rho\to 0} \frac{\mathrm{d}m_{do}}{\mathrm{d}\psi} > 0$. Therefore, $\mathrm{sign}\Big[\lim_{\rho\to 0} \frac{\mathrm{d}w}{\mathrm{d}\psi}\Big] = \mathrm{sign}[y_d - \hat{y}]$, proving the statement.

Next, consider the case of $\rho \to \infty$.] Note that $\frac{\mathrm{d}m_{do}}{\mathrm{d}\psi} > 0$ (Lemma S1.3). Then signing $\frac{\mathrm{d}w}{\mathrm{d}\psi}$ in this case is equivalent to

$$\mathrm{sign}\left[\lim_{\rho\to\infty} \frac{\mathrm{d}w}{\mathrm{d}\psi}\right] = \mathrm{sign}\left[\frac{y_d - y_l}{y_h - y_l} - \lim_{\rho\to\infty}\left(\frac{\frac{\mathrm{d}m_{hn}}{\mathrm{d}\psi}}{\frac{\mathrm{d}m_{do}}{\mathrm{d}\psi}}\right)\right].$$

Next, it follows from from (B.2) that $\frac{\partial m_{hn}}{\partial n} \leq 0$. Implicit function theorem applied to (B.2) then implies that $\lim_{\rho\to\infty}\Big(\frac{dm_{hn}}{d\psi}/\frac{dm_{do}}{d\psi}\Big) \leq \lim_{\rho\to\infty} \frac{\partial m_{hn}}{\partial m_{do}} = 0$. This implies that $\lim_{\rho\to\infty} \frac{\mathrm{d}w}{\mathrm{d}\psi} > 0$. □

Proposition 5

Proof. Proposition S4 shows how the trading gains are split between one searching customer and n *potential* counterparty dealers. Recall that with probability q, the customer is able to capture the full trading gain. Therefore, conditional on finding at least one dealer of her matching type, an hn-buyer expects a profit of $q\Delta_{hd} + (1-q)(R_h - (R_d + \bar{A}\Delta_{hd})) = (q + (1-q)(1-\bar{A}))\Delta_{hd}$, while an lo-seller expects $q\Delta_{dl} + (1-q)((R_d - \bar{A}\Delta_{hd}) - R_d) = (q + (1-q)(1-\bar{B}))\Delta_{dl}$. Substituting in $\bar{A}$ and $\bar{B}$ gives the stated $\gamma(\cdot)$ expression. □

Proposition 6

Proposition 6 only characterizes the equilibrium for the case of $n = n^{\mathrm{SMS}} > 2$, which guarantees that $\pi^* \leq \frac{1}{2}$ by Lemma S1.4. Before proceeding to the proof, we first add the case of $n = 2$:

Proposition (Equilibrium technology choices when $n = 2$). If $\pi^* \leq \frac{1}{2}$, Proposition 6 holds. If $\pi^* > 1/2$, a unique stationary equilibrium exists depending on the asset supply s: There exist thresholds $0 < \hat{s}_{hn,0} < \hat{s}_{hn,1} \leq \hat{s}_{lo,1} < \hat{s}_{lo,0} < 1 + m_d$ so that

	(a) hn-buyers' proba -bility to use SMS, θ_{hn}	(b) lo-sellers' proba -bility to use SMS, θ_{lo}	(c) asset holding by dealers, m_{do}
(1) $0 < s \leq \hat{s}_{hn,0}$	0	1	$g(0, 1, m_{do}) = s$
(2) $\hat{s}_{hn,0} \leq s \leq \hat{s}_{hn,1}$	0	$g(1, \theta_{lo}, m_d - m^*_d) = s$	$m_d - m^*_d$
(3) $\hat{s}_{hn,1} < s < \hat{s}_{lo,1}$	0	0	$g(0, 0, m_{do}) = s$
(4) $\hat{s}_{lo,1} \leq s \leq \hat{s}_{lo,0}$	$g(\theta_{hn}, 1, m^*_d) = s$	0	m^*_d
(5) $\hat{s}_{lo,0} < s < 1 + m_d$	1	0	$g(1, 0, m_{do}) = s$

where $g(x_1, x_2, x_3) = s$ uniquely solves θ_{hn}, θ_{lo}, and m_{do} in columns (a), (b), and (c), respectively. The constant π^* is given in Lemma 3 and $m^*_d := \pi^{-1}(\pi^*)m_d$. The function $g(\cdot)$ and the the thresholds $\{\hat{s}_{hn,0}, \hat{s}_{hn,1}, \hat{s}_{lo,1}, \hat{s}_{lo,0}\}$ are given in the proof.

Proof. To begin with, note that both ν_{lo} and ν_{hn} are only functions of θ^k_{lo} and θ^k_{hn}, respectively; see, e.g., Equation (S20). Following Lemma S1.1, Equation (S18) can be written as $g(\theta_{lo}, \theta_{hn}, m_{do}; s) = 0$. For the case of $\pi^* \leq \frac{1}{2}$, define the four thresholds $\{s_{hn,0}, s_{hn,1}, s_{lo,1}, s_{lo,0}\}$ to be the respective unique solution to $g(\cdot; s) = 0$ for $\{\theta_{lo}, \theta_{hn}, m_{do}\} \in \{\{1, 0, \pi^{-1}(\pi^*)m_d\}, \{1, 1, \pi^{-1}(\pi^*)m_d\}, \{1, 1, (1 - \pi^{-1}(\pi^*))m_d\}, \{0, 1, (1 - \pi^{-1}(\pi^*))m_d\}\}$. For the case of $\pi^* > \frac{1}{2}$, likewise, the four thresholds $\{\hat{s}_{hn,0}, \hat{s}_{hn,1}, \hat{s}_{lo,1}, \hat{s}_{lo,0}\}$ are defined as the respective unique solution to $g(\cdot; s) = 0$ for $\{\theta_{lo}, \theta_{hn}, m_{do}\} \in \{\{1, 0, (1 - \pi^{-1}(\pi^*))m_d\}, \{0, 0, (1 - \pi^{-1}(\pi^*))m_d\}, \{0, 0, \pi^{-1}(\pi^*)m_d\}, \{0, 1, \pi^{-1}(\pi^*)m_d\}\}$. In either case, it is easy to see that the four thresholds indeed exist according to the respective definition. In particular, the monotonicity shown in Lemma S1.1 guarantees the sorting of these thresholds. To complete the proof, for each region of s, the stated values of $\{\theta_{lo}, \theta_{hn}, m_{do}\}$ are first verified to indeed sustain an equilibrium and then shown to be unique in that region. Only the case of $\pi^* \leq \frac{1}{2}$ is discussed below for brevity.

Region 1: $0 < s < s_{hn,0}$. With $\{\theta_{lo}, \theta_{hn}\} = \{1, 0\}$, m_{do} is uniquely pinned down by Equation (S18). Since $s < s_{hn,0}$, Lemma S1.1 implies that $\pi_{do} < \pi^*$. Hence, by Lemma 3, $\zeta^{\text{SMS}}_{hn} < \zeta^{\text{BB}}_{hn}$ but $\zeta^{\text{SMS}}_{lo} > \zeta^{\text{BB}}_{lo}$ and, indeed, $\{\theta_{lo}, \theta_{hn}\} = \{1, 0\}$ sustains an equilibrium.

There are three possible deviations. First, suppose instead that :w $\{\theta_{lo}, \theta_{hn}\} \in (0, 1) \times (0, 1)$. This would require both hn-buyers and lo-sellers be indifferent between the two technologies. That is, $\pi_{do} = \pi_{dn} = \pi^*$ must hold, but this cannot be true because $\pi_{do} < \pi^*$ in this region. Second, suppose $\theta_{lo} = \theta_{hn} = 0$. But by Lemma S1.1, this reduction in θ_{lo} would only reduce m_{do} (for a fixed s) and increase m_{dn}, making lo-sellers prefer SMS more, hence inconsistent with $\zeta^{\text{SMS}}_{lo} < \zeta^{\text{BB}}_{lo}$ as implied by $\theta_{lo} = 0$. Third, suppose $\theta_{lo} = \theta_{hn} = 1$. Likewise, this increase in θ_{hn} would decrease m_{do}, inconsistent with hn-buyers' switch from BB to SMS as a lower m_{do} would only strengthen $\zeta^{\text{SMS}}_{hn} < \zeta^{\text{BB}}_{hn}$. Since none of these alternative values of θ_{lo} and θ_{hn} can sustain the equilibrium, in this range of s, the only possible equilibrium is $\{\theta_{lo}, \theta_{hn}\} = \{1, 0\}$.

Region 2: $s_{hn,0} \leq s \leq s_{hn,1}$. With $\{\theta_{lo}, m_{do}\} = \{1, \pi^{-1}(\pi^*)m_d\}$ in this region, $g(\cdot; s) = 0$ uniquely solves $\theta_{hn} \in [0, 1]$. This is indeed an equilibrium because at $\pi_{do} = \pi^*$, hn-buyers are indifferent between SMS and BB and, hence, any $\theta_{hn} \in [0,1]$ is admissible. On the other hand, $\pi_{do} = \pi^* < \frac{1}{2}$ implies that $m_{do} < m_d/2$ (recall that $\pi(x) \geq x$ by assumption) and so $m_{dn} > m_d/2$. It then follows that $\pi_{dn} > m_{dn}/m_d > \frac{1}{2} > \pi^*$ (because $\pi^* < 1/2$). Therefore, $\zeta_{lo}^{\text{SMS}} > \zeta_{lo}^{\text{BB}}$ by Lemma 3 and $\theta_{lo} = 1$ is sustained.

To rule out other equilibria, consider alternative values. Suppose $\pi_{do} > \pi^*$, implying $\theta_{hn} = 1$. Recall that $s = s_{hn,1}$ is the unique solution to $g(\cdot; s) = 0$ when $\theta_{lo} = \theta_{hn} = 1$ and $m_{do} = \pi^{-1}(\pi^*)m_d$. The monotonicity in Lemma S1.1 would then require $s > s_{hn,1}$, out of this region. Suppose instead $\pi_{do} < \pi^*$, implying $\theta_{hn} = 0$. Then, similarly, the monotonicity in Lemma S1.1 would require $s < s_{hn,0}$, again out of this region. Finally, suppose $\pi_{do} = \pi^*$ but $\theta_{lo} < 1$. Then $\pi_{do} = \pi^* < \frac{1}{2}$ implies that $m_{do} < m_d/2$ and so $m_{dn} > m_d/2$. It then follows that $\pi_{dn} > m_{dn}/m_d > \frac{1}{2} > \pi^*$, implying $\theta_{lo} = 1$, a contradiction.

Region 3: $s_{hn,1} < s < s_{lo,1}$. When $\theta_{lo} = \theta_{hn} = 1$, $s_{hn,1} < s < s_{lo,1}$ ensures that m_{do} as solved from $g(\cdot; s) = 0$ satisfies $\pi^{-1}(\pi^*)m_d < m_{do} < \pi^{-1}(1 - \pi^*)m_d$; and, hence, $\pi_{dn} > \pi^*$. That is, $\zeta^{\text{SMS}} > \zeta^{\text{BB}}$ for both hn and lo, which indeed guarantee that $\theta_{lo} = \theta_{hn} = 1$ as an equilibrium.

Again, consider other values for $\{\theta_{lo}, \theta_{hn}\}$. First, $\{\theta_{lo}, \theta_{hn}\} \in (0,1)^2$ cannot be an equilibrium for the same reason as explained in Region 1. Second, suppose $\{\theta_{lo}, \theta_{hn}\} = \{1, 0\}$. By Lemma S1.1, this reduction in θ_{hn} would result in an increase in m_{do}, but such an increase would only make SMS more attractive for hn-buyers, contradicting the reduction of θ_{hn}. Third, suppose $\{\theta_{lo}, \theta_{hn}\} = \{0, 1\}$. Then similarly by Lemma S1.1, this reduction in θ_{lo} would result in a decrease in m_{do} or an increase in m_{dn}, but such an increase would only make SMS more attractive for lo-sellers, contradicting the reduction of θ_{lo}.

Region 4: $s_{lo,1} \leq s \leq s_{lo,0}$. This region mirrors Region 2 and the proof is omitted for brevity.

Region 5: $s_{lo,0} < s < 1 + m_d$. This region mirrors Region 1 and the proof is omitted for brevity. □

Proposition 7

Proof. We consider the case of $s > s_{hn,1}$ and prove that the ratio defined in (24) weakly decreases in s. The volume share ratio, VS, in this region can be written as

$$\frac{\rho^{\text{SMS}} m_{lo}^{\text{SMS}} \nu_{lo}^{\text{SMS}} + \rho^{\text{SMS}} m_{hn}^{\text{SMS}} \nu_{hn}^{\text{SMS}}}{\left(\rho^{\text{SMS}} m_{lo}^{\text{SMS}} \nu_{lo}^{\text{SMS}} + \rho^{\text{SMS}} m_{hn}^{\text{SMS}} \nu_{hn}^{\text{SMS}}\right) + \left(\rho^{\text{BB}} m_{lo}^{\text{BB}} \nu_{lo}^{\text{BB}} + \rho^{\text{BB}} m_{hn}^{\text{BB}} \nu_{hn}^{\text{BB}}\right)} = \frac{1}{2} + \frac{1}{2} \frac{m_{lo}^{\text{SMS}} \nu_{lo}^{\text{SMS}}}{m_{hn}^{\text{SMS}} \nu_{hn}^{\text{SMS}}}.$$

This is because in the considered region, $\theta_{hn} = 1$. Then the dealer stationarity (S8) reduces to

$$\rho^{\text{SMS}} m_{lo}^{\text{SMS}} \nu_{lo}^{\text{SMS}} + \rho^{\text{BB}} m_{lo}^{\text{BB}} \nu_{lo}^{\text{BB}} = \rho^{\text{SMS}} m_{hn}^{\text{SMS}} \nu_{hn}^{\text{SMS}}. \tag{B.5}$$

We consider three cases next:

- $s < s_{lo,1}$. In this case, $\theta_{lo} = 1$, which means that the dealer stationarity condition (S8) writes as $\rho^{\text{SMS}} m_{lo}^{\text{SMS}} \nu_{lo}^{\text{SMS}} = \rho^{\text{SMS}} m_{hn}^{\text{SMS}} \nu_{hn}^{\text{SMS}}$ implying $VS = 1$.

