When Large Traders Create Noise*

Sergei Glebkin† and John Chi-Fong Kuong‡

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Abstract

We consider a market where large investors do not only trade on information about asset fundamentals. When they trade more aggressively, the price becomes less informative. Other investors who learn from prices, in turn, are less concerned about adverse selection and provide more liquidity, causing large investors to trade even more aggressively. This trading complementarity can engender three unconventional results: i) increased competition among large investors makes all investors worse off, ii) more precise private information reduces price informativeness, creating complementarities in information acquisition, and iii) multiple equilibria emerge. Our results have implications for competition and transparency policies in financial markets.

JEL Classification: G0, G01, G12, G14

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†INSEAD; Email: glebkin@insead.edu

‡INSEAD and CEPR; Email: john.kuong@insead.edu

1 Introduction

Large investors play an increasingly important role in asset markets in the U.S. and around the world. In U.S. equities markets, for example, ownership by the largest 10 institutions has more than quadrupled to 26.5% from 1980 to 2016 (Ben-David, Franzoni, Moussawi, and Sedunov, 2021). In other developed countries, the largest five equity holders hold around 3% to 20% of all shares (Kacperczyk, Nosal, and Sundaresan, 2020). Thus, asset markets nowadays are populated by large investors with significant market power. Moreover, research has shown that these investors often trade for reasons unrelated to stocks' discount rates or future cash flows, that is, fundamentals. In doing so, they cause fluctuations in prices unrelated to fundamentals, or, add noise in prices. A salient example is institutional investors who put increasingly more weight on the environment, social, and governance (ESG) performance of firms in their investment decisions. Stocks that are likely owned more by ESG-conscious investors would thus have less informative stock prices. Such correlation is found in our empirical analysis summarized in Table 1. Existing studies also show that ownership by the largest 10 institutions in the U.S. equities market is positively associated with noisier prices (Ben-David et al., 2021). Mutual funds experiencing large flows adjust their holdings, injecting noise in prices (Coval and Stafford, 2007; Edmans, Goldstein, and Jiang, 2012). Among high-frequency traders (HFTs), trading revenues and risk-adjusted performance are highly concentrated in the fastest five traders (Baron, Brogaard, Hagströmer, and Kirilenko, 2019), and overall HFT activity and presence are associated with lower price informativeness (Weller, 2018; Gider, Schmickler, and Westheide, 2019). We refer to these investors as *noise-creating large investors*.[1]

The existence of noise-creating large investors underlies a potential tension between the two central functions of financial markets, namely, the efficient allocation of assets and the aggregation of dispersed information. When their market power rises, these large investors take their higher price impact into account and scale down trades. This hampers the efficient

[1]In Section 9.1, we discuss the evidence on noise-creating large traders in more detail.

reallocation of assets while, at the same time, reduces the noise they inject into prices. The attenuated noise allows *other* traders to extract more information from prices and thus trade more efficiently. As a result, market power does not need to reduce welfare, in contrast to conventional understanding.[2] As we demonstrate in the paper, the overall impact on market outcomes and welfare ultimately depends on the nature of the interaction between the noise-creating large investors and other investors.

We address the following questions: How does the trading behavior of noise-creating large investors' affect the behavior of other investors? Conversely, how is it affected by other investors' behavior? Under what conditions does an increase in the market power enjoyed by large investors improve welfare? How do these results change when investors rely less on the information contained in prices (e.g., because they have more precise private signals)? Answering these questions not only enriches our understanding of the effect of market power on the workings of financial markets, but it is also essential for devising competition and transparency policies, given how concentrated ownership has become.

To answer these questions, we develop a model in which large investors have private valuations, as in Vives (2011), while others learn from prices, as in Hellwig (1980). For simplicity, we assume that the latter are small and hence behave competitively. Crucially, the model features large investors creating noise from the small investors' point of view: Large investors' valuation v_L is imperfectly correlated with small investors' v_S, and small investors have dispersed signals about v_S. Hence, small investors learn about v_S from prices that are contaminated by v_L. For simplicity, interpret v_S as an asset's fundamentals.[3] The imperfect correlation between v_L and v_S captures the fact that large investors have trading motives other than asset fundamentals.

[2] Noise in asset prices might also matter to welfare if it influences firms' real decisions, as empirically shown in Edmans et al. (2012) for takeovers and in Dessaint, Foucault, Frésard, and Matray (2018) for investments by peer firms. See Bond, Edmans, and Goldstein (2012) for a comprehensive survey on the real effects of the information content of prices.

[3] This assumption is not necessary for our results. We derive our results about informational efficiency under the assumption that v_S has two components: a fundamental value and one a private value. Thus, both small and large investors' trading may contaminate price with noise unrelated to fundamental.

The premise that the large investors have private valuations can be rationalized by ESG concerns and, as discussed in Section 8 in three motivating examples in which large investors are, respectively, high-frequency traders (HFTs) who have private information about some future order flow, mutual funds with demands driven by fund flows, and commodity producers with risky production costs. Notably, in the HFTs example, large and small investors have common values. It is the order flow that HFTs absorb that makes them behave as if they have private values.

The key mechanism uncovered in this paper is a trading complementarity between large and small investors when large investors create noise. When the large investors trade more aggressively, prices reflect more of their own valuation v_L. Knowing this, small investors are less concerned with adverse selection (vis-à-vis other small investors) and are more willing to provide liquidity; that is, they sell (buy) more following an increase (decrease) in price. The improved liquidity in turn encourages large investors to trade more aggressively.[4] We show in Internet Appendix IA.1 that the trading complementarity does not arise when large investors do not exercise their market power and take prices as given.

This trading complementarity underpins three novel insights on the consequences of market power in financial markets. The first concerns the effects of competition on market quality and investor welfare. Consider an increase in competition among large investors caused by a breakup of existing ones. As their market power is reduced, large investors trade more aggressively, resulting in higher liquidity but lower informational efficiency. Our mechanism thus underpins a negative relationship between these two important dimensions of market quality (liquidity and informational efficiency).

Regarding welfare, increased competition among large investors can *reduce* aggregate welfare and even make small investors worse off, when the informational friction is severe, that is,

[4]It is worth noting that our results are not affected by whether large investors are more or less informed about v_S than small investors. Indeed, they create noise because they have trading motives unrelated to asset fundamentals, not because they are uninformed.

when the assets' fundamentals are noisy and investors' signals are imprecise. As discussed above, more competition among large investors leads them to impound more noise into prices, thus making price less informative and small investors' asset allocations less efficient. This unconventional result suggests that competition policy in financial markets should take the informational friction into account.[5]

The second result shows that an improvement in the quality of private information held by investors can *reduce* informational efficiency. This seemingly paradoxical result stems from the aforementioned trading complementarity: Small investors endowed with more precise signals are less concerned with adverse selection and are more willing to provide liquidity. Higher liquidity, in turn, induces large investors to trade more aggressively, thereby injecting more noise into the price. This additional noise can dominate the effect of improved private information, resulting in a net decrease in informational efficiency, if the informational friction is severe enough. This can explain the puzzling evidence in Bai, Philippon, and Savov (2016) and Farboodi, Matray, Veldkamp, and Venkateswaran (2020) that, despite the tremendous advancement in information technology, price informativeness has decreased for stocks outside the S&P 500 index (while increasing for stocks in the index). Indeed, non-index stocks are less covered by analysts and are therefore subject to more severe informational friction.

The above result also implies a potential complementarity in information acquisition. Consider a small investor deciding how much information to acquire about an asset. If other small investors acquire more information, then, provided that informational frictions are severe, the price becomes less informative (due to the large investors impounding more noise into the price), thereby stimulating the acquisition of more information. The novel prediction is that, for stocks with high (low) informational friction, investors' information choices are strategic complements (substitutes).

[5] SEC chairman Gary Gensler has expressed concerns about large trading firms' market power in executing retails trades and their practice of "payment for order flow" (*Financial Times*, "Gensler raises concern about market influence of Citadel Securities," May 6, 2021.). In Section 8, we offer an interpretation of our model, which can be seen as market makers competing for order flows. (See Remark 4.)

The third result concerns the effect of market power on the stability of financial markets. The trading complementarity engenders an amplification mechanism whereby small shocks are magnified to have a disproportionate impact on market outcomes. Furthermore, multiple equilibria can emerge. These results do not arise when large investors take prices as given, suggesting that market power, in combination with informational friction, can be a source of fragility in financial markets.[6]

Putting our three main results together, we show that the presence of large investors creating noise entails rich and novel implications, with several overturning traditional results. Regarding the design of policies, promoting competition reduces investor welfare, and increasing transparency (whereby investors can obtain better information at lower cost) harms informational efficiency of the markets. Furthermore, as market power can both increase welfare and cause fragility, there is a potential trade-off between investor welfare and financial stability to be confronted by regulators. In Section 9.3, we discuss in detail the model's novel implications to transparency and competition policies.

The remainder of the paper is organized as follows. Section 2 reviews the literature. Section 3 presents the model. Section 4 characterizes the equilibrium and the key mechanism. Section 5 studies the effects of competition on welfare and Section 6 the effects of more precise private signals on informational efficiency. Section 7 analyzes multiple equilibria and fragility. Section 8 offers three interpretations of our model. Section 9 discusses the evidence that supports the model's premise, as well as novel empirical and policy implications. Section 10 concludes.

[6] That market power as a source of fragility in financial markets is rarely considered in the literature. Models of trading complementarity typically feature competitive agents (Goldstein, Ozdenoren, and Yuan, 2011, 2013b; Goldstein and Yang, 2015; Cespa and Foucault, 2014). More generally, competition is often linked to financial fragility because i) agents do not internalize the adverse effects of their trades (e.g., fire sales) and contract choices (Eisenbach and Phelan, 2021; Kuong, 2021) and ii) agents shirk on time-consuming risk management efforts in order to preempt others in taking profitable trading opportunities (Bouvard and Lee, 2020).

2 Relation to the Literature

This paper is related to two strands of research. The first is the literature on strategic complementarities in the presence of various kinds of informational frictions in markets. Most studies are cast in a competitive REE framework, which implies that all traders take prices as given. Complementarities have also been found when there is multidimensional information (Goldstein and Yang, 2015), learning across markets (Cespa and Foucault, 2014), learning by financiers (Goldstein, Ozdenoren, and Yuan, 2013b; Glebkin, Gondhi, and Kuong, 2020), agency problems in delegated investment (Huang, 2015), dynamic trading (Cespa and Vives, 2011), and endogenous liquidity trading (Han, Tang, and Yang, 2016). We contribute by adding that market power, when interacting with informational friction, can give rise to complementarities. Below, we review other closely related papers with competitive agents and highlight our contribution.