- $s > s_{lo,0}$. In this case, $\theta_{lo} = 0$, implying $VS = 1/2$.
- $s_{lo,1} \leq s \leq s_{lo,0}$. In this case, m_{do} is a constant, invariant of s, and so both ν_{lo}^{BB} and ν_{lo}^{SMS} are constants as well. Then sign$\frac{\mathrm{d}VS}{\mathrm{d}s} = \text{sign}\frac{\mathrm{d}}{\mathrm{d}s}\big(m_{lo}^{\text{SMS}}/m_{hn}^{\text{SMS}}\big)$. Using again (B.5),
$$\frac{m_{lo}^{\text{SMS}}}{m_{hn}^{\text{SMS}}} = \frac{\rho^{\text{SMS}} m_{lo}^{\text{SMS}} \nu_{hn}^{\text{SMS}}}{\rho^{\text{SMS}} m_{lo}^{\text{SMS}} \nu_{lo}^{\text{SMS}} + \rho^{\text{BB}} m_{lo}^{\text{BB}} \nu_{lo}^{\text{BB}}} = \frac{\rho^{\text{SMS}} \nu_{hn}^{\text{SMS}}}{\rho^{\text{SMS}} \nu_{lo}^{\text{SMS}} + \rho^{\text{BB}} \nu_{lo}^{\text{BB}} \left(\frac{m_{lo}^{\text{BB}}}{m_{lo}^{\text{SMS}}}\right)}$$
Using the stationarity conditions (S4) and (S5),
$$\frac{m_{lo}^{\text{SMS}}}{m_{lo}^{\text{BB}}} = \frac{\lambda_u + \rho^{\text{BB}} \nu_{lo}^{\text{BB}}}{\lambda_u + \rho^{\text{SMS}} \nu_{lo}^{\text{SMS}}} \frac{\theta_{lo}}{1-\theta_{lo}},$$
increasing in θ_{lo}, which is the only variable endogenous of s. Proposition 6 has shown that in this range, θ_{lo} decreases with s. Therefore, by chain rule, sign$\frac{\mathrm{d}V}{\mathrm{d}s} < 0$.

Combining the three cases completes the proof for the claims regarding s. To prove the claims regarding λ_d, note that from Equation (S18), cateris paribus, the left-hand side is monotone increasing in λ_d (the excess supply implies $\nu_{hn} > \nu_{lo}$; see Equation (S22)) but decreasing in s. Therefore, increases in s are equivalent to those in λ_d. Hence, all results about s above also hold for λ_d. □

Proposition 8

Proof. Since $n^{\text{BB}} = 1 < n^{\text{SMS}}$, below the notation n, without the superscript, indicates n^{SMS}. Proposition S4 gives the expression of $\bar{B}^k$ for $k \in \{\text{BB}, \text{SMS}\}$. In particular, $\bar{B}^{\text{BB}} = 1$, and for SMS, $\bar{B}^{\text{SMS}} = \frac{n\cdot(1-\pi_{do})\pi_{do}^{n-1}}{1-(1-\pi_{do})^k}$. Then $\bar{B}^{\text{SMS}}/\bar{B}^{\text{BB}} = \bar{B}^{\text{SMS}}$. By Lemma S1.1, m_{do} is weakly increasing with s and hence so does π_{do}, thus proving the claim. To prove the claims regarding λ_d, note that from Equation (S18), cateris paribus, the left-hand side is monotone increasing in λ_d (the excess supply implies $\nu_{hn} > \nu_{lo}$; see Equation S22) but decreasing in s. Hence, all results about s hold for λ_d. □

Proposition 9 and 10

Proof. Welfare can be written as $w = \frac{1}{r}(y_l s + (y_d - y_l)m_{do} + (y_h - y_l)(\eta - m_{hn}))$. Consider a small change in either $\theta \in \{\theta_{hn}, \theta_{lo}\}$. We then have
$$\text{sign}\left[\frac{\mathrm{d}w}{\mathrm{d}\theta}\right] = \text{sign}\left[(y_d - y_l)\frac{\mathrm{d}m_{do}}{\mathrm{d}\theta} - (y_h - y_l)\frac{\mathrm{d}m_{hn}}{\mathrm{d}\theta}\right].$$
Moreover, following $m_{ho} + m_{hn} = \eta$ and using the expressions (S2) and (S3), we have
$$\frac{\mathrm{d}m_{hn}}{\mathrm{d}\theta} = -\frac{\mathrm{d}m_{hn}}{\mathrm{d}\theta} = \eta\frac{\mathrm{d}m_{do}}{\mathrm{d}\theta} - \frac{1}{\lambda_u + \lambda_d}\frac{\mathrm{d}t}{\mathrm{d}\theta}. \tag{B.6}$$
Combining the above two, we get
$$\text{sign}\left[\frac{\mathrm{d}w}{\mathrm{d}\theta}\right] = \text{sign}\left[(y_d - \hat{y})\frac{\mathrm{d}m_{do}}{\mathrm{d}\theta} + \frac{y_h - y_l}{\lambda_u + \lambda_d}\frac{\mathrm{d}t}{\mathrm{d}\theta}\right], \tag{B.7}$$

where $\hat{y} := \eta y_h + (1-\eta) y_l$. The derivative of $\frac{\mathrm{d}m_{do}}{\mathrm{d}\theta}$ can be signed by the implicit function theorem using the results from Lemma S1.1: $\frac{\mathrm{d}m_{do}}{\mathrm{d}\theta_{lo}} > 0$ and $\frac{\mathrm{d}m_{do}}{\mathrm{d}\theta_{hn}} < 0$. To see how volume t changes with respect to θ, recall from Equations (S2) and (S3) and use $t = \rho m_{lo} \nu_{lo} = \rho m_{hn} \nu_{hn}$ to get

$$t = \frac{\lambda_d (s - m_{do})}{\frac{\lambda_d + \lambda_u}{\rho \nu_{lo}} + 1} \text{ and } t = \frac{\lambda_u (1 + m_{do} - s)}{\frac{\lambda_d + \lambda_u}{\rho \nu_{hn}} + 1}. \tag{B.8}$$

Note that θ_{hn} in the first expression only affects t through m_{do}. Therefore, t is increasing in θ_{lo}. Likewise, θ_{lo} affects t in the second expression only through m_{do}. Hence, t is also increasing in θ_{hn}. That is, $\frac{\mathrm{d}t}{\mathrm{d}\theta} > 0$ for either $\theta \in \{\theta_{lo}, \theta_{hn}\}$.

The case of sufficiently high ρ, i.e., $\rho := \min[\rho^{\mathbf{BB}}, \rho^{\mathbf{SMS}}] \to \infty$: Since $\frac{\mathrm{d}t}{\mathrm{d}\theta} > 0$,

$$\operatorname{sign}\left[\lim_{\rho\to\infty} \frac{\mathrm{d}w}{\mathrm{d}\theta}\right] = \operatorname{sign}\left[(y_d - \hat{y}) \lim_{\rho\to\infty} \left(\frac{\mathrm{d}m_{do}}{\mathrm{d}\theta} / \frac{\mathrm{d}t}{\mathrm{d}\theta}\right) + \frac{y_h - y_l}{\lambda_u + \lambda_d}\right].$$

Hence, one needs to find $\lim_{\rho\to\infty}\left(\frac{\mathrm{d}m_{do}}{\mathrm{d}\theta} / \frac{\mathrm{d}t}{\mathrm{d}\theta}\right)$.

Consider first $\theta = \theta_{lo}$. Then differentiate the second expression of t in (B.8) with respect to $\theta = \theta_{lo}$, to get $\left(\frac{\lambda_d + \lambda_u}{\nu_{hn}} + \rho\right)\frac{\mathrm{d}t}{\mathrm{d}\theta} = \rho \lambda_u \frac{\mathrm{d}m_{do}}{\mathrm{d}\theta}$. Hence, $\lim_{\rho\to\infty}\left(\frac{\mathrm{d}m_{do}}{\mathrm{d}\theta} / \frac{\mathrm{d}t}{\mathrm{d}\theta}\right) = \frac{1}{\lambda_u}$. (Note that $\nu_{hn}^k = 1 - (1 - m_{do}/m_d)^{n^k}$ is always nonzero, because $m_{do} > m_d/2$ in the case of excess supply.) Then $\operatorname{sign}\left[\lim_{\rho\to\infty} \frac{\mathrm{d}w}{\mathrm{d}\theta}\right] = \operatorname{sign}\left[\frac{y_d - \hat{y}}{\lambda_u} + \frac{y_h - y_l}{\lambda_u + \lambda_d}\right] = \operatorname{sign}[y_d - \hat{y} - (y_h - y_l)\eta] = \operatorname{sign}[y_d - y_l] > 0$. (Recall that $y_d \in (\overline{y}_d^{'}, \overline{y}_d^{'}) \subset (y_l, y_h)$ by Corollary **??**).

Consider $\theta = \theta_{hn}$. Then differentiate the first expression of t in (B.8) with respect to $\theta = \theta_{hn}$. As $\rho \to \infty$, the limit of m_{do} may be binding at m_d, resulting in $\nu_{lo} \to 0$. If it is not binding, i.e., if $\lim_{\rho\to\infty} m_{do} < m_d$, then $\nu_{lo}^k > 0$ and $\lim_{\rho\to\infty}\left(\frac{\mathrm{d}m_{do}}{\mathrm{d}\theta} / \frac{\mathrm{d}t}{\mathrm{d}\theta}\right) = -\frac{1}{\lambda_d}$. If it is binding, i.e., $m_{do} \to m_d$ and $\nu_{lo} \to 0$, however $\rho \nu_{lo}$ has a strictly positive limit as folows from (S18). Then, one can again derive $\lim_{\rho\to\infty}\left(\frac{\mathrm{d}m_{do}}{\mathrm{d}\theta} / \frac{\mathrm{d}t}{\mathrm{d}\theta}\right) > -\frac{1}{\lambda_d}$. Therefore, $\operatorname{sign}\left[\lim_{\rho\to\infty} \frac{\mathrm{d}w}{\mathrm{d}\theta}\right] > \operatorname{sign}\left[-\frac{y_d - \hat{y}}{\lambda_d} + \frac{y_h - y_l}{\lambda_u + \lambda_d}\right] = \operatorname{sign}[-y_d + \hat{y} - (y_h - y_l)(1 - \eta)] = \operatorname{sign}[y_h - y_d] > 0$.

The case of sufficiently low ρ, i.e., $\rho := \max\{\rho^{\mathbf{BB}}, \rho^{\mathbf{SMS}}\} \to 0$: For either $\theta \in \{\theta_{lo}, \theta_{hn}\}$, directly calculating $\frac{\mathrm{d}t}{\mathrm{d}\theta}$ from (B.8) and taking the limit yield $\lim_{\rho\to 0} \frac{\mathrm{d}t}{\mathrm{d}\theta} = 0$. Yet, $\lim_{\rho\to 0} \frac{\mathrm{d}m_{do}}{\mathrm{d}\theta} \neq 0$, which follows by taking the limit in the calculations of Lemma S1.1. Hence, $\lim_{\rho\to 0} \frac{\mathrm{d}m_{do}}{\mathrm{d}\theta_{lo}} > 0$ and $\lim_{\rho\to 0} \frac{\mathrm{d}m_{do}}{\mathrm{d}\theta_{hn}} < 0$ remain. Therefore, $\lim_{\rho\to 0} \operatorname{sign}\left[\frac{\mathrm{d}w}{\mathrm{d}\theta}\right] = \operatorname{sign}\left[(y_d - \hat{y}) \lim_{\rho\to 0} \frac{\mathrm{d}m_{do}}{\mathrm{d}\theta}\right]$, proving the statement made in the proposition. □

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Simultaneous Multilateral Search

Supplementary Appendix

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- Section S1 details additional useful results and completes the proofs for the main paper.
- Section S2 compares the main model in the paper with "directed search."
- Section S3 examines the pricing by the dealers.
- Section S4 studies a model extension where each customer can choose her search intensity ρ.
- Section S5 characterizes the transition dynamics equilibrium.
- Section S6 describes an alternative price setting mechanism and shows that it is equivalent to the one adopted in the main model.
- Section S7 collects proofs to the results in S2-S6.

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S1 Additional results and proofs of lemmas

S1.1 Demographics with one search technology

While the system (2)-(7) has only two zero-flow conditions (Equations 6 and 7), the stationarity of all other types of agents is also implied. Apart from the dealer stationarity (8), $-(5)-(6)$ gives $\nu_{lo}m_{lo}\rho - m_{ln}\lambda_u + m_{hn}\lambda_d = 0$, ensuring that the net flow in and out of ln-bystanders is zero. Likewise, $(5)-(7)$ gives $\nu_{hn}m_{hn}\rho - m_{ho}\lambda_d + m_{lo}\lambda_u = 0$, ensuring that the net flow of ho-bystanders is zero.

We also derive some useful expressions for the customer masses. Equations (2), (3), and (5) together imply the stable fractions of the high-type and the low-type customers:

$$m_{ho} + m_{hn} = \frac{\lambda_u}{\lambda_d + \lambda_u} =: \eta \text{ and } m_{lo} + m_{ln} = \frac{\lambda_u}{\lambda_d + \lambda_u} = 1 - \eta. \tag{S1}$$

Then combining the market clearing condition (2) and the lo-seller net flow (6), we obtain

$$m_{lo} = (1-\eta)(s - m_{do}) - \frac{t}{\lambda_u + \lambda_d}, \tag{S2}$$

which intuitively says that the stationary mass of lo-sellers is a fraction $(1-\eta)$ of the residual asset supply $(s - m_{do})$ available to customers, less a term $t/(\lambda_u + \lambda_d)$ due to their active trading. Combining (2) and (6) gives

$$m_{hn} = \eta \cdot (1 + m_{do} - s) - \frac{t}{\lambda_u + \lambda_d}. \tag{S3}$$

Note that $1 + m_{do} - s$, which is the total mass of non-owner customers in this economy. That is, the stationary mass of hn-buyers is the high-type fraction η of all non-owner customers, less the same term due to trading. The above expressions are in fact generic in the search literature. For example, if, as in DGP, customers find each other at intensity ρ without dealers, then Equations (S2) and (S3) still hold with $m_{do} = 0$ and $t = 2\rho m_{hn} m_{lo}$.

S1.2 Demographics with two search technologies

There are six customer population sizes: $\{m_{ho}, m_{ln}, m_{hn}^{\text{SMS}}, m_{hn}^{\text{BB}}, m_{lo}^{\text{SMS}}, m_{lo}^{\text{BB}}\}$; in addition, there are two types of dealers, $\{m_{do}, m_{dn}\}$. For notation simplicity, write

$$m_{hn} = m_{hn}^{\text{SMS}} + m_{hn}^{\text{BB}}; \text{ and } m_{lo} = m_{lo}^{\text{SMS}} + m_{lo}^{\text{BB}}.$$

Then the four (aggregate) customer masses, $\{m_{ho}, m_{ln}, m_{hn}, m_{lo}\}$, must satisfy the conditions (2)-(5) in Section 2.1. The other four conditions are analogous to the stationarity conditions (6) and (7):

$$\text{net flow of } lo\text{-sellers using SMS:} \qquad -\nu_{lo}^{\text{SMS}} m_{lo}^{\text{SMS}} \rho^{\text{SMS}} - \lambda_u m_{lo}^{\text{SMS}} + \theta_{lo} \lambda_d m_{ho} = 0 \tag{S4}$$

$$\text{net flow of } lo\text{-sellers using BB:} \qquad -\nu_{lo}^{\text{BB}} m_{lo}^{\text{BB}} \rho^{\text{BB}} - \lambda_u m_{lo}^{\text{BB}} + (1-\theta_{lo}) \lambda_d m_{ho} = 0 \tag{S5}$$

$$\text{net flow of } hn\text{-buyers using SMS:} \qquad -\nu_{hn}^{\text{SMS}} m_{hn}^{\text{SMS}} \rho^{\text{SMS}} - \lambda_d m_{hn}^{\text{SMS}} + \theta_{hn} \lambda_u m_{ln} = 0 \tag{S6}$$

$$\text{net flow of } hn\text{-buyers using BB:} \qquad -\nu_{hn}^{\text{BB}} m_{hn}^{\text{BB}} \rho^{\text{BB}} - \lambda_d m_{hn}^{\text{BB}} + (1-\theta_{hn}) \lambda_u m_{ln} = 0 \tag{S7}$$

where $\nu_{lo}^k = 1 - \left(1 - \frac{m_{dn}}{m_d}\right)^{n^k}$ and $\nu_{hn}^k = 1 - \left(1 - \frac{m_{do}}{m_d}\right)^{n^k}$ are the probabilities for a customer to find at least one counterparty dealer using technology $k \in \{\text{BB}, \text{SMS}\}$. Compared to Equations (6) and (7) in Section 2.1, the key differences are (i) that every variable here is technology-dependent and superscripted with $k \in \{\text{BB}, \text{SMS}\}$; and (ii) that only a fraction of θ_σ of the newly shocked σ-customer use SMS, while the rest $(1-\theta_\sigma)$ use BB, where $\sigma \in \{hn, lo\}$.