There are competitive REE models that feature noise creation by one group of investors to another. Goldstein, Li, and Yang (2013a) consider a model with two groups of speculators with different investment opportunities. Sockin and Xiong (2015) study a trading model with commodity producers and consumers. Due to heterogeneity of hedging needs (Goldstein, Li, and Yang, 2013a), private values (Sockin and Xiong, 2015), and preferences (Goldstein, Kopytov, Shen, and Xiang, 2022), demands made by one group affect inference from prices by the other. The key difference from our paper is that the traders who create noise are small. Thus, their trade aggressiveness does not vary with the liquidity provision by other traders. As a result, relative to these two papers, our contribution is to derive novel implications of competition on liquidity, informational efficiency, fragility, and welfare.

One unconventional result of our paper is that improvement of transparency could deteriorate price informativeness. With very different mechanisms, Dugast and Foucault (2018) and Banerjee, Davis, and Gondhi (2018) find a similar prediction. Dugast and Foucault show that when the cost of low-precision signals declines, traders acquire more of them and less of time-consuming, high-precision signals. In the differences-of-opinion model of Banerjee et al., improving transparency about asset fundamentals can cause traders to learn more about others' beliefs. While both papers obtain the result that transparency can harm price informativeness, our result applies specifically for stocks with high informational frictions, which corroborates with the evidence in Farboodi et al. (2020) and Bai et al. (2016).

At a conceptual level, our model of noise-creating large traders uncovers a tension between liquidity and informational efficiency. Papers that share this tension include Stein (1987), Dow (2004), and Han et al. (2016), who emphasize the (endogenous) entry of, respectively, speculators, hedgers, and liquidity traders.[7] We enrich their results in the sense that, even without entry, such tension exists because imperfectly competitive traders adjust their trading aggressiveness in response to liquidity provision by other traders. We therefore can provide new analyses about market power's impact on market quality and welfare in financial markets. As we have argued before, the issue of market power possessed by large, institutional investors is empirically and policy-relevant in many asset markets.

The second stream of research we contribute to is the literature on strategic trading. More specifically, our paper is most related to the works on demand function equilibria in which agents have private valuations.[8] Unlike ours, many private values models (e.g., Vives, 2011; Rostek and Weretka, 2012, 2015b; Du and Zhu, 2017; Kyle, Obizhaeva, and Wang, 2017) feature ex-ante symmetric agents, and there is no heterogeneity in price impact. Therefore, the complementarity between large and small traders uncovered in this paper does not arise. Manzano and Vives (2016) consider a setting similar to ours, with the key difference being that the traders within each group receive the same signal (while small traders in ours receive dispersed signals). In equilibrium, therefore, traders in one group not only know their own signal but also learn about the other group's signal perfectly from the price. Hence, there is no interaction between liquidity and informational efficiency, as traders learn the same information

[7]Other works have found that liquidity and informational efficiency reinforce each other. See, e.g., Cespa and Vives (2011), Cespa and Foucault (2014), and Lee (2013).

[8]Papers in which models incorporate common valuations include Kyle (1989), Pagano (1989), Vayanos (1999), Rostek and Weretka (2015a), and Malamud and Rostek (2017).

regardless of the liquidity.

Finally, Kacperczyk et al. (2020) also study the effect of market power on price informativeness. Their model features oligopolistic informed traders who acquire information and trade with competitive uninformed traders. They find that price informativeness is non-monotonic in size of the informed sectors. The crucial difference between their paper and ours is that they assume that uninformed traders do not learn from prices. As a result, their large traders do not create noise and there is no trading complementarity. In contrast, our model predicts that large traders' activities make price noisier, consistent with evidence about institutional investors (Ben-David et al., 2021)) and HFTs (Weller, 2018; Gider et al., 2019). Our model also delivers implications that market power can improve welfare and engender fragility.

3 A Model of Noise-Creating Large Traders

There are two time periods, $t \in \{0, 1\}$. Two trader groups, *large traders* and *small traders*, are trading a risky asset at time $t = 0$. There are $N > 1$ large traders indexed by $i \in \{1, 2, ..., N\}$ as well as a unit continuum of small traders indexed by $j \in [0, 1]$. Hereafter, we shall facilitate the exposition by using male (female) pronouns for large (small) traders. All traders are risk-neutral and have quadratic inventory costs. Traders are identical within each group, and their preferences are characterized as follows. If a large trader purchases x units of the asset (and pays price p) at time $t = 0$, then his utility at time $t = 1$ is

$$u_L = (v_L - p)x - \frac{w_L x^2}{2}, \tag{1}$$

where v_L denotes asset value for a large trader and the term $w_L x^2/2$ represents the inventory cost of holding x units of asset. This cost may be due to regulatory capital requirements, collateral requirements, or risk management considerations. We call $1/w_L$ the *risk-bearing capacity* of a large trader.

Suppose a small trader similarly purchases x units of the asset (and pays price price p) at time $t = 0$; then her utility at time $t = 1$ is

$$u_S = (v_S - p)x - \frac{w_S x^2}{2}, \tag{2}$$

where v_S and $1/w_S$ are, respectively, the asset value and risk-bearing capacity of small traders. The preference specification just given, where risk-neutral traders have quadratic inventory costs and private values, is the same as in for example, Vives (2011), Rostek and Weretka (2012, 2015b), Du and Zhu (2017), and Duffie and Zhu (2017).

Asset values are realized at time $t = 1$ but are uncertain at time $t = 0$. The difference in the values of large and small traders generates gains from trade between groups. The values v_L and v_S are (jointly) normally distributed and imperfectly correlated.[9] That is, $v_k \sim N(\bar{v}_k, 1/\tau_k)$, for $k \in \{S, L\}$, with $\text{corr}(v_L, v_S) = \rho \in [0, 1)$.[10]

The information structure is as follows. Small traders do not know v_L and have dispersed information about v_S. In particular, each small trader j receives a signal $s_j = v_S + \varepsilon_j$, where the ε_j are independent and identically distributed (i.i.d.) as $\varepsilon_j \sim N(0, 1/\tau_\varepsilon)$ and are also independent of v_S and v_L; the parameter τ_ε measures the signal's precision. We assume that large traders know v_L but do not know v_S. Our main results continue to hold when large traders do not know v_L perfectly (see Section IA.4). Also, the results are unchanged if large traders receive dispersed information, however precise, about v_S. In short, how informed large traders are relative to small traders is not crucial. What is crucial is the noise created by large traders: Small traders learn about v_S from prices that are contaminated by v_L.

[9]While we assume common values *within* each group (large and small) in the main model, we show that our main results continue to hold in a model with heterogeneous values considered in Section IA.5.

[10]The model would still be tractable for $\rho < 0$, but in that case, the equilibrium mechanism would feature additional strategic complementarities that are not the focus of this paper. The complementarity is as follows: When other small traders trade more aggressively, a small trader of interest might have incentives to trade more aggressively as well. The reason is that, when the correlation is negative, if other small traders trade more aggressively then the price might become *less* informative to the trader of interest because the information in other traders' signals may be (partly) canceled out by information in the large traders' value. Hence, the trader of interest will weigh the price less and her signal more, thereby increasing her trading aggressiveness as well.

The trading protocol is a uniform-price double auction. Each trader k submits a net demand schedule $x_k(p)$, where $x_k(p) > 0$ ($x_k(p) < 0$) corresponds to a buy order (sell order). The market-clearing price p^* is such that the net aggregate demand is zero,

$$\sum_{i=1}^{N} x_i(p^*) + \int_0^1 x_j(p^*)\, dj = 0. \tag{3}$$

In equilibrium, a trader k is allocated $x_k^* = x_k(p^*)$.

The equilibrium concept is the Bayesian Nash Equilibrium, as in Kyle (1989) and Vives (2011); thus, traders maximize expected utility, given their information and accounting for their price impact, and equilibrium demand schedules are such that the market clears. As in most of the literature, we restrict the analysis to symmetric linear equilibria in which a large trader i and a small trader j have the following demand schedules:

$$x_i = \alpha + \beta \cdot v_L - \gamma \cdot p \quad \text{and} \quad x_j = \alpha_S + \beta_S \cdot s_j - \gamma_S \cdot p. \tag{4}$$

The coefficients (α, β, γ) and $(\alpha_S, \beta_S, \gamma_S)$ are identical for traders within the same group. Note that we rule out trivial (no-trade) equilibria by focusing on equilibria for which $(\beta, \gamma, \beta_S, \gamma_S) \neq 0$.

We conclude this section with a brief discussion of our modeling choices. First, there are noise-creating large investors. As mentioned in the introduction, a prominent example is institutional investors who put weights on ESG-related metrics in their investment decisions. In Section 9.1, we present supportive empirical evidence. We also discuss existing evidence showing that mutual funds and high-frequency traders have market power and contribute to noise in prices. Second, we use heterogeneous private valuations to capture the idea of large investors creating noise to small investors. In Section 8, we show how the model can be interpreted in contexts of trading among i) institutional and retail investors, ii) fast and slow traders, and iii) commodities producers and firms that use commodities as input for production.[11] In short,

[11] In the example of fast and slow traders, there is common valuation. Yet, fast traders behave as though they have private values because they can trade at higher frequencies and so absorb order flow before slow traders.

we believe the key feature of our model is realistic, while the model itself is parsimonious and flexible enough to speak to various segments of financial markets.

Remark 1 (Traders' Preference Specification). We have opted for a framework with risk-neutral investors subject to quadratic inventory costs (*linear-quadratic framework*, hereafter) instead of one with exponential utility. We do so because the linear-quadratic framework is (i) more flexible to adopt different applications (see Section 8), and (ii) more tractable, especially when it comes to welfare analysis, allowing us to derive all our results analytically. In terms of economic forces, the main difference between the two frameworks, as pointed out in Vives (2017), is that the linear-quadratic framework abstracts from the Hirshleifer (1971) effect – the potential impact of information on ex ante risk-sharing opportunities. Since our mechanism focuses on the role of information aggregation and market power, we believe that incorporating such additional forces would enrich but not overturn our results regarding welfare.

Remark 2 (Aggregate Supply of Risky Asset). Equations (1) and (2) suggest that traders start with zero endowment in the risky asset; hence, its aggregate supply is equal to zero. It is easy to reduce a model with positive asset supply to the one presented above. Suppose that all traders in group $k \in \{L, S\}$ traders start with endowment e_k in the risky asset. For simplicity, assume that e_k, $k \in \{L, S\}$ is known to all traders. (Given that v_L and v_S are different, our model does not require extra sources of noise.) Then, traders' utilities can be written as $u_k = (v_k - w_k e_k - p)x - \frac{w_k x^2}{2}$, where we have dropped the constant term $\frac{w_k e_k^2}{2}$, since it does not affect optimization. Thus, adding initial endowment is equivalent to adding a constant $-w_k e_k$ to v_k, or, changing the mean of v_k from $\bar{v}_k$ to $\bar{v}_k - w_k e_k$.