The conditions (2)-(5) and (S4)-(S7) exactly pin down the eight demographic variables:

Lemma S1 (Stationary demographics with technology choice). Given the customers' technology choices $\{\theta_{lo}, \theta_{hn}\} \in [0,1]^2$, Equations (2)-(5) and (S4)-(S7) uniquely pin down the demographics $\{m_{ho}, m_{ln}, m_{hn}^{\text{SMS}}, m_{hn}^{\text{BB}}, m_{lo}^{\text{SMS}}, m_{lo}^{\text{BB}}\} \in [0,1]^6$ and $\{m_{do}, m_{dn}\} \in (0, m_d)^2$.

The resulting expressions are similar to those implied by Lemma 1. In particular, (S4) + (S5) − (S6) − (S7) + (5) gives the trading volume expression

$$t := \sum_k \nu_{lo}^k m_{lo}^k \rho^k = \sum_k \nu_{hn}^k m_{hn}^k \rho^k, \tag{S8}$$

ensuring the stationarity of both dealer types. The h- and l-type customer stationarity (S1) also holds the same, and so do the expressions for the total size of trading customers $m_{lo} = \sum_k m_{lo}^k$ and $m_{hn} = \sum_k m_{hn}^k$. The stationarity of all other types of agents are also ensured: For example, −(5) − (S4) − (S5) gives $-\lambda_u m_{ln} + \sum_k \big(\nu_{lo}^k m_{lo}^k \rho^k + \lambda_d m_{hn}^k\big) = 0$, which ensures the stationarity of ln-bystanders. Likewise, (5) − (S6) − (S7) gives $-\lambda_d m_{ho} + \sum_k \big(\nu_{hn}^k m_{hn}^k \rho^k + \lambda_u m_{ln}^k\big) = 0$, which ensures the stationarity of ho-bystanders.

S1.3 Value functions with two search technologies

Given the technology choices $\{\theta_\sigma\}$, hence also the demographics (Lemma S1), the value functions for all six agent types can be derived analogously to those in Equations (10)-(15). For example, the value functions of an ho-bystander and an ln-bystander must satisfy the HJB equations

$$y_h + \lambda_d \cdot \big(\max\big[V_{lo}^{\text{SMS}}, V_{lo}^{\text{BB}}\big] - V_{ho}\big) - (r + f_c) V_{ho} = 0; \tag{S9}$$

$$\lambda_u \cdot \big(\max\big[V_{ho}^{\text{SMS}}, V_{ho}^{\text{BB}}\big] - V_{ln}\big) - (r + f_c) V_{ln} = 0. \tag{S10}$$

Compared with Equations (10) and (11), the only difference is that upon a preference shock, a newly shocked trading customer can choose which technology to use, hence the term of $\max\big[V_\sigma^{\text{SMS}}, V_\sigma^{\text{BB}}\big]$

in the above HJBs ($\sigma \in \{lo, hn\}$).

The HJB equations for the trading agents are also similar to before:

$$\text{HJB of } lo\text{-sellers using technology } k\text{:} \quad y_l + \lambda_u \cdot (V_{ho} - V_{lo}^k) - (r + f_c)V_{lo}^k + \zeta_{lo}^k \Delta_{dl}^k = 0; \tag{S11}$$

$$\text{HJB of } hn\text{-buyers using technology } k\text{:} \quad \lambda_d \cdot (V_{ln} - V_{hn}^k) - (r + f_c)V_{hn}^k + \zeta_{hn}^k \Delta_{hd}^k = 0; \tag{S12}$$

$$\text{HJB of } do\text{-dealers:} \quad y_d - (r + f_d)V_{do} + \sum_k \zeta_{do}^k \Delta_{hd}^k = 0; \tag{S13}$$

$$\text{HJB of } dn\text{-dealers:} \quad -(r + f_d)V_{dn} + \sum_k \zeta_{dn}^k \Delta_{dl}^k = 0. \tag{S14}$$

Compared to Equations (12)-(15), the only difference is that the trading gains $\{\Delta_{hd}, \Delta_{dl}\}$ and the trading gain intensities $\{\zeta_{lo}, \zeta_{hn}, \zeta_{do}, \zeta_{dn}\}$ are technology specific, superscripted with $k \in \{\text{BB}, \text{SMS}\}$. For completeness, we derive these expressions below.

Using technology k, an lo-seller's reservation value is $R_l^k := V_{lo}^k - V_{ln}$, and that for an hn-buyer is $R_h^k := V_{ho} - V_{hn}^k$. A dealer's reservation value is the same $R_d := V_{do} - V_{dn}$ as before. Then, depending the customer's technology k, the trading gain between an hn-buyer and a do-seller is $\Delta_{hd}^k := R_h^k - R_d$ and that between a dn-buyer and an lo-seller is $\Delta_{dl}^k := R_d - R_l^k$. By Proposition S4, the dealers' respective average ask and bid are:

$$\bar{A}^k = \frac{n^k \frac{m_{do}}{m_d}\left(1 - \frac{m_{do}}{m_d}\right)^{n^k - 1}}{1 - \left(1 - \frac{m_{do}}{m_d}\right)^{n^k}} \quad \text{and} \quad \bar{B}^k = \frac{n^k \frac{m_{dn}}{m_d}\left(1 - \frac{m_{dn}}{m_d}\right)^{n^k - 1}}{1 - \left(1 - \frac{m_{dn}}{m_d}\right)^{n^k}}.$$

Thus, an hn-buyer expects $\zeta_{hn}^k \Delta_{hd}^k$, while a do-dealer expects $\zeta_{do}^k \Delta_{hd}^k$, where the respective trading gain intensities are

$$\zeta_{hn}^k = \rho^k \nu_{hn}^k \cdot \left(q^k + (1 - q^k)(1 - \bar{A}^k)\right) \quad \text{and} \quad \zeta_{do}^k = \frac{m_{hn}^k \rho^k \nu_{hn}^k}{m_{do}}(1 - q^k)\bar{A}^k.$$

Analogously, an lo-seller expects $\zeta_{lo}^k \Delta_{dl}^k$, while a dn-dealer expects $\zeta_{dn} \Delta_{dl}^k$, with intensities

$$\zeta_{lo}^k = \rho^k \nu_{lo}^k \cdot \left(q^k + (1 - q^k)(1 - \bar{B}^k)\right) \quad \text{and} \quad \zeta_{dn}^k = \frac{m_{hn}^k \rho^k \nu_{lo}^k}{m_{dn}}(1 - q^k)\bar{B}^k.$$

Finally, with the above expressions of the value functions and trading gain intensities, we can show that the trading gain remains positive under the same condition of $\overline{y}_d \geq y_d \geq \underline{y}_d$ as defined in Proposition 1. To see why, note that in equilibrium, the trading customers either have a strict preference for one of the technology or are indifferent. Consider lo-sellers, for example. If the preference is strict, then only one of the two HJBs in (S11) is relevant; and if indifference, then the two HJBs reduce to the same one. The same holds for hn-buyers in their two HJBs (S12). Likewise, the max$[\cdot]$ operator in Equations (S11) and (S12) can be dropped in equilibrium. Hence, defining $V_{lo} = \max_k[\{V_{lo}^k\}]$ and $V_{hn} = \max_k[\{V_{hn}^k\}]$, the HJB equations (S9)-(S14) can be reduced to the exactly the same set of (10)-(15) as if there is only one technology. Therefore, solving the same equation system, the same result from Proposition 1 holds. Following the same argument in the paragraph after Proposition 1, the positive trading gains also ensures that both the $ho-$ and ln-customers do stay out of trading—they are indeed bystanders.

S1.4 Additional lemmas

Lemma S1.1. Write the left-hand side of Equation (S18) as a function of $g(\theta_{lo}, \theta_{hn}, m_{do}, s)$. Then (1) $\frac{\partial g}{\partial m_{do}} > 0$, (2) $\frac{\partial g}{\partial \theta_{lo}} < 0$, (3) $\frac{\partial g}{\partial \theta_{hn}} > 0$, and (4) $\frac{\partial g}{\partial s} < 0$. In particular, (5) $m_{do} \downarrow 0$ when $s \downarrow 0$ and $m_{do} \uparrow 1 + m_d$ when $s \uparrow 1 + m_d$ regardless of θ_{lo} and θ_{hn}. Finally, (6) $\frac{\partial m_{do}}{\partial \theta_{lo}} > 0 > \frac{\partial m_{do}}{\partial \theta_{hn}}$.

Lemma S1.2. When there is excess supply, both the search intensity ρ and the capacity n increase m_{do} and reduce m_{dn}, but their effects on customers' matching rates are different: A higher ρ increases ν_{hn} but decreases ν_{lo}, while a larger n increases both ν_{hn} and ν_{lo}.

Lemma S1.3. When there is excess supply, the transparency ψ increase m_{do} and reduce m_{dn}.

Lemma S1.4. The functions $z^{\text{SMS}}(\pi)$ and $z^{\text{BB}}(\pi)$ in Lemma 3 cross at $\pi^* > \frac{1}{2}$ if and only if $n^{\text{SMS}} = 2$ and $\frac{2q^{\text{SMS}}\rho^{\text{SMS}} - q^{\text{BB}}\rho^{\text{BB}}}{(2q^{\text{SMS}}-1)\rho^{\text{SMS}}} > \frac{1}{2}$.

S1.5 Proof of Lemma 1 and Lemma S1

Proof. The proof considers the general case of Lemma S1 with arbitrary θ_{hn} and θ_{lo}. Lemma 1 is then just a special case of $\theta_{hn} = \theta_{lo} = 1$. Where convenient, we will occasionally write $\theta_\sigma^{\text{SMS}} = 1 - \theta_\sigma^{\text{BB}} = \theta_\sigma$. The idea is to first express all other unknowns as monotone functions of m_{do}. The existence and the uniqueness then follow as long as the solution to m_{do} exists and is unique. To begin with, add (S4) and (S5) to get

$$\text{all } lo\text{-seller stationarity:} \qquad -\lambda_u m_{lo} + \lambda_d m_{ho} - \rho m_{lo}\nu_{lo} = 0, \tag{S15}$$

where $m_{lo} := \sum_k m_{lo}^k$ is the total lo-seller mass, $\rho := \max[\rho^{\text{SMS}}, \rho^{\text{BB}}]$, $\nu_{lo} := \frac{1}{\rho m_{lo}} \sum_k \rho^k m_{lo}^k \nu_{lo}^k$ is the (weighted) average matching rate for an lo-seller, and $m_{ho} := \sum_k m_{ho}^k$ is the total ho-bystander mass. Similarly, adding (S6) and (S7) yields

$$\text{all } hn\text{-buyer stationarity:} \qquad -\lambda_d m_{hn} + \lambda_u m_{ln} - \rho m_{hn}\nu_{hn} = 0, \tag{S16}$$

where $m_{hn} := \sum_k m_{hn}^k$, $\nu_{hn} := \frac{1}{\rho m_{hn}} \sum_k \rho^k m_{hn}^k \nu_{hn}^k$, and $m_{ln} := \sum_k m_{ln}^k$. Taking $\{\nu_{lo}, \nu_{hn}\}$ as given, Equations (S1), (S15), and (S16) form a linear system of the four masses $\{m_{ho}, m_{ln}, m_{hn}, m_{lo}\}$, which have the unique solution of

$$\begin{aligned} m_{ho} &= \eta\frac{\lambda_u\nu_{hn} + \rho\nu_{lo}\nu_{hn}}{\lambda_u\nu_{hn} + \lambda_d\nu_{lo} + \rho\nu_{hn}\nu_{lo}}; \quad m_{ln} = (1-\eta)\frac{\lambda_d\nu_{lo} + \rho\nu_{lo}\nu_{hn}}{\lambda_u\nu_{hn} + \lambda_d\nu_{lo} + \rho\nu_{hn}\nu_{lo}}; \\ m_{hn} &= (1-\eta)\frac{\lambda_u\nu_{lo}}{\lambda_u\nu_{hn} + \lambda_d\nu_{lo} + \rho\nu_{hn}\nu_{lo}}; \quad m_{lo} = \eta\frac{\lambda_d\nu_{hn}}{\lambda_u\nu_{hn} + \lambda_d\nu_{lo} + \rho\nu_{hn}\nu_{lo}}. \end{aligned} \tag{S17}$$

Plug in the expressions of m_{ho} and $m_{lo} = \sum_k m_{lo}^k$ into the market clearing condition (2) to get

$$\eta \frac{(\lambda_u + \lambda_d)\nu_{hn} + \rho\nu_{lo}\nu_{hn}}{\lambda_u\nu_{hn} + \lambda_d\nu_{lo} + \rho\nu_{lo}\nu_{hn}} + m_{do} - s = 0. \tag{S18}$$

This is an equation with unknowns $\{m_{do}, \nu_{hn}, \nu_{lo}\}$. It remains to express ν_{hn} and ν_{lo} as (monotone) functions of m_{do}.

Consider ν_{lo} for example. Note that (S4) and (S5) imply that

$$m_{lo}^k = \frac{\lambda_d m_{ho}\theta_{lo}^k}{\lambda_u + \rho^k\nu_{lo}^k} \tag{S19}$$

where $\theta_{lo}^{\mathrm{BB}} := 1 - \theta_{lo}$ and $\theta_{lo}^{\mathrm{SMS}} := \theta_{lo}$. Hence, from the earlier definition,

$$\nu_{lo} = \frac{\sum_k \rho^k m_{lo}^k \nu_{lo}^k}{\rho m_{lo}} = \frac{\sum_k \rho^k m_{lo}^k \nu_{lo}^k}{\rho\sum_k m_{lo}^k} = \frac{\sum_k \frac{\rho^k\theta_{lo}^k\nu_{lo}^k}{\lambda_u+\rho^k\nu_{lo}^k}}{\rho\sum_k \frac{\theta_{lo}^k}{\lambda_u+\rho^k\nu_{lo}^k}}, \tag{S20}$$

which is monotone increasing in both ν_{lo}^k for $k \in \{BB, SMS\}$. Recall from the definition $\nu_{lo}^k := 1 - (1 - \pi_{dn})^{n^k}$ that both ν_{lo}^k are monotone decreasing in m_{do}. Therefore, so is ν_{lo}. In the same way, both ν_{hn}^k are monotone increasing in m_{do} and so is ν_{hn}.