4 Equilibrium

Liquidity and informational efficiency

We begin the analysis with definitions of the two fundamental aspects of market quality, namely, liquidity and informational efficiency.

Liquidity $\mathcal{L}$ is measured by *market depth*, defined as the reciprocal of price impact λ, as in Kyle (1989)[12],

$$\mathcal{L} \equiv \frac{1}{\lambda} = (N-1)\gamma + \gamma_S. \tag{5}$$

By definition, $1/\lambda$ is the price sensitivity of the residual supply of the asset. Equation (5) holds, as there are $(N-1)$ large traders with sensitivity γ and a unit mass of small traders with sensitivity γ_S contributing to the price sensitivity of the residual supply. Equation (5) implies that liquidity is directly related to the price sensitivities γ and γ_S, an implication that enables defining *liquidity provision* as follows: *A trader who increases (decreases) the price sensitivity of his demand provides more (less) liquidity.*

Next, we define informational efficiency. We work with two measures. The first one, *revelatory price efficiency* (RPE), as introduced in Bond et al. (2012), measures the extent to which prices reveal the amount of information necessary for decision-makers to take value-maximizing actions. Since large traders know their values perfectly, only the information about small traders' values, v_S, contributes to RPE. Formally, RPE is defined as follows:

$$\mathcal{I}^{RPE} \equiv \frac{\mathrm{Var}(v_S)}{\mathrm{Var}(v_S|s_j, p)}.$$

$\mathcal{I}^{RPE}$ captures the reduction in variance of small traders' values that is due to learning.

The second measure, *forecasting price efficiency* (FPE), is most closely related to empirical

[12]All our results continue to hold if we define liquidity as price elasticity of aggregate demand, equal to $N\gamma + \gamma_S$, which is a standard measure of liquidity in competitive REE models.

measures of price informativeness. It measures the extent to which prices reveal information about the fundamental value of the asset, which we denote by v. Formally, FPE is defined as follows[13]:

$$\mathcal{I}^{FPE} \equiv \frac{\text{Var}(v)}{\text{Var}(v|s_j, p)}.$$

To compute $\mathcal{I}^{FPE}$ in our model, we need to specify the relationship between investors' values (v_k, where $k \in \{S, L\}$) and asset fundamental value v. To this end, we assume that $v_k = v + \tilde{u}_k$, where $\tilde{u}_k$ are investors' private values. Moreover, we allow the private values to be correlated with v by setting $\tilde{u}_k = (a_k - 1)v + u_k$, so $v_k = a_k v + u_k$. Thus, when $a_k > 1$ ($a_k < 1$), the investors' private value is positively (negatively) correlated with asset fundamentals. The correlation structure depends on applications. For example, if investors' private values stem from ESG concerns and better ESG performance (higher $\tilde{u}_k$) tend to come at the expense of worse financial performance (lower v), then the correlation is negative. In particular, $a_L < a_S$ reflects the idea that large investors have stronger preferences for ESG outcomes than small ones.

While the specific structure of v_S and v_L help the discussion of $\mathcal{I}^{FPE}$, it restricts the generality of the model. Thus, we state it as an assumption below, which is only invoked when we establish results concerning $\mathcal{I}^{FPE}$. This allows us to distinguish the results that hold in the specific structure from those that hold more generally.

Assumption 1. *The values are given by $v_S = a_S v + u_S$ and $v_L = a_L v + u_L$, where $a_S, a_L > 0$, and v, u_S and u_L are jointly normally distributed and independent, with $v \sim N(\bar{v}, \tau_v^{-1})$, $u_S \sim N(0, \tau_{u,S}^{-1})$, and $u_L \sim N(0, \tau_{u,L}^{-1})$.*

[13]Our results also hold if we define FPE as $\mathcal{I}' \equiv \frac{\text{Var}(v)}{\text{Var}(v|p)}$. In particular, under $\mathcal{I}'$, it is easier to achieve one of our novel results that informational efficiency can decrease in investors' private signal precision τ_ϵ. The reason is that the private signals only indirectly affect $\mathcal{I}'$ via the information aggregated by prices. In contrast, a more precise signal s_j directly improves $\mathcal{I}^{FPE}$ as $\text{Var}(v|s_j, p)$ decreases.

Equilibrium characterization

The key to our results is the complementarity between how aggressively large traders trade, captured by the coefficient β in their demand, and how much liquidity small traders provide, reflected in their demand's price sensitivity γ_S. For clarity, we break it down into two parts and analyze them separately. In the first part, large traders' demand schedules are treated as exogenous. Lemma 1 states how an exogenous increase in their trading aggressiveness β affects informational efficiency $\mathcal{I}^{RPE}$ and the amount of liquidity provided by small traders γ_S. In the second part, small traders' demand schedules are assumed to be exogenous. Lemma 2 then shows how an increase in their liquidity provision γ_S affects liquidity $\mathcal{L}$ and large traders' trading aggressiveness β. The equilibrium is analyzed in Theorem 1.

Lemma 1 (Large traders' trading aggressiveness worsens informational efficiency and encourages small traders to provide liquidity)**.** *Fix the parameters (α, β, γ) in large traders' demand schedules. There exists a sufficient static π that is infomationally equivalent to the price p, such that $\pi = v_S + \zeta/\sqrt{\tau_\pi}$, where $\zeta \sim N(0,1)$ and is independent of v_S, and the precision $\tau_\pi \equiv \mathrm{Var}[\pi|v_S]^{-1} = \frac{\tau_L}{1-\rho^2}\left(\rho\sqrt{\frac{\tau_S}{\tau_L}} + \frac{\beta_S}{N\beta}\right)^2$. Informational efficiency can then be written as*

$$\mathcal{I}^{RPE} = \frac{\tau_S + \tau_\varepsilon + \tau_\pi}{\tau_S}, \tag{6}$$

and the price sensitivity of small trader j's demand is

$$\gamma_S = \frac{1}{w_S} - \frac{1}{w_S} \underbrace{\frac{\partial E[v_S|s_j, p]}{\partial p}}_{>0,\ information\ effect}.$$

Both τ_π and $\frac{\partial E[v_S|s_j,p]}{\partial p}$ are positive and decreasing in β, ceteris paribus.

Lemma 1 establishes two steps in the trading complementarity, which is illustrated by Figure 1. The first step shows that because large traders create noise in the price from the small traders' perspective, more aggressive trading reduces the informational efficiency, measured by the RPE. The second step illustrates how small traders change the amount of liquidity they provide as a consequence. Since informational efficiency is reduced, an increase in price becomes less of a signal of an increase in asset value v_S ($\frac{\partial E[v_S|s_j,p]}{\partial p}$ becomes smaller). Hence, small traders are willing to sell more of the asset (price sensitivity γ_S increases) in response to an increase in the price.

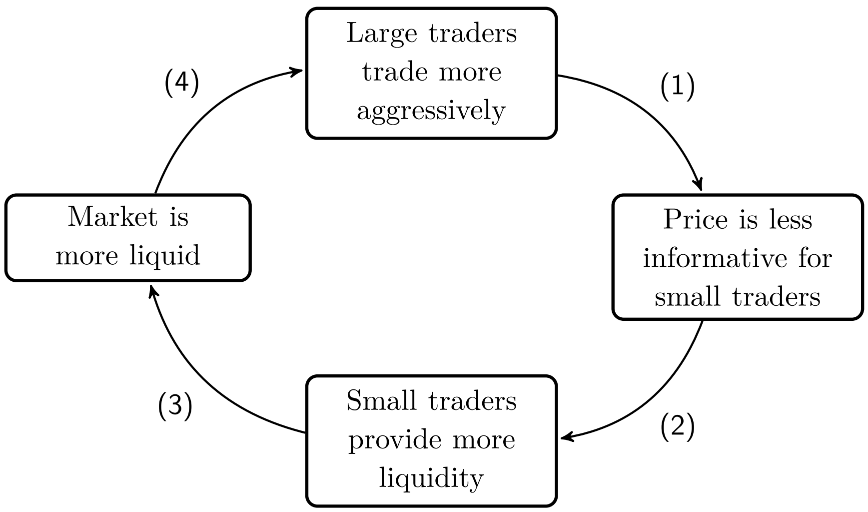


Figure 1: Trading complementarity when large traders create noise.

Next, we turn to the other direction of the trading complementarity and show that small traders' liquidity provision makes large traders trade more aggressively.

Lemma 2 (Small traders' liquidity provision enhances liquidity and encourages large traders to trade more aggressively)**.** *Fix the parameters* $(\alpha_S, \beta_S, \gamma_S)$ *in small traders' demand schedules. The demand of a large trader* i *is given by* $x_i = (v_L - p)/(w_L + \lambda)$, *and his trading aggressiveness is given by*

$$\beta = \frac{1}{w_L + \lambda}.$$

Ceteris paribus, an increase in γ_S *reduces price impact* λ, *whereby increases liquidity* $\mathcal{L}$ *and* β.

Lemma 2 describes the third and fourth steps in Figure 1. It is intuitive that small traders' liquidity provision makes the market more liquid, reducing the price impact faced by the large traders. The large traders then take advantage of the decreased price impact by trading more aggressively.

In sum, the two parts of our proposed mechanism generate a new type of complementarity. As large traders trade more aggressively, prices become less informative to small traders. Less informative prices induce small traders to provide more liquidity, which then induces large traders to trade even more aggressively. The overall equilibrium is characterized in the following theorem.

Theorem 1 (Equilibrium characterization)**.** *There exists at least one equilibrium. All equilibrium variables can be expressed in closed form by way of an endogenous variable* $\delta \equiv \sqrt{\tau_\pi/\tau_\varepsilon}$. *In particular, price impact can be expressed as*

$$\lambda(\delta) = \frac{N w_S}{\sqrt{\tau_L}}\left(\delta\sqrt{\tau_\varepsilon(1-\rho^2)} - \rho\sqrt{\tau_S}\right)\left(\delta^2 + \frac{\tau_S + \tau_\varepsilon}{\tau_\varepsilon}\right) - w_L.$$

The expressions for other equilibrium variables are presented in Appendix A.3. The equilibrium δ *is the solution to the sixth-order polynomial equation*

$$\lambda(\delta)(w_L + N w_S + \lambda(\delta)) - w_S\left(1 + \delta\left(\delta - \frac{\rho}{\sqrt{1-\rho^2}}\sqrt{\frac{\tau_S}{\tau_\varepsilon}}\right)\right)(w_L + 2\lambda(\delta)) = 0, \tag{7}$$

such that $\lambda(\delta) > -w_L/2$.

We end this section by providing sufficient conditions for the equilibrium uniqueness.

Proposition 1. *The equilibrium is unique for large enough* N.

We have shown that trading complementarity arises when large traders create noise. Proposition 1 states that when the large traders have little market power, the complementarity is sufficiently weak and hence the equilibrium is unique. When there are many large traders, each of them has little price impact. Additional liquidity provided by small traders would not affect large traders' trading aggressiveness much.

In the next three sections, we explore various implications of the trading complementarity.

5 Competition and Welfare

Suppose that due to splits or entries, large traders become more competitive. Will small traders be better off and will aggregate welfare then increase? The answer is yes, according to the conventional understanding of competition, which forces larger traders to offer better terms to small traders. In this section, we show that this conventional wisdom is no longer necessarily valid when large traders create noise, especially when the informational frictions are severe.

In general, there are two ways to increase the degree of competition among large traders: either by breaking up existing traders or by entry of new ones. While all our results regarding competition hold in both formulations, breakup of large traders is our preferred analysis because it does not change the total risk-bearing capacity of large traders and thus helps to isolate the effect of competition. Formally, a breakup constitutes an increase in the number of large traders N, while the cost parameter w_L is defined as $w_L \equiv N/c_L$, where the constant c_L is equal to the aggregate risk-bearing capacity of large traders (N/w_L). In this way, we keep constant the total amount of potential noise to be injected by large traders. We maintain the assumption that $w_L = N/c_L$ hereafter.

Next, we turn to welfare. Denote $\mathcal{U}_L$ and $\mathcal{U}_S$ as the ex-ante expected utility of a large and a small trader, respectively. Then the social welfare is defined as the sum of all traders' ex-ante utilities, $\mathcal{W} \equiv N\mathcal{U}_L + \mathcal{U}_S$.

To clearly show the impact of competition on welfare, it is useful to compute the welfare in the frictionless, *first-best* (FB) benchmark and then study the distortions caused by the frictions. In the first-best benchmark, all traders take prices as given and know their values perfectly; that is, there is neither market power nor informational friction. Traders bid according to their marginal utilities, so that $x_j = (v_S - p)/w_S$ for all $j \in [0, 1]$ and $x_i = (v_L - p)/w_L$ for all

$i \in \{1, 2, ..., N\}$. It is then immediate to show that the equilibrium allocations to small and large traders are given by

$$x_S^{\text{FB}} = \frac{v_S - v_L}{w_S + w_L/N} \quad \text{and} \quad x_L^{\text{FB}} = -\frac{x_S}{N},$$

respectively. Welfare is then given by

$$\mathcal{W}^{\text{FB}} = \frac{E[(v_L - v_S)^2]}{2(w_S + w_L/N)}.$$

The next proposition characterizes *welfare loss* (WL)—that is, the difference between the first-best and the equilibrium welfare. It shows how market power and noise distort allocations and cause inefficiency. We note that under breakup or merger, changes in N do not affect w_L/N and hence $\mathcal{W}^{\text{FB}}$. Therefore, the comparative statics of WL with respect to N are the same as that of $\mathcal{W}$.

Proposition 2. *The welfare loss* $\text{WL} \equiv \mathcal{W}^{\text{FB}} - \mathcal{W}$ *can be expressed as*

$$\text{WL} = \frac{w_S + w_L/N}{2} E[(x_S^{\text{FB}} - \langle x_S \rangle)^2] + \frac{1}{2} w_S E[(x_j - \langle x_S \rangle)^2]. \tag{8}$$

Here, $\langle x_S \rangle \equiv \int_0^1 x_j \, dj$*, the average allocation to small traders, is given by*

$$\langle x_S \rangle = \psi \cdot (x_S^{\text{FB}} + b), \tag{9}$$

where

$$\psi = \frac{w_S + w_L/N}{w_S + (w_L + 1/\mathcal{L})/N}, \quad b = \frac{\langle v_S \rangle - v_S}{w_S + w_L/N}, \quad \langle v_S \rangle = \int_0^1 E[v_S | p, s_j] \, dj.$$

The allocation to a small trader j *is given by*

$$x_j = \langle x_S \rangle + \beta_S \varepsilon_j. \tag{10}$$

Expression (8) for welfare loss, as derived in Vives (2017), incorporates two sources of inefficiency. The first term in (8) captures the welfare loss due to the deviation of the average allocation $\bar{x}_S$ from the x_S^{FB}. The deviation, as characterized in (9), is specific to this model and is closely related to the two aspects of market quality. First, a lack of informational efficiency causes the average small trader's forecast $\langle v_S \rangle$ to differ from the true value v_S, and that difference contributes to a *bias* b in (9). Second, a lack of liquidity causes large traders to reduce their demand, so the allocation is scaled down by a factor $\psi \in (0, 1)$. We refer to ψ as *scaled liquidity* as it increases with $\mathcal{L}$ and approaches zero (unity) as $\mathcal{L}$ approaches zero (infinity).

The second term in (8) captures the welfare loss due to the deviations of small traders' allocation from the average allocation $\langle x_S \rangle$. Such deviations, which are absent in the first-best, reflect the imperfect risk-sharing among small traders. As shown in (10), small traders in equilibrium hold allocation dispersed around the average because they put some weight on the signals and thus the idiosyncratic noise therein. Indeed, the deviations increase in the weight on signal β_S and the noise in the signal ϵ_j.

By identifying the equilibrium distortions, we can decompose the welfare loss in different components and study how each of these components is affected by competition.

Proposition 3. *The welfare loss can be decomposed into four terms,* $\text{WL} = \text{WL}_1 + \text{WL}_2 + \text{WL}_3 + \text{WL}_4$, *where*

$$\text{WL}_1 \equiv (1-\psi)^2 \mathcal{W}^{\text{FB}}, \quad \text{WL}_2 \equiv \frac{\psi^2 E[(v_S - \langle v_S \rangle)^2]}{2(w_S + w_L/N)},$$
$$\text{WL}_3 \equiv -\left(w_S + \frac{w_L}{N}\right)\psi(1-\psi)\operatorname{Cov}(b, x_S^{\text{FB}}), \quad \text{WL}_4 \equiv \frac{w_S}{2} E[(x_j - \langle x_S \rangle)^2].$$

WL_1 *is decreasing in* N*, whereas* WL_4 *is increasing in* N*. If* $\operatorname{Var}(v_S|s_j, p)^{-1} > 2\tau_\varepsilon$*, which is implied by* $\tau_\varepsilon < \tau_S$*, then* WL_2 *increases in* N*. Suppose that* $(\rho^2 - 1)\,\tau_\epsilon + \rho\sqrt{\tau_L \tau_S} - \tau_S > 0$*, then for sufficiently small* $h = w_L/(N w_S)$ *there exists equilibrium in which* WL *increases in* N*.*

Such equilibrium is unique for N large enough.

Proposition 3 is the key result in understanding how competition affects welfare loss and thus equilibrium welfare. The decomposition is derived as follows. Consider the deviation of $\langle x_S \rangle$ from the first-best allocation x_S^{FB}. According to (9), that deviation can be expressed as

$$x_S^{\mathrm{FB}} - \langle x_S \rangle = (1-\psi) x_S^{\mathrm{FB}} - \psi \cdot b. \tag{11}$$

The first term displayed in Proposition 3, WL_1, is proportional to $E[(1-\psi)^2 (x_S^{\mathrm{FB}})^2]$ and corresponds to the first term in equation (11). This term decreases with N and is in line with the standard result in industrial organization (see, e.g., Tirole 1988; Ausubel, Cramton, Pycia, Rostek, and Weretka 2014): with more competition, large traders offer better terms to small traders, and thus more trades between the groups occur.

The second term WL_2 is central and most noteworthy to this paper. It is proportional to $E[\psi^2 b^2]$ and corresponds to the second term in equation (11). Importantly, it increases with competition when the informational frictions are severe enough, for example, when the private signals are noisy enough. The intuition is as follows. Endowed with dispersed signals, small traders rely on the price to infer the asset value v_S, and such reliance leads to a common bias b due to the noise created by the large traders. When the large traders become more competitive, they trade more aggressively, and, by the trading complementarity, noise and liquidity reinforce each other, increasing both b and ψ. Finally, if the private signals are imprecise enough, the small traders rely heavily on the price and then it is guaranteed that they suffer from the increased bias.

The third term WL_3 is proportional to $\mathrm{Cov}((1-\psi)x_S^{\mathrm{FB}}, \psi \cdot b)$ and is due to the interaction between the two terms in (11). It states that, if bias b is (on average) positive when x_S^{FB} is positive (i.e., if $\mathrm{Cov}(b, x_S^{\mathrm{FB}}) > 0$), then the bias can partly compensate for the "scaling down" effect due to illiquidity and thereby reduce the corresponding loss in welfare. Numerical simulations reveal that WL_3 can either decrease or increase with N, depending on the parameters chosen.

The fourth and last term WL_4 is proportional to $E[(x_j - \langle x_S \rangle)^2]$ and arises from the dispersion of individual allocations x_j around the average allocation $\langle x_S \rangle$. This term is decreasing in informational efficiency: The higher the informational efficiency is, the less small traders load on their signals and on the noise in those signals. Since competition diminishes informational efficiency, it follows that WL_4 increases with N.

The analysis and discussion above suggest that competition could harm all investors' welfare. In particular, the channel of welfare loss that is closest to the heart of our mechanism, WL_2, should be more dominant when large traders and noise are more relevant. We confirm this intuition and show that the welfare loss increases with competition when $h = w_L/(Nw_S)$ is small (i.e., when large traders have relatively bigger risk-bearing capacity) and when $\tau_\varepsilon < \tau_S$ and τ_L is small (i.e., when small investors face high levels of informational friction). Figure 2 illustrates this unconventional result regarding competition and the importance of informational friction. Panel (a) shows that all traders are worse off when competition increases, and Panel (b) shows that when informational friction is small (τ_ε is high), competition is no longer welfare-decreasing.