Now return to Equation (S18). Since both ν_{lo} and ν_{hn} can be expressed as a unique function in m_{do}, (S18) is an equation of a single unknown m_{do}. To prove the existence of the solution, consider the limits of the support of $m_{do} \in [0, m_d]$. As $m_{do} \downarrow 0$, both $\nu_{lo}^k \uparrow 1$ while both $\nu_{hn}^k \downarrow 0$, and as a result, $\nu_{lo} \uparrow 1$ and $\nu_{hn} \downarrow 0$. The left-hand side of (S18), therefore, reaches $-s < 0$. Reversely, as $m_{do} \uparrow m_d$, $\nu_{lo} \downarrow 0$ and $\nu_{hn} \uparrow 1$, the left-hand side of (S18) reaches $1 + m_d - s > 0$ (as it is assumed that $0 < s < 1 + m_d$). Therefore, by continuity, the solution to m_{do} always exists.

To prove uniqueness, examine the derivative of the left-hand side of (S18) with respect to m_{do}:

$$-\eta\lambda_d \frac{(\lambda_u + \lambda_d + \rho\nu_{hn})\nu_{hn}}{(\lambda_u\nu_{hn} + \lambda_d\nu_{lo} + \rho\nu_{hn}\nu_{lo})^2}\frac{\partial\nu_{lo}}{\partial m_{do}} + \eta\lambda_d \frac{(\lambda_u + \lambda_d + \rho\nu_{lo})\nu_{lo}}{(\lambda_u\nu_{hn} + \lambda_d\nu_{lo} + \rho\nu_{hn}\nu_{lo})^2}\frac{\partial\nu_{hn}}{\partial m_{do}} + 1 > 0, \tag{S21}$$

where the inequality holds because ν_{lo} decreases, while ν_{hn} increases, in m_{do}. That is, the left-hand side of (S18) is strictly monotone increasing in m_{do}. Hence, there exists one and only one m_{do} that solves (S18). Therefore, the demographics equation system always has a unique solution. □

S1.6 Proof of Lemma 2

Proof. Calculate the difference between m_{hn} and m_{lo} using the expressions (S3) and (S2) to get

$$m_{hn} - m_{lo} = \eta + m_{do} - s = \eta\lambda_d \cdot \frac{\nu_{lo} - \nu_{hn}}{\lambda_u\nu_{hn} + \lambda_d\nu_{lo} + \rho\nu_{hn}\nu_{lo}}, \tag{S22}$$

where the last equality follows Equation (S18). Therefore, $\mathrm{sign}[m_{hn} - m_{lo}] = \mathrm{sign}[\nu_{lo} - \nu_{hn}]$. Recall that $\nu_{lo} = 1 - (1 - \pi_{dn})^n$ and $\nu_{hn} = 1 - (1 - \pi_{do})^n$, from which it follows that $\nu_{lo} > \nu_{hn}$ if and only if $m_{dn} > m_{do}$. Given that $m_{dn} + m_{do} = m_d$, therefore, $m_{hn} > m_{lo}$ if and only if $m_{do} < m_d/2$. Use again $m_{hn} - m_{lo} = \eta + m_{do} - s$, which is negative if and only if $s > \eta + m_{do} > \eta + m_d/2$. □

S1.7 Proof of Lemma 3

Proof. Consider **first** the case of $\rho^{\text{SMS}} q^{\text{SMS}} n^{\text{SMS}} < \rho^{\text{BB}} q^{\text{BB}} n^{\text{BB}}$. The proof first establishes the single-crossing of $z^{\text{SMS}}(\pi)$ and $z^{\text{BB}}(\pi)$ at some $\pi^* \in (0,1)$. The general idea is to characterize the shapes of $z^{\text{BB}}(\pi)$ and $z^{\text{SMS}}(\pi)$. In particular, it will be shown that z^{BB} is linearly increasing in π, while z^{SMS} is sigmoid-shaped in π, starting below z^{BB} for sufficiently small π; and the two satisfy $z^{\text{BB}}(0) = z^{\text{SMS}}(0) = 0$ and $z^{\text{BB}}(1) < z^{\text{SMS}}(1)$. Therefore, there is always one and only one intersection point $\pi^* \in (0,1)$.

Consider z^{BB} first. With $n^{\text{BB}} = 1$, $z^{\text{BB}} = q^{\text{BB}} \rho^{\text{BB}} \pi$, which is linearly increasing from 0 at $\pi = 0$ to $q^{\text{BB}} \rho^{\text{BB}}$ at $\pi = 1$. Next, consider z^{SMS}. For notation simplicity, the superscripts SMS on n, ρ, and q are omitted when there is no confusion. With $n = n^{\text{SMS}} > 1$, $z^{\text{SMS}} = \big(1 - (1-\pi)^{n-1}(1 - \pi + (1-q)n\pi)\big)\rho$, whose first-order derivative with respect to π is $\frac{\partial z^{\text{SMS}}}{\partial \pi} = -n\rho(1-\pi)^{n-2}(\pi(1-n) + q(\pi n - 1))$, which is positive. To see why, note that the bracketed term, $\pi(1-n) + q(\pi n - 1)$ is linear in π and is negative for both $\pi = 0$ and $\pi = 1$ and so it is negative for all π. Thus, $z^{\text{SMS}}(\pi)$ is strictly monotone increasing on $\pi \in (0,1)$. Its second-order derivative with respect to π is $\frac{\partial^2 z^{\text{SMS}}}{\partial \pi^2} = (n-1)n\rho(1-\pi)^{n-3}(\pi - n\pi + (\pi n - 2)q + 1)$, which is positive if and only if $\pi < \frac{1-2q}{n-1-nq}$. Note that $\frac{1-2q}{n-1-nq} > 0$, because $\rho^{\text{SMS}} q^{\text{SMS}} n^{\text{SMS}} < \rho^{\text{BB}} q^{\text{BB}} n^{\text{BB}}$ implies $q = q^{\text{SMS}} < 1/n^{\text{SMS}} \leq 1/2$. Summarizing the above, $z^{\text{SMS}}(\cdot)$ is sigmoid-shaped for $\pi > 0$: it is monotone increasing, initially convex, but eventually concave.

Now note that in the lower end, $z^{\text{SMS}}|_{\pi \downarrow 0} = z^{\text{BB}}|_{\pi \downarrow 0} = 0$. Further, the slope of $z(\cdot)$ satisfies $\lim_{\pi \downarrow 0} \frac{\mathrm{d}z}{\mathrm{d}\pi} = n\rho q$. Therefore, the assumption $\rho^{\text{SMS}} q^{\text{SMS}} n^{\text{SMS}} < \rho^{\text{BB}} q^{\text{BB}} n^{\text{BB}}$ ensures that for π sufficiently small, $z^{\text{SMS}} < z^{\text{BB}}$. On the upper end of $\pi \uparrow 1$, $z^{\text{SMS}} \to \rho^{\text{SMS}} \geq \rho^{\text{BB}} \geq q^{\text{BB}} \rho^{\text{BB}}$, where the first inequality follows (18) and the second follows $q^{\text{BB}} \in [0,1]$. That is, z^{SMS} exceeds z^{BB} eventually. Therefore, there exists a unique $\pi^* \in (0,1)$ at which $z^{\text{SMS}}(\pi^*) = z^{\text{BB}}(\pi^*)$.

Next, it is clear that V^k_σ is monotone increasing in ζ^k_σ, where $k \in \{\text{BB}, \text{SMS}\}$ and $\sigma \in \{lo, hn\}$. Hence, comparing the value functions is equivalent to comparing the trading gain intensities $\{\zeta^k_\sigma\}$; i.e., the technology choice (19) is equivalent to (22). With the single-crossing property established above, it then follows that the comparison of the $\{\zeta^k_\sigma\}$ is equivalent to (21).

Finally, consider the case of $\rho^{\text{SMS}} q^{\text{SMS}} n^{\text{SMS}} \geq \rho^{\text{BB}} q^{\text{BB}} n^{\text{BB}}$. The only change is that the slope of $z^k(\pi)$ at the lower end now is higher for SMS than for BB. Thus, the only intersection possible is at $\pi = 0$, i.e., $z^{\text{SMS}} > z^{\text{BB}}$ for all $\pi \in (0,1)$, i.e., SMS is always preferred and, hence, $\theta_{hn} = \theta_{lo} = 1$. □

S1.8 Proof of Lemma S1.1

Proof. (1) $\frac{\partial g}{\partial m_{do}}$ has been evaluated in (S21) in the proof of Lemma S1. (2) Note that θ_{lo} affects $g(\cdot)$ only through ν_{lo}, which is given by (S20). Carefully simplifying, it can be found that $\frac{\partial \nu_{lo}}{\partial \theta_{lo}} =$

$\frac{(\nu_{lo}^{\text{SMS}}-\nu_{lo}^{\text{BB}})(\lambda_u+\nu_{lo}^{\text{SMS}})(\lambda_u+\nu_{lo}^{\text{BB}})}{(\lambda_u+(1-\theta_{lo})\nu_{lo}^{\text{SMS}}+\theta_{lo}\nu_{lo}^{\text{BB}})^2} > 0$ where the inequality holds because $\nu_{lo}^{\text{SMS}} > \nu_{lo}^{\text{BB}}$ always holds (with $\rho^{\text{SMS}} \geq \rho^{\text{BB}}$ and $n^{\text{SMS}} > n^{\text{BB}} = 1$). The partial derivative of $g(\cdot)$ with respect to ν_{lo} is $\frac{\partial g}{\partial \nu_{lo}} = -\frac{(\lambda_d+\lambda_u+\nu_{hn})\lambda_d\nu_{hn}}{(\lambda_u\nu_{hn}+(\lambda_d+\nu_{hn})\nu_{lo})^2} < 0$. Therefore, by chain rule, $\frac{\partial g}{\partial \theta_{lo}} < 0$. (3) can be proved similarly by showing that $\frac{\partial \nu_{hn}}{\partial \theta_{hn}} > 0$ and that $\frac{\partial g}{\partial \nu_{hn}} > 0$. The details are omitted for brevity. (4) is straightforward as $\frac{\partial g}{\partial s} = -1$. (5) By implicit function theorem, $g(\cdot) = 0$ implies that m_{do} strictly increases in s; see (1) and (4) above. The limit values as $s \downarrow 0$ or $s \uparrow 1$ can then be easily verified, regardless of θ_{lo} and θ_{hn}. (6) directly follows the implicit function theorem by (1)-(3). □

S1.9 Proof of Lemma S1.2

Proof. The key equation is (S18) in the proof of Lemma 1. Define the left-hand side as $f(m_{do}, \rho, n)$. Recall that Equation (S21) has shown that $\frac{\partial f}{\partial m_{do}} > 0$. In addition, simple calculus gives $\text{sign}\left[\frac{\partial f}{\partial \rho}\right] = \text{sign}[\nu_{lo} - \nu_{hn}]$. Since excess supply is assumed, i.e., $m_{hn} < m_{lo}$, Equation (S22) gives $\nu_{lo} < \nu_{hn}$. Hence, $\frac{\mathrm{d}m_{do}}{\mathrm{d}\rho} = -\frac{\partial f}{\partial \rho} / \frac{\partial f}{\partial m_{do}} > 0$, i.e., a higher ρ increases m_{do} and, because $m_{dn} = m_d - m_{do}$, decreases m_{dn}. It then also follows that a higher ρ increases $\nu_{hn} = 1 - (1 - \pi_{do})^n$ but decreases $\nu_{lo} = 1 - (1 - \pi_{dn})^n$.

Consider the effect of a larger n next. In that case it is more convenient to work with an equivalent version of (S18):

$$\eta(1 + m_{do} - s)\left(\frac{\lambda_d + \lambda_u}{\nu_{lo}} + \rho\right) - (1 - \eta)(s - m_{do})\left(\frac{\lambda_d + \lambda_u}{\nu_{hn}} + \rho\right) = 0. \tag{S23}$$

Define the left-hand side of (S23) as $g(m_{do}, \rho, n)$. Since $\frac{\partial g}{\partial m_{do}} > 0$ we have $\text{sign}\left[\frac{\mathrm{d}m_{do}}{\mathrm{d}n}\right] = -\text{sign}\left[\frac{\partial g}{\partial n}\right]$ and it remains to sign $\frac{\partial g}{\partial n}$. Taking π_{do} and π_{dn} as given, then $\text{sign}\left[\frac{\partial g}{\partial n}\right] = \text{sign}[hh(1 - \pi_{dn}) - hh(1 - \pi_{do})]$, where

$$hh(x) := (\rho(1 - x^n) + \lambda_d + \lambda_u)^{-1} \frac{\log x}{x^{-n} - 1}.$$

Further, one can show that $hh(x)$ is decreasing in x. Then, because $\pi_{do} > \pi_{dn}$, $hh(1 - \pi_{dn}) < hh(1 - \pi_{do})$. It follows that $\frac{\mathrm{d}m_{do}}{\mathrm{d}n} > 0$, i.e., a higher n increases m_{do} and, hence, decreases m_{dn}.