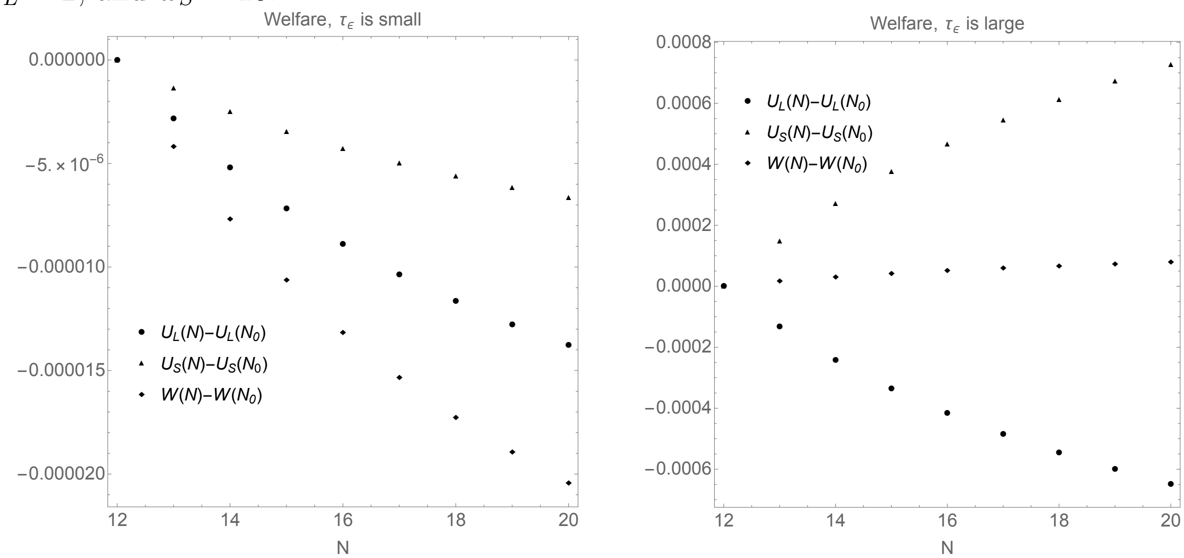
6 Quality of Private Information and Informational Efficiency

Suppose that small traders have signals of better quality (i.e., higher τ_ε). Will the price then be more informative for them? The intuitive answer is yes because the price that aggregates those more precise signals should likewise be more informative. In this section, we show that this conventional wisdom is no longer necessarily valid when large traders create noise.

We proceed with a formal result that shows that better signals lead to less informational

Figure 2: Effect of the extent of competition on welfare.

The graphs plot aggregate welfare $(\mathcal{W}(N) - \mathcal{W}(N_0))$, small traders' welfare $(\mathcal{U}_S(N) - \mathcal{U}_S(N_0))$, and large traders' welfare $(\mathcal{U}_L(N) - \mathcal{U}_L(N_0))$ as functions of N when $\tau_\varepsilon = 0.01$ (Panel (a)) and $\tau_\varepsilon = 1$ (Panel (b)). The welfare measures are normalized to zero at $N = N_0 = 12$. Other parameter values are $\bar{v}_L = \bar{v}_S = 0$, $a_S = 1$, $a_L = 0.6$ $\tau_v = 2$, $\tau_{u,S} = 0.2$, $\tau_{u,L} = 33$, $w_L = N/c_L$, $c_L = 2$, and $w_S = 10$.



(a) (b)

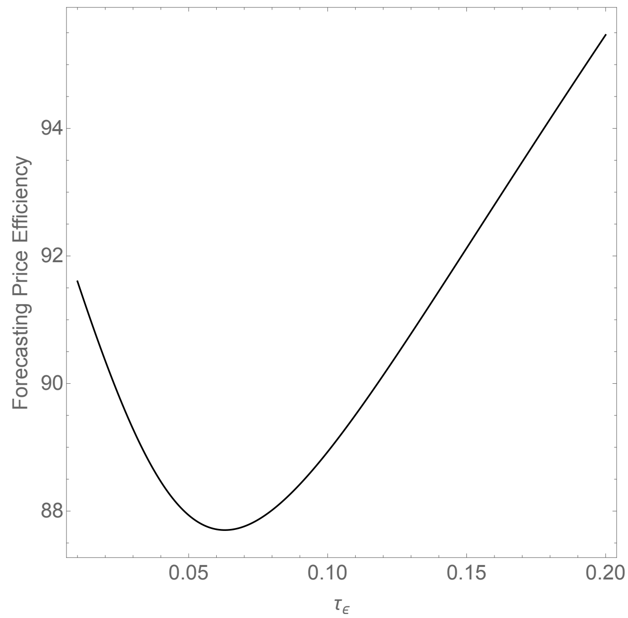
efficiency. In order to define fundamentals and measure informational efficiency properly, here we invoke Assumption 1, which puts more structure on the investors' values.

Proposition 4 (Better signals, less informational efficiency)**.** *Suppose that Assumption 1 holds. Suppose that* $2 \leq N < \bar{N} \equiv 2a_S/a_L$ *and* $h = w_L/(Nw_S) < \bar{h} \equiv 2 - \frac{2}{N} - \frac{a_L}{a_S}$. *For small enough* τ_ϵ, τ_v, *and* $\tau_{u,L}$, *there exists an equilibrium in which both* $\mathcal{I}^{FPE}$ *and* $\mathcal{I}^{RPE}$ *decrease as signal precision* τ_ε *increases. Such equilibrium is unique, provided that condition* (26) *in the Appendix holds.*

The unconventional result stated in Proposition 4 follows from the trading complementarity. When the signals become more precise, small traders face less adverse selection and provide more liquidity. The improved liquidity makes large traders trade more aggressively, injecting more noise in the price. The less informative price in turn reduces the small traders' adverse selection problem and thus further enhances liquidity. Such liquidity and noise feedback mechanism, as

Figure 3: Effect of precision τ_ε on informational efficiency $\mathcal{I}^{FPE}$.

The graph plots informational efficiency $\mathcal{I}^{FPE}$ as a function of τ_ε. Parameter values: $N = 9$, $\bar{v}_L = \bar{v}_S = 0$, $a_L = 0.8$, $\tau_{u,S} = 1$, $\tau_{u,L} = 1$, $\tau_v = 0.01$, $\rho = 0.9$, $w_L = 0.2$, and $w_S = 2$.



illustrated in Figure 1, can worsen informational efficiency to an extent that dominates the positive effect brought by the improved signals.

The conditions stated in Proposition 4 ensure that the trading complementarity is strong enough to achieve the unconventional result. The condition that $h = \frac{w_L}{N w_S}$ is small enough can be understood as large traders representing a significant part of the market. That is, their aggregate risk-bearing capacity $1/w_L$ is large compared to the small traders' capacity $1/w_S$. This condition reinforces steps (1) and (2) in Figure 1. Condition $N < \bar{N}$ ensures that large traders have market power, reinforcing the link between market liquidity and their trade aggressiveness (step (4) in Figure 1). The remaining conditions, τ_ε, $\tau_{u,L}$, and τ_v are small enough, and correspond to severe informational friction. They imply that the small traders face substantial uncertainty in the asset values ex-ante and rely heavily on learning from the price. They therefore reinforce steps (2) and (3) in Figure 1. Figure 3 provides a numerical illustration that informational efficiency decreases in signal precision when the informational friction is severe, that is, when τ_ε is low.[14]

We also note that the condition $2 \leq N < 2a_S/a_L$ can only hold if $a_L < a_S$. The condition ensures that the equilibrium of interest discussed above exists.[15] The condition of $a_L < a_S$ also captures an economically interesting application in which large investors having stronger preferences for ESG outcomes than small ones. See the discussion preceding Assumption 1.

Remark 3 (On Endogenous Information Acquisition). Suppose that the quality τ_ε of private information is not exogenously given; instead, small traders choose the precision τ_ε, subject to an increasing cost $C(\tau_\varepsilon)$. Now, provided that the conditions of Proposition 4 are satisfied, there is a complementarity in information acquisition. If other small traders acquire more information, by Proposition 4, informational efficiency will decrease, thereby encouraging the small trader of interest to acquire more information as well. This complementarity implies that the main result of this section is not only robust but could also be reinforced when information is endogenous.

7 Multiple Equilibria and Market Fragility

In this section, we study the implications of equilibrium multiplicity.

The trading complementarity depicted in Figure 1 suggests that expectations can be self-fulfilling and lead to multiple equilibria. When large traders expect small traders to supply more liquidity, they respond by trading more aggressively and inject more noise into the price. As a result, small traders indeed find it optimal to provide more liquidity. Likewise, when

[14]We note that the strong complementarity needed to generate the unconventional result might also generate multiplicity of equilibria. There is a tension between the condition $\frac{w_L}{Nw_S} < \bar{h}$ for the result and the condition (26) for uniqueness. One can nonetheless show that the set of parameters satisfying both conditions is non-empty. In fact, the set of parameters used for Figure 3 is an example.

[15]The intuition for why $a_L < a_S$ is needed is as follows. In the equilibrium described in Proposition 4, as τ_ϵ gets smaller, the price impact becomes very large. As a result, large traders trade very little, and the price mostly reflects small traders value, $p \approx v_S$. The market-clearing condition implies that $cov(\langle x_S \rangle, v) + cov((v_L - p)/(w_L + \lambda), v) = 0$. Since we have $cov(\langle x_S \rangle, v) > 0$ (small traders buy when the signals tells them v is high, and vice versa when v is low), we need $cov(v_L - p, v) < 0$ for the market to clear. For that we have $cov(v_L - p, v) \approx cov(v_L - v_S, v) = \tau_v^{-1}(a_L - a_S)$, which is negative iff $a_L < a_S$.

small traders expect large traders to trade more aggressively, they supply more liquidity, which encourages large traders to trade more aggressively. The market is therefore *fragile* in the sense that market outcomes such as liquidity and informational efficiency can vary substantially simply due to changes in traders' expectations.

There are two insights from the analyses in this section. First, market power, in the presence of informational friction, is a source of fragility in financial markets. Second, when multiple equilibria exist, they can be ranked by liquidity and in the reversed order by informational efficiency. The latter unveils a tension between equilibrium liquidity and informational efficiency.

We proceed by characterizing the sufficient conditions for equilibrium multiplicity, which also underpins the first insight.

Proposition 5. *For any $N > 4$, there exist constants $\bar{w}$, $\underline{\tau}_2$, and $\bar{\tau}_2 > \underline{\tau}_2$ such that, if*

$$w_L < \bar{w} \quad \textit{and} \quad \underline{\tau}_2 < \tau_L < \bar{\tau}_2,$$

then there exist at least three distinct equilibria.

The condition $\underline{\tau}_2 < \tau_L$ ensures that the price is not too noisy, and thus small traders rely on it for inferences. This reliance implies that changes in price informativeness affect how much liquidity the small traders provide. The condition $\tau_L < \bar{\tau}_2$ ensures that the price is not so informative that changes in the amount of noise injected by large traders still matter. Finally, the conditions $w_L < \bar{w}$ ensure that large traders constitute a substantial fraction of the market; that is, their aggregate risk-bearing capacity N/w_L is large, and hence they have a significant effect on price informativeness. In short, it is the *combination* of market power and informational frictions that generate fragility. Recall from Proposition 1 that, if either of these forces is weakened, then the equilibrium is unique.