Since higher n increases m_{do} it follows immediately that higher n increases $\nu_{hn} = 1 - (1 - \pi_{do})^n$. To see the effect of n on ν_{lo} we first prove that trading volume is increasing in n. The trading volume can be written as $t = \rho m_{hn}\nu_{hn}$ (Equation 8). Equation (S3) gives another link between t and m_{hn}. Combining the two gives

$$t = \frac{(1 + m_{do} - s)\lambda_u\rho}{(\lambda_d + \lambda_u)\nu_{hn}^{-1} + \rho},$$

which is increasing in n both directly and through the dependence of m_{do} on n. Thus, trading volume increases in n. Now, writing $t = \rho m_{lo}\nu_{lo}$ and using (S2) we obtain $\nu_{lo} = \frac{\lambda_d+\lambda_u}{\frac{\lambda_d\rho(s-m_{do})}{t}-\rho}$ from which it follows that ν_{lo} increases in n. □

S1.10 Proof of Lemma S1.3

Proof. Consider (S23). Define the left-hand side of (S23) as $g(m_{do}, \psi)$. It is straightforward to show that $\text{sign}(\partial g/\partial\psi) = \text{sign}\Big[\frac{\partial}{\partial\psi}\log\Big(\frac{\lambda_d+\lambda_u}{\nu_{lo}} + \rho\Big) - \frac{\partial}{\partial\psi}\log\Big(\frac{\lambda_d+\lambda_u}{\nu_{hn}} + \rho\Big)\Big]$. Denoting $x := m_{do}/m_d$, and $\nu(\pi) := 1-(1-\pi)^n$ we can express $\nu_{hn} = \nu(\pi(x;\psi))$ and $\nu_{lo} = \nu(\pi(1-x;\psi))$. For $x > 1/2$ (which holds in the excess supply case) one can show that $\frac{\partial\pi(x,\psi)}{\partial\psi} < \frac{\partial\pi(1-x,\psi)}{\partial\psi}$. Additionaly, $\pi(x;\psi) > \pi(1-x;\psi)$. Then, an explicit calculation of $\frac{\partial}{\partial\psi}\log\Big(\frac{\lambda_d+\lambda_u}{\nu(\pi(x;\psi))} + \rho\Big)$ implies that $\frac{\partial}{\partial\psi}\log\Big(\frac{\lambda_d+\lambda_u}{\nu(\pi(x;\psi))} + \rho\Big) > \frac{\partial}{\partial\psi}\log\Big(\frac{\lambda_d+\lambda_u}{\nu(\pi(1-x;\psi))} + \rho\Big)$. It then follows that $\frac{\partial g}{\partial\psi} < 0$ and by implicit function theorem $\frac{\mathrm{d}g}{\mathrm{d}\psi} > 0$. □

S1.11 Proof of Lemma S1.4

Proof. Consider first the case $n^{\text{SMS}} > 2$. To establish that $\pi^* \leq \frac{1}{2}$, note that $z^{\text{SMS}}(\pi)$ is monotone increasing in n^{SMS} and in q^{SMS}. Therefore, fixing $z^{\text{BB}}(\pi) = q^{\text{BB}}\rho^{\text{BB}}\pi$, the intersection π^* must be higher as n^{SMS} and q^{SMS} reduce. Likewise, fixing $z^{\text{SMS}}(\pi)$, π^* must be higher when the product of $q^{\text{BB}}\rho^{\text{BB}}$ increases. Since $q^{\text{BB}} \in [0,1]$ and $\rho^{\text{BB}} \leq \rho^{\text{SMS}}$, the maximum of this product is $q^{\text{BB}}\rho^{\text{BB}} \leq \rho^{\text{SMS}}$. Therefore, the maximum π^* is the solution to $z^{\text{SMS}}(\pi; n^{\text{SMS}} = 3, q^{\text{SMS}} = 0) - \rho^{\text{SMS}}\pi = 0$. Solving this equation gives the unique interior solution of $\pi^* = \frac{1}{2}$. For the case $n^{\text{SMS}} = 2$, we plug $n^{\text{SMS}} = 2$ into $z^{\text{SMS}}(\pi)$ and solve for π^* in (a linear equation) $z^{\text{BB}}(\pi) = q^{\text{BB}}\rho^{\text{BB}}\pi$. Doing so yields $\pi^* = \frac{2q^{\text{SMS}}\rho^{\text{SMS}} - q^{BB}\rho^{BB}}{(2q^{\text{SMS}}-1)\rho^{\text{SMS}}}$. □

S2 Comparison with directed search

This appendix analyzes a model of OTC trading with "directed search" (DS), where dealers continuously post quotes and, after observing them, each customer directs her search to one chosen dealer. This is a realistic feature of corporate bonds trading, as dealers sometimes broadcast their indicative bids and asks to customers on electronic platforms (Section III.B of Bessembinder, Spatt, and Venkataraman, 2020). The similarity of this paper and the DS literature is that both allow endogenous trading gain shares accruing to customers and dealers. See, e.g., Guerrieri, Shimer, and Wright (2010), Lester, Rocheteau, and Weill (2015), Chang (2018) and, for a review, Wright et al. (2020). The purpose is to compare the findings of DS model in this appendix with those of SMS from Section 2. The key result is that the steady state equilibrium under DS can be proxied by the SMS equilibrium either (i) when the dealers inventory transparency ψ is sufficiently higher; or (ii) when the search capacity n is sufficiently large.

S2.1 A model of directed search

The model setup follows Section 1, except for the parts of "search" and "price determination," which are modified as follows: All dealer owners (type *do*) constantly post ask quotes, while all dealer non-owners (type *dn*) post bid quotes. Customers observe these quotes and direct their searches to chosen dealers at independent Poisson processes with the same intensity ρ. Customers and their chosen dealers meet pairwise and once met, the pair exchanges the asset at the dealer quoted price.

Denote by ν_{hn} the probability for a *hn-do* match and by ν_{lo} for a *lo-dn* match. Assume that

$$\nu_{hn} = 1 \text{ if } m_{do} > 0 \text{ and } \nu_{lo} = 1 \text{ if } m_{dn} > 0. \tag{S24}$$

This is because at any instant, there is only an infinitesimal amount of customers searching ($\rho m_{hn} \mathrm{d}t$ buyers and $\rho m_{lo} \mathrm{d}t$ sellers), which is negligible compared to the vast mass of quoting dealers (m_{do} and m_{dn}, if positive). Effectively, the assumption (S24) lets the dt-measure customers find dealers with certainty as long as the measures of counterparties are strictly positive. In the case when some dealer masses become zero, i.e., $m_{do} \to 0$ or $m_{dn} \to 0$, the matching probabilities will be solved endogenously.

We shall look for a steady-state, symmetric dealer pricing equilibrium, characterized by the following time-invariant variables: (i) the demographics $\{m_{ho}, m_{hn}, m_{lo}, m_{ln}, m_{do}, m_{dn}\}$; (ii) the matching probabilities $\{\nu_{hn}, \nu_{lo}\}$ in case the corresponding dealer mass is zero; and (iii) the symmetric ask and bid, p_a and p_b, by the *do*- and *dn*-dealers, respectively. Note that since the quotes are symmetric, the customers randomly direct their searches to all counterparty dealers.

S2.1.1 Demographics and matching probabilities

The six demographic variables $\{m_{ho}, m_{hn}, m_{lo}, m_{ln}, m_{do}, m_{dn}\}$ and the two matching probabilities $\{\nu_{hn}, \nu_{lo}\}$ can be pinned down by the six equations (2)-(7), which hold as before, plus the two conditions given in (S24):

Proposition S1 (Demographics under DS). In a steady-state, the matching probabilities satisfy

$$\nu_{hn} = \min\left[1, \frac{m_{lo}}{m_{hn}}\right] \text{ and } \nu_{lo} = \min\left[1, \frac{m_{hn}}{m_{lo}}\right], \tag{S25}$$

where the ratios $\frac{m_{lo}}{m_{hn}}$ and $\frac{m_{hn}}{m_{lo}}$ depend on the asset supply s:

$$\text{sign}\left[\frac{m_{lo}}{m_{hn}} - 1\right] = -\mathbb{1}_{\{0<s<\eta\}} + \mathbb{1}_{\{\eta+m_d<s<1+m_d\}}. \tag{S26}$$

Given the $\{\nu_{hn}, \nu_{lo}\}$ above, the steady-state demographics exist and are the unique solution to the linear equation system (2)-(7).

The key insight from (S25) is that the matching probabilities $\{\nu_{hn}, \nu_{lo}\}$ only depend on the relative sizes of *hn*-buyers and *lo*-sellers in the market. Perhaps surprisingly, the dealer sizes m_{do} and m_{dn} do not show up, as if the customers are directly searching for counterparties, skipping the dealers. To see why, recall that (2)-(7) also imply the dealer stationarity condition (8): $\rho m_{hn}\nu_{hn} = \rho m_{lo}\nu_{lo}$, which balances the asset inflow to and the outflow from the dealers (otherwise the dealer sizes will change overtime). Note that it can be equivalently interpreted as the *hn*-buyers are directly matched with the *lo*-sellers. That is, the dealers are merely passing the asset from the sellers to the buyers, not affecting the matching probabilities $\{\nu_{hn}, \nu_{lo}\}$.

One might wonder why ν_{hn} and ν_{lo} can be less than one: Since there is always only a $\mathrm{d}t$ amount—zero measure—of customers searching for dealers, by assumption (S24), should not the matching probabilities always be one? It turns out that $\nu_{hn} < 1$ and $\nu_{lo} < 1$ precisely when the measure of the corresponding dealers is also zero. Consider, for example, the case of $m_{do} = 0$, which implies that $m_{dn} = m_d - m_{do} > 0$. Then, by assumption (S24), $\nu_{lo} = 1$ and there is always an amount of $\rho m_{lo} \nu_{lo} \mathrm{d}t$ trades occurring between *dn*-buyers and *lo*-sellers. These trades then create a "transient" fringe of *do*-sellers (of the same magnitude of $\mathrm{d}t$) who then quote asks to the searching *hn*-buyers. The dealer stationarity condition (8) above then requires:

$$\rho m_{hn} \nu_{hn} = \rho m_{lo} \nu_{lo} \implies \nu_{hn} = \frac{m_{lo} \nu_{lo}}{m_{hn}} = \frac{m_{lo}}{m_{hn}}.$$

Note that it must be $\nu_{hn} < 1$ in this case (as verified in the proof of Proposition S1); that is, *all* such transient *do*-sellers are sought after by *hn*-buyers. Otherwise, some of the *do*-sellers will accumulate overtime, making the dealer masses nonstationary.

S2.1.2 Value functions and prices

The value functions can be found using the same set of HJB equations (10)-(15) as in Section 2.2. Hence, Proposition 1 continues to hold for directed search. In particular, the condition for positive trading gains remains the same. The only differences are in the trading gain intensities, $\{\zeta_\sigma\}$.

We first consider the interior equilibrium where $m_{do} \in (0, m_d)$. In this case, there is perfect competition among dealers and so $\zeta_{dn} = \zeta_{do} = 0$. To see why, consider, for example, the *do*-sellers' quote p_a to the *hn*-buyers. Since the amount of searching *hn*-buyers is vanishingly small ($\rho m_{hn} \mathrm{d}t$), the standard Bertrand price competition applies to the *do*-sellers, giving $p_a = R_d$ as the only symmetric price in equilibrium. Similar argument implies that in this case the *dn*-buyers quote $p_b = R_d$. Summing up, the dealers quotes are given by $p_a = p_b = R_d$, and the trading gain intensities are given by $\zeta_{lo} = \rho \nu_{lo}$, $\zeta_{hn} = \rho \nu_{hn}$, and $\zeta_{do} = \zeta_{dn} = 0$.

Next, in the corner equilibrium of $m_{do} = 0$, any *do*-dealer exists only transiently: Whenever a *dn*-dealer has bought the asset (from an *lo*-seller), he immediately trades again with an *hn*-buyer, becoming *dn*-dealer again. Therefore, $\Delta_{hd} = 0$ in this case. This follows from Equation (14) if one divides it by ζ_{do} and sets $\zeta_{do} = \infty$ (because a *do*-dealer trades immediately without waiting). Then $p_a = R_h = R_d$. The Bertrand competition argument for $m_d > 0$ amount of *dn*-buyers still implies that $p_b = R_d$. Likewise, in the corner equilibrium of $m_{dn} = 0$, we have $\Delta_{dl} = 0$ and $p_a = p_b = R_d$.

S2.2 Comparing DS with SMS

In DS, a customer can first observe all dealers' quotes and then direct her search to a chosen dealer. In SMS, a customer can reach only n dealers (where n is set by the platform like RFQ), not knowing the types of the dealers, only with noisy signals of quality ψ. This appendix concludes with the following convergence result:

Proposition S2 (Convergence of SMS to DS). The equilibrium demographics, value functions, and prices in the SMS model converge to those in DS either (i) when $\psi \to 1$ under the "random matching with signal" specification with $n \geq 2$; or (ii) when $n \to \infty$ under the general specification.

Intuitively, one should expect as the search capacity $n \to \infty$, the customer can almost surely find at least one counterparty dealer. Likewise, if the signal quality (dealer inventory transparency) $\psi \to 1$, the customer can always direct her quote to the correct dealers. Therefore, in both limits of SMS, the customers' searches converge to those in DS.

S3 Price dispersion with homogenous dealers

This appendix studies the dealers pricing strategies under SMS. Despite the homogeneity of dealers, in equilibrium, there still is price dispersion in their quotes. Consider a customer just contacted n dealers via SMS. First, there is probability q that she can make a TIOLIO to the dealers. In this case, it is optimal for her to set the price of the TIOLIO at the dealers' reservation value, i.e., $p = R_d$.

Second, there is probability $1 - q$ that the n dealers independently quote to the customer. For concreteness, suppose the customer is an *hn*-buyer. In this case, a quoting dealer must be a *do*-seller and he would like to capture the full surplus by setting $p \uparrow R_h$. However, he faces potential competition from the other $(n-1)$ dealers, as their asking quotes might be lower than his. Yet not all of the other $(n-1)$ dealers are necessarily also *do*-sellers. The quoting *do*-seller therefore engages in a price competition with *unknown number of competitors.*

Such price competition differs from the standard Bertrand price competition, in which every *do*-seller quotes his reservation price of R_d and the *hn*-buyer gets the full surplus Δ_{hd}. Instead, every *do*-seller has an incentive to charge a higher price, $R_d + \alpha\Delta_{hd}$ for some $\alpha \in [0, 1]$. (When $\alpha = 1$, $R_d + \alpha\Delta_{hd} = R_h$, which is the *hn*-buyer's reservation value.) This is because he might actually be the only *do*-seller among the n contacted dealers, in which case his quote is the only price available to the searching *hn*-buyer. As long as $\alpha \leq 1$, the buyer will accept it[4] and the dealer can pocket the difference $\alpha\Delta_{hd}$ as his profit. In a Nash equilibrium, however, the fraction α cannot be deterministic, as the undercutting argument of Bertrand competition will lead to $\alpha \downarrow 0$. Yet, it

[4] To see this, note that by accepting an offer $p = R_d + \alpha\Delta_{hd}$, the customer-buyer becomes *ho*-bystander and gets a continuation value of $V_{ho} - p$. If instead he rejects the offer, his value remains as V_{hn}. This customer-buyer will accept the offer as long as $V_{ho} - p \geq V_{hn}$, a condition equivalent to $\alpha \leq 1$.

would be strictly better off to quote some $\alpha > 0$ if all the potential competitors were to quote $\alpha \downarrow 0$. The heuristic discussion above is formalized in the proof of the following proposition.

Proposition S3 (Dealers' equilibrium quoting). Suppose a customer contacts n (≥ 1) dealer(s). With probability $1-q$, each dealer independently makes a TIOLIO to the customer. Within symmetric strategies, there is a unique mixed-strategy equilibrium for the dealers. Define $F(x;\pi,n) := \frac{1}{\pi} - \big(\frac{1}{\pi} - 1\big)x^{-\frac{1}{n-1}}$ for $(1-\pi)^{n-1} \leq x \leq 1$, $\pi \in (0,1)$, and $n \in \mathbb{N}$. Then,

- a *do*-seller asks $R_l + \alpha\Delta_{hd}$, where α is random with c.d.f. $F(\alpha;\pi_{do},n)$; and
- a *dn*-buyer bids $R_h - \beta\Delta_{dl}$, where β is random with c.d.f. $F(\beta;\pi_{dn},n)$.

Note that when $n = 1$, the c.d.f. $F(\cdot)$ becomes degenerate with $F(x) = \mathbb{1}_{\{x\geq 1\}}$.

The proposition above implies that a quoting *do*-seller expects a trading price of $R_l + \bar{\alpha}\Delta_{hd}$ and a quoting *dn*-buyer expects $R_h - \bar{\beta}\Delta_{dl}$, where

$$\bar{\alpha} := \mathbb{E}[\alpha] = (1-\pi_{do})^{n-1} \quad \text{and} \quad \bar{\beta} := \mathbb{E}[\beta] = (1-\pi_{dn})^{n-1}. \tag{S27}$$

To see this, consider a quoting *do*-seller and note that under the mixed-strategy equilibrium, he must be indifferent across all possible $\alpha \in [0,1]$. In particular, the only situation for quoting $\alpha = 1$ to "win" is that there are no other competing *do*-sellers; that is, with probability $(1-\pi_{do})^{n-1}$. Therefore, when contacted, a quoting *do*-seller expects a profit of $\bar{\alpha}\Delta$, where $\bar{\alpha}$ can be interpreted as his expected trading gain share. Likewise, a quoting *dn*-buyer expects $\bar{\beta}\Delta$.

Proposition S3 characterizes a contacted dealer's quoting strategy. From a searching customer's perspective, however, the expected trading price has a different distribution, because she can pick the best quote and because there might be no quote if none of the contacted dealers is of the matching type. Consider an *hn*-buyer for example. He contacts n dealers knowing that the number of counterparties he will actually find, N_{do}, is random and follows a binomial distribution of n draws and success rate π_{do}. Each of these N_{do} dealers then quotes a random price according to $F(\alpha;\pi_{do},n)$, following Proposition S3. (The *hn*-buyer can safely ignore the other $n - N_{do}$ dealers' quotes, as they also want to buy.) The *hn*-buyer then picks the lowest ask among the N_{do} available quotes. Conditional on that $N_{do} \geq 1$, the c.d.f. of this minimum ask is $1 - (1 - F(\alpha;\cdot))^{N_{do}-1}$. (When $N_{do} = 0$, the *hn*-buyer finds no ask quote and there is no trade.) Averaging across all possible $N_{do} \in \{1,...,n\}$, the corollary below gives the expectation of this minimum ask quote.

Proposition S4 (Trading prices). Define $G(x;\pi,n) := \frac{1-(1-\pi)^n x^{-\frac{n}{n-1}}}{1-(1-\pi)^n}$ with support $(1-\pi)^{n-1} \leq x \leq 1$, for some $\pi \in (0,1)$ and $n \in \mathbb{N}$. Then

- a searching *hn*-buyer expects to trade with probability $(1-(1-\pi_{do})^n)$ at price $A := QR_d + (1-Q)(R_d + a\Delta_{hd})$, and
- a searching *lo*-seller expects to trade with probability $(1-(1-\pi_{dn})^n)$ at price $B := QR_d + (1-Q)(R_d - b\Delta_{dl})$,

where Q is a Bernoulli draw with success rate q, and a and b are random variables with respective c.d.f. $G(x; \pi_{do}, n)$ and $G(x; \pi_{dn}, n)$. Note that when $n = 1$, $a = b = 1$ almost surely.

Proposition S4 implies that trades have price dispersion in equilibrium. Formally, the price dispersion of each customer type's trades can be evaluated as the variances of their trading prices, i.e., $\text{var}[A]$ for customer-buying trades and $\text{var}[B]$ for customer-selling. Such dispersions arise due to the unknown number of competitors, an intrinsic feature of SMS: The contacted dealers' types are unknown to each other. In the current stylized model, such types boil down to the dealers' inventory holdings (do vs. dn). In real-world trading, agents' other characteristics (like risk-aversion, patience, wealth, and relationship with customers, etc.) can enrich their possible types. As long as such a friction remains, price dispersion will be a robust feature in equilibrium. The models by Duffie, Dworczak, and Zhu (2017) and Lester et al. (2018) also feature similar price setting mechanisms. The key novelty here is that such price dispersion is endogenously parametrized by the equilibrium demographics of counterparties, through $\pi_{do} = \pi\left(\frac{m_{do}}{m_d}\right)$ and $\pi_{dn} = \pi\left(\frac{m_{do}}{m_d}\right)$.

The flexible specification of the matching rate $\pi(\cdot; \psi)$, as in Equation (1), allows to study how dealers' inventory transparency $\psi \in (\frac{1}{2}, 1]$ affects the equilibrium prices. Under (1), the matching rate π monotonically increases in ψ. Intuitively, knowing dealers' inventories better, customers can find matching counterparties more easily. This leads to the following comparative statics:

Proposition S5 (Inventory transparency and price dispersion). In SMS, when dealers' inventory transparency ψ is sufficiently high, the price dispersions $\text{var}[A]$ and $\text{var}[B]$ monotonically decreases with the dealers' inventory transparency ψ.

Intuitively, when the transparency is higher, customers can direct their searches to matching dealers more precisely. Knowing this, the contacted n dealers will quote the prices more competitively, i.e., closer to their reservation values R_d. This competition effect of SMS ($n \geq 2$), therefore, reduces the magnitude of price dispersion.

This result contrasts with the finding from Cujean and Praz (2015), who show that the opposite happens in a setting where higher inventory transparency exposes agents to wider, more extreme "predatory quotes," thus adding to the price dispersion. Since agents only meet in pairs in their framework, the quoting side is effectively a monopolist (subject to risk aversion and information asymmetry) and there is no price competition from others quoters. In other words, transparency does not prompt price competition in their framework. Instead, in the SMS setup of this paper, a searching customer reaches out to multiple dealers and transparency, therefore, has the direct effect on price competition.

S4 Endogenous search intensity

This appendix studies customers' costly searching by allowing them to choose the search intensity ρ endogenously, subject to certain cost function. Specifically, instead of an exogenous common ρ, a customer of type-σ can choose to search with intensity ρ_σ by incurring a quadratic flow cost of $\frac{1}{2\kappa}\rho_\sigma^2$

per unit of time. The exogenous parameter κ (> 0) represents how advanced the technology is. (The higher κ is, the less costly is searching.) The objective is to study when κ increases, whether such a more advanced technology creates a similar "dealer bottleneck" to the one created by the search capacity n (Section 2.3). For this objective, the analyses below only concern the case of SMS only (not how customers might choose between SMS and BB).

S4.1 Demographics and value functions

In this subsection, we assume that there is a symmetric-strategy steady state equilibrium, where all customers of the same type $\sigma \in \{hn, lo\}$ choose the same search intensity ρ_σ. (The next subsection pins them down endogenously through customers' optimization.) As in Section 2.1, the six endogenous demographic parameters, $\{m_{hn}, m_{ho}, m_{ln}, m_{lo}, m_{do}, m_{dn}\}$, are pinned down exactly by the following six conditions:

$$\begin{aligned}
&\text{market clearing:} && m_{ho} + m_{lo} + m_{do} = s \\
&\text{total customer mass:} && m_{ho} + m_{ln} + m_{hn} + m_{lo} = 1 \\
&\text{total dealer mass:} && m_{do} + m_{dn} = m_d \\
&\text{high/low type stability:} && (m_{lo} + m_{ln})\lambda_u \mathrm{d}t = (m_{ho} + m_{hn})\lambda_d \mathrm{d}t \\
&\text{net flow of } lo\text{-sellers:} && -\rho_{lo} m_{lo} \nu_{lo} - \lambda_u m_{lo} + \lambda_d m_{ho} = 0 \\
&\text{net flow of } hn\text{-buyers:} && -\rho_{hn} m_{hn} \nu_{hn} - \lambda_d m_{hn} + \lambda_u m_{ln} = 0
\end{aligned}$$

which correspond to Equations (2)-(7). The only difference is that the search intensities ρs in the last two equations are now type-specific.

Likewise, the six value functions, $\{V_{hn}, V_{ho}, V_{ln}, V_{lo}, V_{do}, V_{dn}\}$, follow the HJB equation system below:

$$\begin{aligned}
0 &= y_h + \lambda_d \cdot (V_{lo} - V_{ho}) - (r + f_c)V_{ho} \\
0 &= \lambda_u \cdot (V_{hn} - V_{ln}) - (r + f_c)V_{ln} \\
0 &= y_l + \lambda_u \cdot (V_{ho} - V_{lo}) - (r + f_c)V_{lo} + \zeta_{lo}\Delta_{dl} - \frac{\rho_{lo}^2}{2\kappa} \\
0 &= \lambda_d \cdot (V_{ln} - V_{hn}) - (r + f_c)V_{hn} + \zeta_{hn}\Delta_{hd} - \frac{\rho_{hn}^2}{2\kappa} \\
0 &= y_d - (r + f_d)V_{do} + \zeta_{do}\Delta_{hd} \\
0 &= -(r + f_d)V_{dn} + \zeta_{dn}\Delta_{dl}
\end{aligned}$$

which correspond to Equations (10)-(15), except that the two trading types of customers, $\sigma \in \{lo, hn\}$, now also incur quadratic flow costs, $\frac{\rho_\sigma^2}{2\kappa}$, as shown in the third and the fourth equations above. The trading gains still have the same expressions as before:

$$\Delta_{dl} = R_d - R_l = (V_{do} - V_{dn}) - (V_{lo} - V_{ln}) \text{ and } \Delta_{hd} = R_h - R_d = (V_{ho} - V_{hn}) - (V_{do} - V_{dn});$$

and so do the trading gain intensities, except that they are also scaled by the type-specific ρs:

$$\zeta_{lo} = \rho_{lo}\nu_{lo}\gamma_{lo},\ \zeta_{dn} = \frac{m_{lo}}{m_{dn}}\rho_{lo}\nu_{lo}(1-\gamma_{lo});\ \text{and}\ \zeta_{hn} = \rho_{hn}\nu_{hn}\gamma_{hn},\ \zeta_{do} = \frac{m_{hn}}{m_{do}}\rho_{hn}\nu_{hn}(1-\gamma_{hn}).$$

S4.2 Customers' optimal choices of search intensity

Consider an lo-seller for example. She chooses *her own* ρ_{lo} to maximizes *her own* value function V_{lo}. Using the first and the third equations from the HJB system above, one obtains a quadratic equation of her ρ_{lo} and her V_{lo}:

$$0 = y_l + \frac{\lambda_u}{\lambda_d + r}y_h - \left(1 + \frac{\lambda_u}{\lambda_d + r}\right)rV_{lo} + \zeta_{lo}\Delta_{dl} - \frac{\rho_{lo}^2}{2\kappa},$$

noting that ζ_{lo} is proportional to ρ_{lo} and Δ_{dl} is linear in V_{lo}. By implicit function theorem, therefore, the lo-seller's first-order condition can be found as

$$\frac{\mathrm{d}V_{lo}}{\mathrm{d}\rho_{lo}} = \frac{r + \lambda_d}{(r + \lambda_d + \lambda_u)r + (r + \lambda_d)\zeta_{lo}}\left(\nu_{lo}\gamma_{lo}\Delta_{dl} - \frac{\rho_{lo}}{\kappa}\right) = 0 \implies \rho_{lo} = \kappa\nu_{lo}\gamma_{lo}\Delta_{dl}. \tag{S28}$$

A few comments are in order for the above. First, due to the continuum of agents, the lo-seller's choice of ρ_{lo} has no impact on the aggregate demographics or the valuation of anyone else. That is why the only endogenous variables, from this lo-seller's point, are her own V_{lo} and her own ρ_{lo} in the above equation. Second, (S28) is a *nonlinear* equation that pins down the solution to the optimal ρ_{lo}, because $\Delta_{dl} = R_d - (V_{lo} - V_{ln})$ is still endogenous of V_{lo} and, hence, also ρ_{lo}. Third, the second-order condition can easily be shown to hold, by envelope theorem, as $\frac{\mathrm{d}^2V_{lo}}{\mathrm{d}\rho_{lo}^2} < 0$, thus ensuring that the solution to (S28) indeed maximizes V_{lo}. Similarly, an hn-buyer's first order condition to solve for her search intensity ρ_{hn} is pinned down by

$$\rho_{hn} = \kappa\nu_{hn}\gamma_{hn}\Delta_{hd}, \tag{S29}$$

which again is a nonlinear equation of ρ_{hn}, because $\Delta_{hd} = (V_{ho} - V_{hn}) - R_d$ is endogenous of V_{hn} and hence also ρ_{hn}.

Due to the nonlinearity, the first-order conditions (S28) and (S28) are solved numerically and the solution seems to always exist and is stable. Taken together, therefore, the six-equation demographic system, the six-equation value function system, and the two first-order conditions jointly determine the 14 variables of the steady state equilibrium: the six demographic variables, the six value functions, and the two search intensities.

S4.3 Does a better search technology create "bottleneck"?

In the baseline model studied in Section 2.3, the dealer bottle neck arises only when the search capacity n increases but not with the search intensity ρ. This is because the common ρ in the baseline symmetrically increases both the asset inflow to and the outflow from the dealers, unlike the asymmetric effects from n.[5] It is natural to ask, given the endogenous choices of ρ_{hn} and ρ_{lo} in this extension (and that they are unlikely to be exactly the same), whether the search technology κ can create similar bottlenecks.

Given the nonlinear first-order conditions (S28) and (S29), the effects are first examined numerically. It is found that the impact of a higher technology κ on the trading customers' sizes, m_{hn} and m_{lo}, turn out to be very robust and consistent across various parametrizations attempted: they always monotonically reduce with κ. In other words, no bottleneck is found and the asset allocation is unanimously improved towards the Walrasian allocation. Figure S1(a) illustrates the effects under a specific set of parameter values.

To understand why the search technology κ does not create the bottleneck, consider the customers' optimal choices of ρs. Apply a small shock of $\mathrm{d}\kappa$ to their first-order conditions (S28) and (S29) to get

$$\frac{\mathrm{d}\rho_{lo}}{\rho_{lo}} = \frac{\mathrm{d}\kappa}{\kappa} + \frac{\mathrm{d}(\nu_{lo}\gamma_{lo}\Delta_{dl})}{\nu_{lo}\gamma_{lo}\Delta_{dl}} \quad \text{and} \quad \frac{\mathrm{d}\rho_{hn}}{\rho_{hn}} = \frac{\mathrm{d}\kappa}{\kappa} + \frac{\mathrm{d}(\nu_{hn}\gamma_{hn}\Delta_{hd})}{\nu_{hn}\gamma_{hn}\Delta_{hd}}.$$

That is, the effects of κ on ρs can be decomposed into a direct scaling effect, $\mathrm{d}\kappa/\kappa$, and an indirect effect (the other term). The direct scaling effect is exactly the same for the *hn*-buyers and the *lo*-sellers and, therefore, does not create the asymmetry needed for the bottleneck. Only when the indirect effect is significant enough might the increase in κ create asymmetry on the two sides of the market, resulting in the bottleneck. However, from the various numerical parametrization attempted, this indirect effect seems to be always small. Indeed, from Figure S1(b), it can be seen that both $\log \rho_{lo}$ and $\log \rho_{hn}$ are monotonically increasing with $\log \kappa$ along the 45-degree line, suggesting the elasticity being very close to unity. The negligibly small curvature concurs with the dominance of the direct scaling effect over the residual indirect effects. In other words, when the technology κ improves, it is as if directly scaling both ρs.