Next, we turn to the tension between equilibrium liquidity and informational efficiency.

Proposition 6. *Suppose the model's parameters are such that there exist multiple equilibria.*

Consider two equilibria, A and B, and suppose that liquidity is lower in equilibrium A than in equilibrium B. Then $\mathcal{I}^{RPE}$ is greater in equilibrium A than in equilibrium B. Suppose in addition that Assumption 1 holds. Then $\mathcal{I}^{FPE}$ is also greater in equilibrium A, provided that $h = w_L/(Nw_S)$ is small enough.

This proposition establishes that equilibria can be ranked by liquidity in one order and by informational efficiency in the reversed order. This negative relationship between equilibrium liquidity and informational efficiency, as far as we know, is not commonly found in other models of fragility and hence can be served as a distinct empirical prediction of our mechanism.

One might be interested in the welfare rankings of equilibria A and B. Our numerical analysis shows that equilibrium A (the one with lower liquidity) tends to have lower welfare in the environment, with relatively less severe informational frictions (so that liquidity, not informational frictions, affects the welfare the most). The opposite is true when the informational frictions are relatively high. Thus, in our model, liquidity is an imperfect proxy for welfare: It is not always true that higher liquidity and higher welfare go hand-in-hand.

8 Interpretations of the Model

In this section, we provide three interpretations of the model. The goal of the exercise is to map the general results of the model into more concrete applications.

8.1 A model with institutional and retail investors

The first interpretation consists of large institutional investors trading with small retail investors in stock markets. There is ample evidence that institutional investors, unlike retail investors, can affect prices and take their price impact into account when trading.[16] The differences in valuations of the stocks could be motivated by the fact that institutional investors' demands

[16]See, e.g., Griffin, Harris, and Topaloglu (2003).

are affected by fund flows. As shown in Coval and Stafford (2007) and subsequent papers, mutual funds tend to sell (buy) assets following outflows (inflows). To capture this, consider the environment from Section 3. Assume that the asset's fundamental value is v and that $v_S = v$, whereas $v_L = v + u$. Here, the private value component u is meant to capture flow concerns of institutional investors. Thus, a positive realization of u would generate additional buying pressure from large traders, corresponding to fund inflows, while a negative realization u corresponds to outflows. The flows u are known to institutional investors but not known to retail traders at $t = 0$. As institutional investors are often better informed than retail investors, we assume that the former know v perfectly.

8.2 A model with fast and slow traders

The second interpretation is about a model with fast and slow traders. There are three dates, $t \in \{0, 1/2, 1\}$. Two trader groups, *high-frequency traders* (HFTs) and *conventional (slow) traders* trade two assets: a risky asset (a stock) and a risk-free asset (a bond). The stock pays a terminal dividend $v \sim N(0, \tau_v)$ at time $t = 1$. The bond is the numeraire; hence its net return is zero. There are $L > 2$ HFTs, indexed by $i \in \{1, 2, ..L\}$, and a unit continuum of slow traders, indexed by $j \in [0, 1]$. Below we will map the HFTs to large traders in the model from Section 3 and conventional traders to small traders.

We emphasize two properties that distinguish HFTs from conventional traders in practice: (i) they trade more frequently and (ii) they often employ strategies that exploit order flow predictability. To capture (i), we assume that HFTs can trade at time $t = 0$ and $t = 1/2$, while slow traders can trade only at $t = 0$. To capture (ii), we assume that there is an exogenous order flow $Z_{1/2} \sim N(0, \tau_{z_{1/2}})$ coming to the market at time $t = 1/2$ and that HFTs have a common signal $\eta = Z_{1/2} + \epsilon_Z$ about this future order flow.[17] The order flow $Z_{1/2}$ generates gains from trade both at $t = 1/2$ and $t = 0$. We also assume that HFTs have existing inventory

[17] Our qualitative results will not change if large traders have dispersed information about $Z_{1/2}$.

$Z_0 \sim N(0, \tau_{z_0})$, which could, for example, come from the order flow absorbed prior to $t = 0$. We assume that Z_0, $Z_{1/2}$ and v are jointly normally distributed and independent of each other.[18] Conventional traders are endowed with dispersed signals about the fundamental v: $s_j = v + \epsilon_j$, where ϵ_j's are i.i.d. $\epsilon_j \sim N\left(0, \frac{1}{\tau_\epsilon}\right)$, and also independent of all other random variables in the model. We assume that HFTs know v.[19]

Remark 4 (Trading at $t = 1/2$). We motivate the assumption that only HFTs trade at $t = 1/2$ by their superior trading speed. Alternatively, one can view the trading at $t = 1/2$ as trading in a market that conventional traders do not have access to. For example, many market makers get order flow from the brokers directly—a practice known as payment for order flow. Trading at $t = 1/2$ can be viewed as competition for order flow $Z_{1/2}$ among market makers.

The traders are risk-neutral and have quadratic inventory costs. They are fully rational, that is, Bayesian, and take their price impact into account. Consider a large trader i who enters period $t = 1/2$ with inventory x_0^i. A large trader i solves the following problem at $t = 1/2$:

$$\max_{x(p)} v \cdot (x + x_0^i) - px - \frac{w(x + x_0^i)^2}{2}, \tag{12}$$

where the term $\frac{w(x+x_0^i)^2}{2}$ represents an inventory cost of holding $x + x_0^i$ units of asset. At time $t = 1/2$, a large trader maximizes (1) by submitting the supply schedule $x(p)$, taking his price impact into account. As we show below, his value function at $t = 1/2$ will depend on his inventory x_0^i after the trade at $t = 0$ as well as the vector of inventories of other large traders, x_0^{-i} and the order flow Z. We denote this value function by $V_i(x_0^i, x_0^{-i}, Z)$.

At time $t = 0$ large trader i solves

$$\max_{x(p)} v \cdot (Z_0 + x) + E[V_i(Z_0 + x, x_0^{-i}(p), Z)|\eta] - px - \frac{w(Z_0 + x)^2}{2}.$$

He takes into account his price impact as well as the impact of his trade on allocations to other

[18] Our results can be easily generalized to allow for correlation between Z_0, $Z_{1/2}$, and v.

[19] As with the model from Section 3, the main results still hold if HFTs do not know v. See Section IA.4.

traders $x_0^{-i}(p)$.

Similarly, a conventional trader i solves the following problem at $t = 0$:

$$\max_{x(p)} \left(E[v|p, s_i] - p\right) x - \frac{w_S x^2}{2}. \tag{13}$$

The trading is structured as a uniform-price double auction. Each trader k submits his net demand schedule $x_k(p)$, where $x_k(p) > 0$ ($x_k(p) < 0$) corresponds to a buy (sell) order. The market-clearing price p^* is one at which the net aggregate demand is zero. Thus, at $t = 1$ we have

$$\sum_{i=1}^{N} x_i\left(p^*\right) + \int_0^1 x_j\left(p^*\right) dj = 0. \tag{14}$$

At $t = 1/2$ we have

$$\sum_{i=1}^{N} x_i\left(p^*\right) = Z. \tag{15}$$

The equilibrium concept is Bayesian Nash, as in Kyle (1989) and Vives (2011): In every period, traders maximize expected utility, given their information and accounting for their price impact; equilibrium demand schedules are such that the market clears.

We are now ready to state the central result of this section.

Proposition 7. *Consider two economies. Call economy A the one described in this section and economy B the one in Section 3, with conventional traders being small traders, with $v_S = v$ and inventory costs $w_S x^2/2$ and HFTs being large traders, with $v_L = v - wZ_0 - wE[Z_{1/2}|\eta]\left(\frac{2}{L} - \frac{1}{L(L-1)}\right)$ and inventory costs wx^2. Economies A and B are equivalent in terms of traders' time-0 equilibrium demands and measures of liquidity and informational efficiency. The aggregate time-0 welfare in economies A and B are different by a constant that does not depend on N.*

The equivalence established above implies that all results already established for economy B are also applicable to the economy A. In particular, even though welfare in economy A

differs from that in B, the results still apply because the difference does not depend on N. We note that the model described here features traders with common values, and yet it is equivalent to the model with private values in Section 3. The HFTs trade *as though* their values consist of a common value component v and a private value component, which stems from the HFTs' hedging needs: $-wZ_0$ captures their desire to hedge existing inventories and $-wE[Z_{1/2}|\eta]\left(\frac{2}{L}-\frac{1}{L(L-1)}\right)$ for future order flow. The latter term highlights that a novel source of noise in prices: HFTs make today's price noisier by incorporating the randomness in future order flow $Z_{1/2}$ and the noise in the order flow signal η.

8.3 A commodity market

The third interpretation involves large and small traders trading a commodity (e.g., crude oil or aluminium). The large traders are commodity *producers*. The small traders are *firms* that buy the commodity to produce a final good.

The production technology employed by commodity producers is characterized by the *convex* cost function

$$v_L \cdot y + \frac{w_L}{2}y^2, \tag{16}$$

where $v_L \sim N(\bar{v}_L, 1/\tau_{v_L})$ is a cost shock that is known to producers but not to firms. Thus, producers are better informed about their own production technology than firms are. Producers are risk-neutral and maximize their profit as follows:

$$p \cdot y - \left(v_L y + \frac{w_L}{2}y^2\right).$$

The term y in this expression is the amount of the commodity sold or the net supply. The net demand of producers is $x = -y$, and substituting this equation into the preceding display yields

$$(v_L - p)x - \frac{w_L}{2}x^2. \tag{17}$$

This profit expression conforms with the utility equation (1).

Firms $j \in [0,1]$ have a production technology characterized by the *concave* production function of the final good,

$$Y(x) \equiv v_S \cdot x - \frac{w_S}{2} x^2, \tag{18}$$

where $v_S \sim N(\bar{v}_S, 1/\tau_{v_S})$ is a productivity shock common to all firms. Such shock drives the aggregate output of the economy and thus can be interpreted as the strength of the economy. Firms have dispersed information about the economy's strength. In particular, each firm j is endowed with a signal

$$s_j = v_S + \varepsilon_j,$$

where $\varepsilon_j \sim N(0, 1/\tau_\varepsilon)$ is independent of all other random variables in the model. Firms are risk-neutral and maximize their expected profits,

$$p_g \left(v_S \cdot x - \frac{w_S}{2} x^2 \right) - p \cdot x, \tag{19}$$

where $p_g = 1$ is the price of the final good (endogenized in what follows) and p is the commodity's price. The expression (19) conforms with the utility equation (2).