As such, it is unsurprising to see that κ creates no bottleneck: The stationarity condition requires that the asset inflow into and out of dealers equate each other, i.e.,

$$\rho_{lo} m_{lo} \nu_{lo} = \rho_{hn} m_{hn} \nu_{hn}, \tag{S30}$$

which follows the last three equations of the demographics system. As κ increases, as illustrated above, the dominant effect is that it directly scales both ρ_{hn} and ρ_{lo}, by the same proportion: $\frac{\mathrm{d}\rho_{lo}}{\rho_{lo}} \approx \frac{\mathrm{d}\kappa}{\kappa} \approx \frac{\mathrm{d}\rho_{hn}}{\rho_{hn}}$. Therefore, just like in the baseline (where $\rho_{hn} = \rho_{lo} = \rho$), the same scaling effect symmetrically affects both the inflow and the outflow, offsetting each other.

[5] As a recap, the intuition for the dealer bottleneck is as follows: When the search capacity n increases, it improves matching by *asymmetrically* raising both matching rates ν_{lo} and ν_{hn}. (The short side's ν increases much less than the long side's.) In the steady state, the asset inflow to and outflow from the dealers must equalize, i.e., $\rho m_{lo}\nu_{lo} = \rho m_{hn}\nu_{hn}$. Since the νs increase asymmetrically with n, to sustain the equality between the inflow and the outflow, the customer sizes m_{lo} and m_{hn} must change asymmetrically as well. In particular, the short-side customer size must increase, while the long-side decrease. Unlike n, ρ symmetrically scale both the inflow and the outflow and therefore does not create the bottleneck.

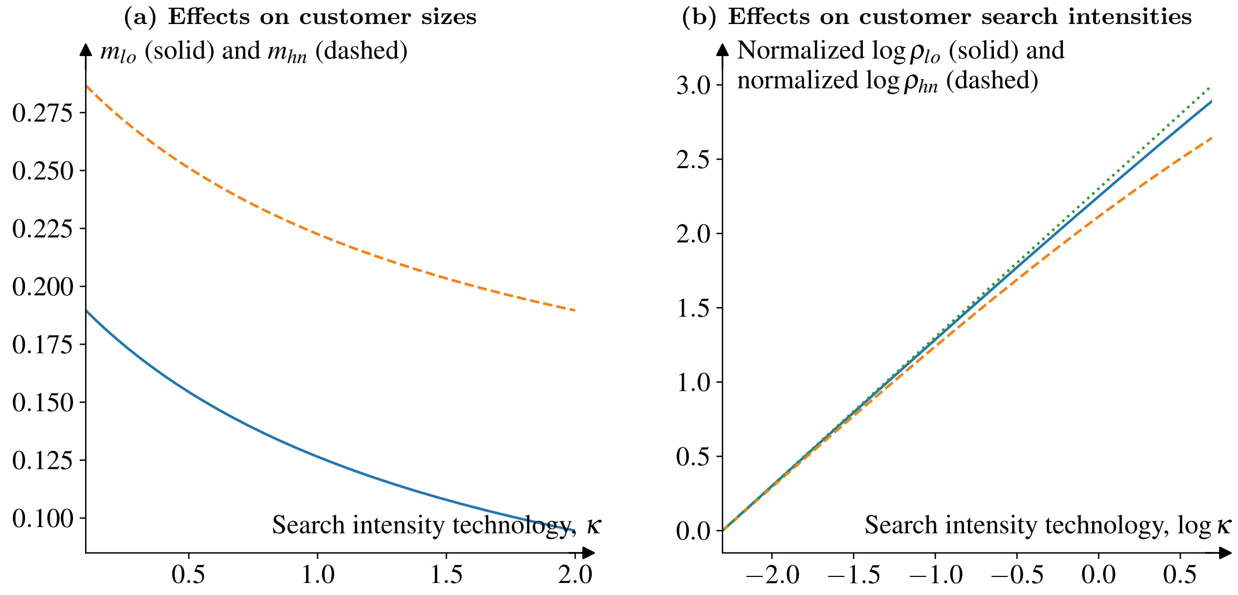


Figure S1: The effects of improving the search intensity technology κ. This figure illustrates how the search intensity technology κ affects the customer sizes in Panel (a) and their choices of the search intensity ρs in (b). The solid (blue) lines and the dashed (orange) lines indicate for lo and hn customers, respectively. In particular, Panel (b) plots the log of the ρs against the log of κ, by normalizing as such that both $\log \rho_{lo} = \log \rho_{hn} = 0$ at the minimum κ selected here. The dotted line in Panel (b) is the 45-degree line. Apart from κ, the primitive parameters are set at $n = 3$, $\lambda_u = \lambda_d = 1.0$, $m_d = 0.1$, and $s = 0.45$. (The other parameters do not affect the objectives plotted here.)

To conclude, the above analysis suggests that indeed the bottleneck effect is a rather unique feature of the search capacity n (as it enters asymmetrically on the two sides of the market through the equilibrium demographics). The bottleneck does not arise with the search intensity ρ (or κ) because of the way it proportionally enters the inflow and outflow equality condition (S30). It is worth noting that our analysis is largely numerical, due to the nonlinear first-order conditions. We therefore do not fully rule out the possibility that bottleneck might arise with ρs under certain parametrizations, where the indirect effects of κ in ρs might dominate, thus overturning the symmetry and creating the bottleneck. However, we have not yet found such examples. (We have also experimented some other convex functional forms of $c(\rho)$ for the flow cost.)

S5 Transition dynamics

This appendix characterizes the non-stationary equilibria of the model. Doing so allows us to examine the transition of the endogenous model elements, like the demographics and the welfare, in between shocks and the eventual steady states. In particular, Section S5.3 shows that when the discount rate is low, comparing welfare only in steady states (as we do in Section 2.3.3 and 3.3) is equivalent to comparing also welfare during transition dynamics.

S5.1 Demographics

First, consider the dynamics of high-type customers, $\eta \equiv m_{ho} + m_{hn}$. The change of η over time, $\dot{\eta}$ is given by inflows $(1-\eta)\lambda_u$ minus the outflows $\eta\lambda_d$. Thus, we can write the flow as an ordinary differential equation (ODE):

$$\dot{\eta} = (1-\eta(t))\lambda_u - \eta(t)\lambda_d.$$

Solving it with initial condition $\eta(0) = \eta_0$ yields the solution

$$\eta(t) = \eta^* + (\eta_0 - \eta^*)\exp(-(\lambda_u + \lambda_d)t), \tag{S31}$$

where $\eta^* = \frac{\lambda_u}{\lambda_u + \lambda_d}$.

Second, one can express all customer masses in terms of m_{do}, η and m_{lo}. From the non-time-varying conditions $m_{ho} + m_{hn} = \eta$, $m_{hn} + m_{ln} = 1 - \eta$, and $m_{ho} + m_{lo} = s - m_{do}$, we obtain

$$m_{ho}(t) = s - m_{do}(t) - m_{lo}(t), \tag{S32}$$

$$m_{hn}(t) = m_{lo}(t) + \eta(t) - s + m_{do}(t), \tag{S33}$$

$$m_{ln}(t) = 1 - m_{lo}(t) - \eta(t). \tag{S34}$$

It remains to characterise the dynamics of m_{lo} and m_{do}. The equations are similar to the stationary case, but we equalise the difference between inflows and outflows to time derivative of these masses (which are zero in steady states):

$$\dot{m_{lo}} = m_{ho}(t)\lambda_d - m_{lo}(t)\lambda_u - \rho m_{lo}(t)\nu_{lo}(t), \tag{S35}$$

$$\dot{m_{do}} = \rho m_{lo}(t)\nu_{lo}(t) - \rho m_{hn}(t)\nu_{hn}(t). \tag{S36}$$

The probabilities $\nu_{lo}(t)$ and $\nu_{hn}(t)$ can be expressed through $m_{do}(t)$ exactly as in the stationary case. We now summarise the above discussion.

Lemma S2. Consider the system of ODEs (S35) and (S36), given the initial conditions $m_{lo}(0) = m_{lo,0}$ and $m_{do}(0) = m_{do,0}$. The solution to this system, combined with (S31)−(S34) fully characterises the demographics in a nonstationary equilibrium, where gains from trade between customers and dealers exist at all times.

The proof follows the preceding analysis.

Solving the demographics reduces to solving the system of two non-linear first-order ODEs (S35) and (S36), which can be done numerically, using standard methods. We verify that the gains from trade are positive numerically, as we explain in the next subsection.

S5.2 Value functions

Our derivations are very similar to the stationary case. Consider an *ho*-bystander. His value function at time t consists of utility flows over a short time interval dt plus the discounted future value function, given that there is no exit shock

$$V_{ho}(t) = y_h dt + \lambda_d (V_{lo}(t) - V_{ho}(t)) + \underbrace{e^{-(r+f_c)dt}}_{1-(r+f)dt} V_{ho}(t+dt)$$

Taking the limit as $dt \to 0$ we obtain the HJB

$$0 = \dot{V}_{ho} + y_h + \lambda_d \cdot (V_{lo}(t) - V_{ho}(t)) - (r + f_c)V_{ho}(t). \tag{S37}$$

The HJB is similar to the stationary case. The only difference is that now we have to add $\dot{V}_{ho}$ at the right-hand side.

Proceeding similarly we obtain the remaining HJBs

$$0 = \dot{V}_{ln} + \lambda_u \cdot (V_{hn}(t) - V_{ln}(t)) - (r + f_c)V_{ln}(t), \tag{S38}$$

$$0 = \dot{V}_{lo} + y_l + \lambda_u \cdot (V_{ho}(t) - V_{lo}(t)) - rV_{lo}(t) + \zeta_{lo}(t)\Delta_{dl}(t), \tag{S39}$$

$$0 = \dot{V}_{hn} + \lambda_d \cdot (V_{ln}(t) - V_{hn}(t)) - (r + f_c)V_{hn}(t) + \zeta_{hn}(t)\Delta_{hd}(t), \tag{S40}$$

$$0 = \dot{V}_{do} + y_d - (r + f_d)V_{do}(t) + \zeta_{do}(t)\Delta_{hd}(t), \tag{S41}$$

$$0 = \dot{V}_{dn} - (r + f_d)V_{dn}(t) + \zeta_{dn}(t)\Delta_{dl}(t). \tag{S42}$$

Here the trading gains Δ_{hd} and Δ_{dl} as well as trading gains rates ζ_τ, $\tau \in \{lo, hn, do, dn\}$ are defined exactly as in the stationary case. The above analysis leads to the following lemma:

Lemma S3. Suppose that the reservation values satisfy $0 < R_l(t) < R_d(t) < R_h(t)$. Then the value functions are the solution to the system of linear ODEs (S37)-(S42).

To check that there are gains from trade one solves for reservation values, which follow the following linear ODE system.

$$0 = \dot{R}_h + y_h - rR_h(t) - \zeta_{hn}(t)(R_h(t) - R_d(t)) - \lambda_d(R_h(t) - R_l(t)), \tag{S43}$$

$$0 = \dot{R}_l + y_l + \lambda_u(R_h(t) - R_l(t)) - rR_l(t) + \zeta_{lo}(t)(R_d(t) - R_l(t)), \tag{S44}$$

$$0 = \dot{R}_d + y_d - rR_d(t) + \zeta_{do}(t)(R_h(t) - R_d(t)) - \zeta_{dn}(t)(R_d(t) - R_l(t)). \tag{S45}$$

We summarise our approach to numerically solving for the non-stationary equilibria. First, we solve for demographics using the results of Lemma S2. Second, we solve for value functions using Lemma S3. Third, we verify that gains from trade are positive everywhere along the equilibrium path, by solving (S43)-(S45).

S5.3 Welfare

The dynamics of welfare is given by

$$0 = \dot{w} + y_l m_{lo} + y_d m_{do} + y_h m_{ho} - rw$$

Solving the above, together with transversality condition $\lim_{t\to\infty} \exp(-\beta t)w(t) = 0$ yields a solution

$$w(t) = \int_t^\infty \exp(-r(\tau - t))(y_l m_{lo}(\tau) + y_d m_{do}(\tau) + y_h m_{ho}(\tau))d\tau.$$

We present the central result of this section below.

Proposition S6 (Welfare comparison with and without the transition dynamics). Consider two equilibrium paths. Suppose both of these paths reach steady states. Then, for small enough discount rate r the comparison of welfare for these two paths is equivalent to comparing welfare in the corresponding steady states.

Note that throughout we assume that the trading gains remain positive, under the condition outlined in Proposition 1 (by, e.g., varying the exist rates f_c and f_d).

S5.4 The bottleneck and the transition dynamics

In this section we do the following numerical exercise. We increase the search capacity n, holding other parameters of the model fixed. We trace the transition dynamics as the economy reaches a new steady state. (The economy could be not in the steady state initially.) While we demonstrates our findings for a particular set of parameters in the figures below, these figures are typical, and our insights appear to be general.

Consider Figure S2 below. There, we compare the welfare flows, defined as

$$y(t) := y_l m_{lo}(t) + y_d m_{do}(t) + y_h m_{ho}(t)$$

of the two equilibrium paths that are only different in terms of the search capacity n (n vs. $n+1$). The welfare flow reflects the total flow utility obtained by all traders in the economy at time t. Note that the social welfare at time t is the present value of welfare flows, discounted to time t, i.e.,

$$w(t) = \int_t^\infty \exp(-r(\tau - t))y(\tau)\mathrm{d}\tau.$$

The figure plots the difference $y(t; n+1) - y(t; n)$, in a situation when there is a bottleneck. Note that this difference is positive initially. This is because higher n improves the matching and allows gains from trade to realise more often. Note also that the difference becomes negative as the economy approaches the steady state and the bottleneck builds up. Thus, traders in the economy enjoy the transition dynamics initially, but end up in a less efficient steady state.