We close the model by assuming that the final good is sold to consumers $l \in [0,1]$, who have a linear Marshallian utility function over the amount z of the final good consumed and over the remaining cash $m = m_0 - p_g z$ left after purchasing the final good,

$$u_l(z, m) = z + m_0 - p_g z,$$

where m_0 represents each consumer's endowment of cash. The existence of a continuum of consumers implies that they are price takers and that the final good's price is equal to their marginal utility; thus, indeed, $p_g = 1$.

The setting considered here is a natural framework for the study of commodities markets.

The linear-quadratic specification of the cost and of the production functions is common in the commodities literature.[20] The information structure—with a cost shock known to producers but not to firms and where firms have dispersed information regarding the strength of the economy—is the same as in Sockin and Xiong (2015). Our setting generalizes Sockin and Xiong (2015) by allowing producers to have market power, which is clearly relevant in commodities markets.[21]

9 Discussion

In this section, we discuss evidence consistent with the core mechanism and the key empirical and normative implications of our results.

9.1 Evidence on noise-creating large traders

The premise of the paper is that large traders make prices less informative. Our primary example is large institutional investors who increasingly put more weights on firms' ESG metrics when constructing their investment portfolios. A notable example is the Net Zero Asset Managers initiative, which is an international group of asset managers, with a total of $57.5 trillion in assets under management, committed to achieve net zero alignment by 2050 or sooner.[22] In this context, the premise of our mechanism is that the demand of these large, ESG-conscious investors would add noises to stock prices. We present supportive evidence in Appendix B

[20]See, e.g., Grossman (1977), Kyle (1984), Stein (1987), and Goldstein and Yang (2017).

[21]In the crude oil market, e.g., OPEC accounts for more than 40% of world production (Fantini (2015)); in the aluminum market, the six largest producers account for more than 40% of world production (Nappi 2013).

[22]See https://www.netzeroassetmanagers.org/. More broadly, recent surveys on active asset managers find that ESG is important and is not mainly for financial reasons. In a survey in 2021 by PwC, 79% of active asset managers say that "ESG risks are an important factor in investment decision-making." In another survey by Natixis, the two most cited motivations for ESG are "to align investment strategies with organizational values" and "to influence corporate behavior." Sources: https://www.pwc.com/gx/en/issues/reinventing-the-future/take-on-tomorrow/download/sbpwc-2021-10-28-Economic-realities-ESG.pdf and https://www.im.natixis.com/us/resources/2021-esg-investor-insight-report-executive-overview.

by showing that firms with higher ESG scores, presumably being held more by ESG-conscious investors, have stock prices that are less informative about fundamentals.[23]

There is existing evidence consistent with noises created by large institutional investors. Ben-David et al. (2021) show that ownership by the largest 10 institutions in the U.S. equities market is associated with more noise in prices. Furthermore, Ben-David et al. show that, along with concentration, these effects have become more pronounced over time. These largest institutional investors include mutual funds, and it is a widely used argument that fund redemption leads to fire sales of portfolio stocks, adding noise in prices (Coval and Stafford, 2007; Edmans et al., 2012).

Other likely candidates for noise-creating large investors are high-frequency traders. Using Swedish equities data, Baron et al. (2019) document substantial and persistent concentration in terms of trading revenues and volume among HFTs. In addition, the five fastest HFTs consistently have higher risk-adjusted performance than others, suggesting that competition is imperfect. Meanwhile, Weller (2018) uses price-jump ratio at earnings announcement and Gider et al. (2019) use the measure of price informativeness developed in Bai et al. (2016) to show that HFTs activities reduce price informativeness about firm fundamentals.

9.2 Empirical predictions

Our paper delivers two main set of testable predictions. The first set of predictions is in regards to the market power of noise-creating large traders. Proposition 8 in the Appendix stipulates that an increase in large traders' market power (a decrease in N), due to, for example, a merger between two large institutional investors, leads to higher price informativeness and lower liquidity. To the best of our knowledge, these predictions have not been directly tested.

The second set of predictions is about the effect of quality of private information on informational efficiency. Proposition 4 suggests that an increase in quality of private signals

[23] See also Goldstein et al. (2022) for a competitive REE model with financial and green investors.

could lower price informativeness when informational friction is large enough. This result helps explain the evidence presented by Farboodi et al. (2020) and Bai et al. (2016); these authors show that, despite the prices of stocks in the S&P 500 index becoming more informative in recent decades, the price informativeness of stocks that are *not* in that index has fallen. This evidence is puzzling when one considers that technological progress has made information about all stocks more easily available (so the quality τ_ε of private information should have increased, on average), which suggests that price informativeness should likewise have increased for all stocks. One implication of our model is that the opposite may be true for stocks that are less transparent—namely, those with lower quality of private information (τ_ε) and of public information (τ_S). Stocks of that type are likely to be smaller, less liquid, and less "glamorous" than those covered by the S&P 500 index.

9.3 Implications on transparency and competition policies

Our paper bears implications for transparency policies, which aim to reduce informational frictions, and competition policies, which promote efficient trading by reducing large traders' market power. Our key message is that in order to achieve any one of the policy goals, both market characteristics, namely, informational friction and market power, have to be considered. In particular, policies that promote transparency could reduce price informativeness when large traders have substantial market power.[24] Also, promoting competition could lead to lower welfare if informational friction is severe enough.

Regulations such as the Sarbanes–Oxley Act and Regulation Fair Disclosure (Reg FD) have been made with the aim of increasing the transparency publicly traded firms to make market information widely accessible. We interpret improved transparency as an increase in the precision of investors' signals in our model. The argument is that better disclosure of

[24]Goldstein and Yang (2019) also show that better disclosure of public signal can harm price informativeness when there are multiple dimensions of information. Our mechanism is different, as we have single dimension of information and emphasize the effect of market power.

firms' balance sheet in the Sarbanes-Oxley Act allows investors to better evaluate the firms' fundamental values. In addition, as Reg FD prohibits selective disclosure to certain investors, such as large institutional investors, it can be seen as leveling the playing field and improving the small investors' information. Then, Proposition 4 implies that transparency can harm price informativeness, thus paradoxically hurting small investors' overall ability to predict firm fundamentals. This suggests that when designing transparency promoting policy, the industrial organization aspect of financial markets should be taken into account.

It is a widely held view that competition is beneficial for welfare, and this view underlies antitrust policies worldwide.[25] In light of the increasing concentration in financial markets, regulators like the SEC have expressed their concerns of market power in financial markets (see footnote 5). However, our paper shows that this received wisdom need not be valid. In such circumstances, Proposition 3 states that (see also Figure 2) increased competition can reduce the welfare not only of large traders with market power but also of small traders—that is, by making prices less informative for them. Meanwhile, the result in Section 7 also highlights a novel financial stability motive of reducing market power in financial markets. Overall, both of these unconventional welfare and fragility results arise when informational friction in the financial market is severe enough, highlighting the importance of considering informational friction in the design of competition policies.

9.4 Robustness and extensions

We end this section with a brief discussion of the additional analyses about the robustness of our results and extensions of the model. These analyses are reported in the Internet Appendix.

We establish the critical importance of large traders' being strategic, that is, not taking prices as given (Section IA.1). Then, we show that our mechanism remains at work when $\rho \to 1$, that is, large traders creating little noise to small traders (Section IA.2). Finally, we

[25] See, e.g., the "Guide to Antitrust Laws," available on the U.S. Federal Trade Commission website (https://bit.ly/3wMKGKI).

demonstrate that our main results continue to hold under the following modifications of the model: when small traders receive dispersed signals about asset fundamental v instead of their value v_S (Section IA.3), when large traders also learn from prices (Section IA.4), when traders' private values are heterogeneous (Section IA.5), when there is an extra round of trading (Section IA.6).

10 Conclusion

Many financial markets nowadays are dominated by a handful of large investors. The market power possessed by these investors has attracted the attention of regulators worldwide. In this paper, we provide a framework to analyze these large investors' impact on the functioning of financial markets and welfare. Crucially, we posit that these large investors sometimes trade for non-fundamental reasons, a phenomenon supported by evidence. As a result, they create noise for other, small investors who glean information from prices.

Our analysis focuses on the interaction between noise-creating large investors and small investors, from which novel empirical and normative implications are derived. There is a complementarity in their trading behavior: When large investors trade more aggressively, more noise is injected in the price. The resulting less informative prices induce small investors to provide more liquidity, which feeds back into more aggressive trading by large investors. We show that this complementarity can lead to two unconventional implications, namely, competition *reduces* welfare; and better information *harms* informational efficiency. Thus, our results suggest that competition policy should take into account of information friction and transparency policy should depend on the market power of large investors.

The model can be extended in several directions. Incorporating multiple assets would allow one to examine cross-asset trading complementarity and its implication. It would also be of interest to explore a dynamic extension, possibly adding an inter-temporal dimension to the feedback mechanism. These extensions are left for future research.

A Proofs

We start with the following lemma that is a useful building block for our results.

Lemma 3. *Large traders' values v_L can be decomposed as follows:*

$$v_L = A + Bv_S + C\zeta,$$

where $B = \rho\sqrt{\tau_S/\tau_L}$, $C = \sqrt{(1-\rho^2)/\tau_L}$, and $A = \bar{v}_L - B\bar{v}_S$. Also, $\zeta \sim N(0,1)$ is independent of v_S.

Proof of Lemma 3. One can check by direct calculation that $\zeta = 1/C(v_L - A - Bv_S)$ has a mean of 0, a variance of 1, and a covariance (with v_S) of 0. The combination of zero covariance and joint normality implies independence. ■

A.1 Proof of Lemma 1

Proof of Lemma 1. The price is informationally equivalent to $\beta_S v_S + N\beta v_L$. After substituting v_L from Lemma 3 and undertaking some rearrangement, we obtain that the price is informationally equivalent to $\pi = v_S + (1/\sqrt{\tau_\pi})\zeta$, where $\zeta = 1/C(v_L - A - Bv_S)$ (see Lemma 3) and

$$\tau_\pi = \frac{\tau_L}{1-\rho^2}\left(\rho\sqrt{\frac{\tau_S}{\tau_L}} + \frac{\beta_S}{N\beta}\right)^2. \tag{20}$$

The formula for informational efficiency now follows directly from the projection theorem. It can be seen from the formulas that both τ_π and $\mathcal{I}^{RPE}$ are decreasing in β.