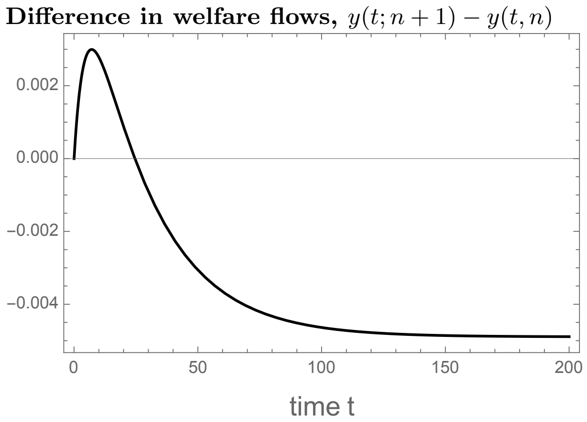


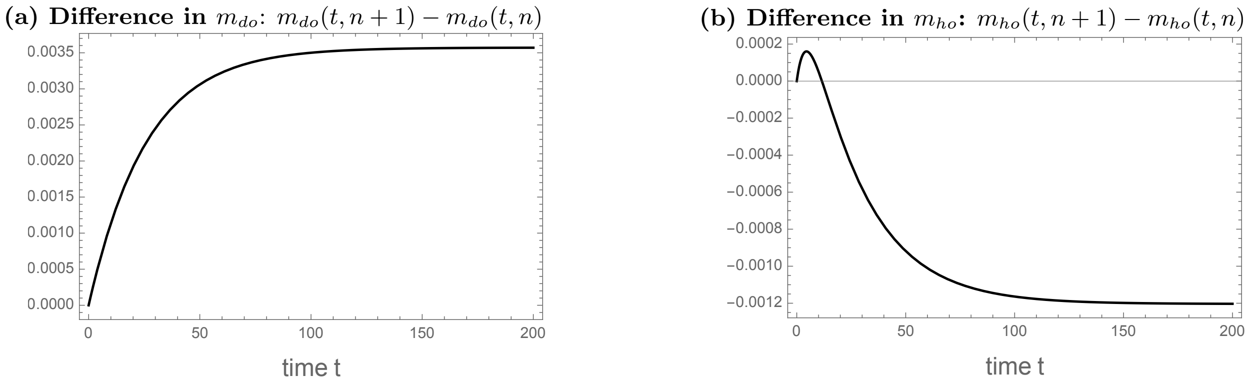
Figure S2: Welfare flows in non-stationary equilibria. This figure plots the dynamics of the difference of welfare flows $y(t;n+1)-y(t,n)$. We set $n=3$. The other parameters are set at $s=0.6$, $m_d=\lambda_d=\lambda_u=\lambda_d=r=0.1$, and $\rho=0.01$. The initial conditions are given by $\eta(0)=0.5$, $m_{do}(0)=0.06$ and $m_{lo}(0)=0.26$. This figure is typical: for the model parameters where we have bottleneck, the plot starts at zero, is positive initially, and then crosses zero to reach a negative horizontal asymptote.

TO illustrate the build up of the bottleneck we also look at the transition dynamics of an incremental change in m_{do}, $m_{do}(t;n+1)-m_{do}(t;n)$ in Figure S3 Panel (a). One can see that the increase in n leads to a steady incremental increase in m_{do}, manifesting the buildup of the bottleneck. Panel (b) shows the change in the mass of the "most efficient" asset holders m_{ho}, i.e., $m_{ho}(t;n+1)-m_{ho}(t;n)$. One can see that it is positive, initially. This is because higher n means that buyers can meet matching dealers more often, implying better asset flow from dealers to buyers. This echoes the intuition above that better matching should improve efficiency. But note that such improvement in efficiency only lasts for a very short period, during which the *masses of traders are roughly unchanged*. Over time, however, the equilibrium allocation effect kicks in and the masses adjust towards the steady state. The bottleneck then builds up, leading to incremental decrease in m_{ho}.

S6 An alternative price-setting mechanism

This appendix considers an alternative price-setting mechanism, where a customer can negotiate the price with the dealer *after* the RFQ auction. Specifically, when a customer is in contact with n dealers, the game proceeds in two steps:

- First, the customer runs a first-price auction among the n dealers.
- Second, the customer bargains with the dealer who has just won the auction, keeping the option to trade at the dealer's quote from the auction. The bargaining protocol is as follows:


(a) Difference in m_{do}: $m_{do}(t,n+1) - m_{do}(t,n)$

(b) Difference in m_{ho}: $m_{ho}(t,n+1) - m_{ho}(t,n)$

Figure S3. The parameters are set at $n = 3$, $s = 0.6$, $m_d = \lambda_d = \lambda_u = \lambda_d = r = 0.1$, and $\rho = 0.01$. The initial conditions are given by $\eta(0) = 0.5$, $m_{do}(0) = 0.06$ and $m_{lo}(0) = 0.26$.

with probability q (resp., $1 - q$) the customer (resp. the dealer) make TIOLIOs to the counterparty. (If there is a tie from the auction, the customer chooses one dealer at random.)

In fact, the above price-setting mechanism is equivalent to the one described in the main model ("Price determination" on p. 8):

Proposition S7 (Same dealer quoting). Dealers' equilibrium quoting and the split of trading gains is as described in the Proposition S3 and 5, respectively.

S7 Collection of proofs

Proposition S1

Proof. The expressions in (S25) follow the dealer stationarity condition (8). Note that the assumption (S24) can be equivalently written as$(\nu_{hn} - 1)m_{do} = 0$ and $(\nu_{lo} - 1)m_{dn} = 0$, which, together with (2)-(7), give three sets of solutions to the six demographic variables and the two matching probabilities. The three solutions one-to-one map into the three regions of the asset supply s: $0 < \eta < \eta + m_d < 1 + m_d$. The equations (S25) and (S26) can be easily verified using the three regions of the solution. □

Proposition S2

Proof. Consider first demographics. Recall that $\nu_{hn} = \nu(\pi_{do}; n) = 1 - (1 - \pi_{do})^n$. Whenever $m_{hn} > 0$, $\pi_{do} \in (0, 1)$ and in the limit of $n \to \infty$, $\nu_{hn} \to 1$. When $m_{hn} = 0$, then $\pi_{do} = 0$ and $\nu_{hn} = 0$. The same holds for the limit of $\psi \to 1$ (with $n \geq 2$), under the specific functional form of $\pi(\cdot; \psi)$. The limits for ν_{lo} follow analogously. Therefore, Equation (S24) holds in both limits, which then proves the convergence of all demographic variables together with the demographic conditions (2)-(7).

For value functions and prices, in both limit as $n \to \infty$ and $\psi \to 1$ (with $n \geq 2$), we have perfect competition among dealers, whenever $m_{do} \in (0, m_d)$. Thus, $p_a = p_b = R_d$ and the value function system is the same as in DS. In the case of $m_{do} = 0$ ($m_{do} = m_d$) we have $\Delta_{hd} = 0$ ($\Delta_{dl} = 0$) in the two limits ($n \to \infty$ and $\psi \to 1$) and so, again, prices and values are as in DS. □

Proposition S3

Proof. The proof only focuses on a contacted *do*-seller's symmetric quoting strategy. The same analysis applies to *dn*-buyers and is omitted. Consider first the trivial case of $n = 1$. A contacted *do*-seller then knows that he is the only one quoting. It is then trivial that with probability $(1 - q)$, he will quote the highest possible ask price, i.e., the *hn*-buyer's reservation value $R_h = R_d + \Delta_{hd}$. This can be viewed as a degenerate mixed strategy with c.d.f. $F(\alpha)$ converging to a unity probability mass at $\alpha = 1$ as stated in the proposition.

Next consider $n \geq 2$. Given the reservation values, it suffices to restrict the ask quote within $[R_d, R_h]$. Without loss of generality, a *do*-seller's strategy can be written as $R_d + \alpha\Delta_{hd}$ by choosing $\alpha \in [0, 1]$. Suppose α has a c.d.f. $F(\alpha)$ with possible realizations $[0, 1]$ (some of which might have zero probability mass). The following four steps pin down the specific form of $F(\cdot)$ so that it sustains a symmetric equilibrium.

Step 1: There are no probability masses in the support of $F(\cdot)$. If at $\alpha^* \in (0, 1]$ there is some non-zero probability mass, any *do*-seller has an incentive to deviate to quoting with the same probability mass but at a level infinitesimally smaller than α^*. This way, he converts the strictly positive probability of tying with others at α^* to winning over others. (The undercut costs no expected revenue as it is infinitesimally small.) If at $\alpha^* = 0$ there is non-zero probability mass, again, any *do*-seller will deviate, this time to an α just slightly above zero. This is because allocating probability mass at zero brings zero expected profit. Deviating to a slightly positive α, therefore, brings strictly positive expected profit. Taken together, there cannot be any probability mass in $\alpha \in [0, 1]$. Note that any pure symmetric-strategy equilibria are ruled out.

Step 2: The support of $F(\cdot)$ is connected. The support is not connected if there is $(\alpha_1, \alpha_2) \subset [0, 1]$ on which there is zero probability assigned and there is probability density on α_1. If this is the case, then any *do*-seller will deviate by moving the probability density on α_1 to any $\alpha \in (\alpha_1, \alpha_2)$. Such a deviation is strictly more profitable because doing so does not affect the probability of winning (if one wins at bidding α_1, he also wins at any $\alpha > \alpha_1$) and because $\alpha > \alpha_1$ is selling at a higher price.

Step 3: The upper bound of the support of $F(\cdot)$ is 1. The logic follows Step 2. Suppose the upper bound is $\alpha^* < 1$. Then, allocating the probability density at α^* to 1 is a profitable deviation: It does not affect the probability of winning and upon winning sells at a higher price.

Step 4: Deriving the c.d.f. $F(\cdot)$. Suppose all other *do*-sellers, when contacted, quote according to some same distribution $F(\cdot)$. Consider a specific seller called i. Quoting $R_d + \alpha\Delta_{hd}$, i gets to trade with the searching buyer if, and only if, such a quote is the best that the buyer receives. The buyer examines all quotes received. For each of the $n-1$ contacts, with probability $1-\pi_{do}$ the dealer is not a *do*-seller and in this case i's quote beats the no-quote. With probability π_{do}, the contacted investor is indeed another *lo*-seller, who quotes at α'. Then, only with probability $\mathbb{P}(\alpha < \alpha') = 1-F(\alpha)$ will i's quote win. Taken together, for each of the $n-1$ potential competitor, i wins with probability $(1-\pi_{do}) + \pi_{do}(1-F(\alpha))$, and he needs to win all these $n-1$ times to capture the trading gain of $\alpha\Delta_{hd}$. That is, i expects a profit of $(1-\pi_{do}F(\alpha))^{n-1}\alpha\Delta_{hd}$. In particular, at the highest possible $\alpha = 1$, the above expected profit simplifies to $(1-\pi_{do})^{n-1}\Delta_{hd}$, because $F(1) = 1$. In a mixed-strategy equilibrium, i must be indifferent of quoting any values of α in the support. Equating the two expressions above and solving for $F(\cdot)$, one obtains the c.d.f. stated in the proposition. It can then be easily solved that the lower bound of the support must be at $(1-\pi_{do})^{n-1}$, where $F(\cdot)$ reaches zero. This completes the proof. □

Proposition S4

Proof. Consider a searching *hn*-buyer, for example. He contacts n dealers but knows that the number of counterparties he will actually find, N, is a random variable that follows a binomial distribution with n draws and success rate π_{do}. Each of these N counterparties then quotes a random price according to $F(\alpha; \pi_{do}, n)$, as stated in Proposition S3. The searching buyer chooses the lowest ask across the N available quotes. The c.d.f. of this minimum α is $1-(1-F(\alpha;\cdot))^{N-1}$ for $N \geq 1$. Since the probability of $N \geq 1$ is $(1-(1-\pi_{do})^n)$, one obtains the conditional c.d.f. as stated in the proposition. The same applies to a searching *lo*-seller. □

Proposition S5

Proof. Consider the limit of $\psi \to 1$ (with $n \geq 2$). For any $m_{do} > 0$ and $m_{dn} > 0$, $\pi_{do} = \pi\left(\frac{m_{do}}{m_d}\right) \to 1$ and $\pi_{dn} = \pi\left(\frac{m_{dn}}{m_d}\right) \to 1$. Therefore, the $G(\cdot)$ function in Proposition S4 converges to $G(x) = 1$ throughout the support of $x \in (0, 1]$. The equilibrium trading prices, therefore, become $A \to R_d$ and $B \to R_d$ almost surely, and $\text{var}[A] \to 0$ and $\text{var}[B] \to 0$ accordingly. That is, in the limit of $\psi \to 0$, there is no price dispersion. Since the variances are nonnegative, by continuity, therefore, both $\text{var}[A]$ and $\text{var}[B]$ must decrease with ψ for sufficiently large ψ. □

Proposition S6

Proof. Consider one path and calculate the welfare:

$$\begin{aligned}
rw(t) &= r\int_t^\infty \exp(-r(\tau-t))(y_l m_{lo}(\tau)+y_d m_{do}(\tau)+y_h m_{ho}(\tau))d\tau \text{ // letting } y := r\tau\\
&= \int_{rt}^\infty \exp(-y+rt)(y_l m_{lo}(y/r)+y_d m_{do}(y/r)+y_h m_{ho}(y/r))dy\\
&\xrightarrow{r\to 0} \int_0^\infty \exp(-y)(y_l m_{lo}(\infty)+y_d m_{do}(\infty)+y_h m_{ho}(\infty))dy\\
&= y_l m_{lo}(\infty)+y_d m_{do}(\infty)+y_h m_{ho}(\infty)
\end{aligned}$$

The last line gives the welfare (in flow terms) in the steady state. The proposition follows. □

Proposition S7

Proof. We start with dealers' quoting. The proof only focuses on a contacted *do*-seller's symmetric quoting strategy. Without loss of generality, a *do*-seller's strategy can be written as $R_d + \alpha\Delta_{hd}$ by choosing $\alpha \in [0,1]$. Suppose α has a c.d.f. $F(\alpha)$ with possible realizations $[0,1]$ (some of which might have zero probability mass). The proof, identical to the one for the Proposition S3 can show that: (i) dealers will quote buyers' reservation value when $n = 1$, (ii) for $n > 2$ dealers would follow mixed-strategies with no point masses and continuous supports, (iii) the upper bound of support of $F(\cdot)$ is one. The only non-trivial step is the following:
Step 4: Deriving the c.d.f. $F(\cdot)$. Suppose all other *do*-sellers, when contacted, quote according to some same distribution $F(\cdot)$. Consider a specific seller called i. Quoting $R_d + \alpha\Delta_{hd}$, i gets to trade with the searching buyer if, and only if, such a quote is the best that the buyer receives. The buyer examines all quotes received. For each of the $n-1$ contacts, with probability $1-\pi_{do}$ the dealer is not a *do*-seller and in this case i's quote beats the no-quote. With probability π_{do}, the contacted investor is indeed another *lo*-seller, who quotes at α'. Then, only with probability $\mathbb{P}(\alpha < \alpha') = 1 - F(\alpha)$ will i's quote win. Taken together, for each of the $n-1$ potential competitor, i wins with probability $(1-\pi_{do}) + \pi_{do}(1-F(\alpha)) = 1-\pi_{do}F(\alpha)$, and he needs to win all these $n-1$ times to capture the trading gain of $\alpha\Delta_{hd}$. When the dealer wins, he trades at a markup α, with probability $1-q$. Indeed, with probability q the customer will extract the full trading gain from her at the bargaining stage. That is, i expects a profit of

$$(1-\pi_{do}F(\alpha))^{n-1}\alpha\Delta_{hd}(1-q).$$

In particular, at the highest possible $\alpha = 1$, the above expected profit simplifies to

$$(1-\pi_{do})^{n-1}\Delta_{hd}(1-q),$$

because $F(1) = 1$. In a mixed-strategy equilibrium, i must be indifferent of quoting any values of α in the support. Equating the two expressions above and solving for $F(\cdot)$, one obtains the c.d.f.

stated in the proposition. It can then be easily solved that the lower bound of the support must be at $(1-\pi_{do})^{n-1}$, where $F(\cdot)$ reaches zero.

Having established that dealers' quoting is unchanged under the new trade protocol, the split of the trading gains follows immediately. This completes the proof. □

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