The optimal demand of a small trader j can be written as $x_j = (E[v_s|s_j,p] - p)/w_S$. It then follows that $\gamma_S = \frac{1}{w_S}\left(1 - \frac{\partial E[v_s|s_j,p]}{\partial p}\right)$. One can write $E[v_s|s_j,p] = \frac{\tau_\pi}{\tau_S+\tau_\varepsilon+\tau_\pi}\pi + ...$, where "..." stands for terms that do not depend on p. One can also write $\pi = \frac{\gamma_S + N\gamma}{\beta_S + N\beta\rho\sqrt{\tau_S/\tau_L}}p + ...$, from which it follows (after some rearrangement) that

$$\gamma_S = \frac{\frac{1}{w_S} - \frac{1}{w_S}\frac{\tau_\pi}{\tau_S+\tau_\varepsilon+\tau_\pi}\frac{N\gamma}{\beta_S+N\beta\rho\sqrt{\tau_S/\tau_L}}}{1 + \frac{1}{w_S}\frac{\tau_\pi}{\tau_S+\tau_\varepsilon+\tau_\pi}\frac{1}{\beta_S+N\beta\rho\sqrt{\tau_S/\tau_L}}}.$$

Thus we can see that γ_S is decreasing in β. ■

A.2 Proof of Lemma 2

Proof of Lemma 2. The first-order condition for a large trader i yields (see, e.g., Kyle 1989; Vives 2011) $x_i = (v_L - p)/(w_L + \lambda)$; here $1/\lambda$ is the slope of the residual supply, $1/\lambda = \gamma_S + (N-1)\gamma$. The second-order condition is satisfied if and only if $\lambda > -w_L/2$. Hence

$\beta = \gamma = 1/(w_L + \lambda)$, and λ is determined by

$$\frac{1}{\lambda} = \frac{N-1}{w_L + \lambda} + \gamma_S.$$

It is easy to show that this equation's solution that satisfies $\lambda > -w_L/2$ is decreasing in γ_S. We can therefore conclude that also β is decreasing in γ_S. ■

A.3 Proof of Theorem 1

Proof of Theorem 1. The first-order conditions from Lemmas 1 and 2 can be summarized as follows:

$$x_j = \frac{E[v_S|s_j, p] - p}{w_S} \quad \text{and} \quad x_i = \frac{v_L - p}{w_L + \lambda}.$$

The second-order condition for large traders, $\lambda > -w_L/2$, must also hold.

According to Lemma 1,

$$\gamma_S = \frac{\frac{1}{w_S} - \frac{1}{w_S}\frac{\tau_\pi}{\tau_S+\tau_\varepsilon+\tau_\pi}\frac{N\gamma}{\beta_S+N\beta\rho\sqrt{\tau_S/\tau_L}}}{1 + \frac{1}{w_S}\frac{\tau_\pi}{\tau_S+\tau_\varepsilon+\tau_\pi}\frac{1}{\beta_S+N\beta\rho\sqrt{\tau_S/\tau_L}}}.$$

Given that $E[v_s|s_j, p] = \frac{\tau_\varepsilon}{\tau_S+\tau_\varepsilon+\tau_\pi} s_j +$ (terms that do not depend on s_j), we can also derive the equality $\beta_S = \frac{1}{w_S}\frac{\tau_\varepsilon}{\tau_S+\tau_\varepsilon+\tau_\pi}$. The first-order conditions for large traders imply that $\beta = \gamma = 1/(w_L + \lambda)$.

Next we express the coefficients β_S, γ_S, β, and γ through the endogenous variable $\delta = \sqrt{\tau_\pi/\tau_\varepsilon}$. It is immediate that

$$\beta_S(\delta) = \frac{1}{w_S}\frac{\tau_\varepsilon}{\tau_S + \tau_\varepsilon(1+\delta^2)}.$$

Theorem 1's expression for $\lambda(\delta)$ follows if we substitute $\beta_S = \beta_S(\delta)$ and $\beta = 1/(w_L + \lambda)$ into (20) and express λ. The terms $\beta(\delta)$ and $\gamma(\delta)$ are related to δ as

$$\beta(\delta) = \gamma(\delta) = \frac{1}{w_L + \lambda(\delta)}.$$

From that expression it follows, with regard to $\gamma_S(\delta)$, that

$$\gamma_S = \frac{\frac{1}{w_S} - \frac{1}{w_S}\frac{\tau_\varepsilon\delta^2}{\tau_S+\tau_\varepsilon(1+\delta^2)}\frac{N\gamma(\delta)}{\beta_S(\delta)+N\beta(\delta)\rho\sqrt{\tau_S/\tau_L}}}{1 + \frac{1}{w_S}\frac{\tau_\varepsilon\delta^2}{\tau_S+\tau_\varepsilon(1+\delta^2)}\frac{1}{\beta_S(\delta)+N\beta(\delta)\rho\sqrt{\tau_S/\tau_L}}}. \tag{21}$$

It remains to derive expressions for $\alpha(\delta)$ and $\alpha_S(\delta)$. The first-order condition for a large trader implies that $\alpha(\delta) = 0$. Given that $E[v_s|s_j, p] = \frac{\tau_S}{\tau_S+\tau_\varepsilon+\tau_\pi}\bar{v}_S +$(terms that depend on s_j and p), we have

$$\alpha_S(\delta) = \frac{1}{w_S}\frac{\tau_S}{\tau_S + \tau_\varepsilon(1+\delta^2)}\bar{v}_S.$$

The polynomial equation (7) for δ can be obtained by rearranging $\frac{1}{\lambda(\delta)} = \gamma_S(\delta) + (N-1)\gamma(\delta)$.[26]

We now prove that there is at least one solution to (7) such that $\lambda > -w_L/2$. Consider a unique δ^* satisfying $\lambda(\delta^*) = -w_L/2$. We can show that the polynomial (7) evaluated at $\delta = \delta^*$ is negative; at the same time, the polynomial's leading coefficient is positive. Hence the polynomial becomes positive for large enough δ. By the intermediate value theorem, there exists a $\delta^{**} > \delta^*$ such that the polynomial is zero. Since $\lambda(\delta)$ is increasing for $\delta > \delta^*$, we have $\lambda(\delta^{**}) > -w_L/2$. ■

A.4 Proof of Proposition 1

Proof of Proposition 1. The equilibrium δ solves the following system of equations:

$$\lambda = \frac{Nw_S}{\sqrt{\tau_L}}\sqrt{\tau_\varepsilon(1-\rho^2)}\left(\delta - \frac{\rho}{\sqrt{1-\rho^2}}\sqrt{\frac{\tau_S}{\tau_\varepsilon}}\right)\left(\delta^2 + \frac{\tau_S+\tau_\varepsilon}{\tau_\varepsilon}\right) - w_L, \tag{22}$$

$$\lambda(w_L + Nw_S + \lambda) - w_S\left(1 + \delta\left(\delta - \frac{\rho}{\sqrt{1-\rho^2}}\sqrt{\frac{\tau_S}{\tau_\varepsilon}}\right)\right)(w_L + 2\lambda) = 0, \tag{23}$$

$$\lambda > -\frac{w_L}{2}. \tag{24}$$

The idea behind the proof is as follows. Equation (22) gives the explicit expression for λ as a function of δ. We then combine (22) and (23) to obtain an explicit expression for $\delta(\lambda)$. Finally, we determine how many times the two functions intersect.

We derive an explicit expression for δ through λ by using (23) to write $\delta(\delta - (\rho/\sqrt{1-\rho^2})\sqrt{\tau_S/\tau_\varepsilon})$ as a function of λ and δ, which we can substitute into (22); we then derive δ from the resulting expression. Following these steps yields

$$\delta = \frac{(2\lambda + w_L)(N\rho w_S(\tau_S+\tau_\varepsilon)\sqrt{\tau_S/\tau_\varepsilon} + \sqrt{\tau_L}\sqrt{\tau_\varepsilon}(\lambda + w_L))}{N\sqrt{1-\rho^2}(\lambda\tau_\varepsilon(\lambda + Nw_S + w_L) + \tau_S w_S(2\lambda + w_L))}. \tag{25}$$

One can show that this function is decreasing for large enough N. The function $\lambda(\delta)$ given by (22) increases with δ for $\delta > \rho/\sqrt{1-\rho^2})\sqrt{\tau_S/\tau_\varepsilon}$. So for such δ, the functions $\lambda(\delta)$ and $\delta(\lambda)$ intersect at most once. There is no solution to the system with $\delta \le (\rho/\sqrt{1-\rho^2})\sqrt{\tau_S/\tau_\varepsilon}$ because, in that case, $\lambda(\delta) < -w_L$, so (24) does not hold. ■

[26] Perhaps the shortest path to the polynomial (7) is as follows. Write $\gamma_S = \frac{1}{w_S} - \frac{1}{w_S}\frac{\tau_\pi}{\tau_S+\tau_\varepsilon+\tau_\pi}\frac{N\gamma+\gamma_S}{\beta_S + N\beta\rho\sqrt{\tau_S/\tau_L}}$. Then observe that $\frac{1}{w_S}\frac{\tau_\pi}{\tau_S+\tau_\varepsilon+\tau_\pi} = \beta_S\tau_\pi/\tau_\epsilon = \delta^2\beta_S$. Then, use (20) to express $\beta_S/N\beta = \delta/\sqrt{\frac{\tau_L/\tau_\varepsilon}{1-\rho^2}} - \rho\sqrt{\frac{\tau_S}{\tau_L}}$. After simplification, one gets $\gamma_S = \frac{1}{w_S} - \delta(\delta-\phi)(N\gamma+\gamma_S)$, where $\phi = \frac{\rho}{\sqrt{1-\rho^2}}\sqrt{\frac{\tau_S}{\tau_\varepsilon}}$. Then, substitute $N\gamma + \gamma_S = 1/\lambda + 1/(w_L+\lambda)$ and $\gamma_S = 1/\lambda - (N-1)/(w_L+\lambda)$. After rearrangement, one obtains (7).

A.5 Proof of Proposition 2

Proof of Proposition 2. First, we write realized total welfare as

$$\begin{aligned}\mathrm{RTW} &\equiv v_S \int_0^1 x_j\, dj - \frac{w_S}{2}\int_0^1 (x_j)^2\, dj + v_L \sum_{i=1}^N x_i - \frac{w_L}{2}\sum_{i=1}^N x_i^2 \\ &= (v_S - v_L)\langle x_S\rangle - \frac{w_S}{2}\int_0^1 (x_j)^2\, dj - \frac{w_L}{2N}\langle x_S\rangle^2.\end{aligned}$$

Note that $\int_0^1 (x_j)^2\, dj = \int_0^1 (x_j - \langle x_S\rangle)^2\, dj + \langle x_S\rangle^2$ and $v_S - v_L = x_S^{\mathrm{FB}}(w_S + w_L/N)$; hence, after applying expectations and some rearranging, the preceding expression for RTW transforms to (8).

Given the aggregate demands of large and small traders, the equilibrium price can be expressed as $p = \langle v_S\rangle - w_S\langle x_S\rangle = v_L + (w_L + \lambda)\langle x_S\rangle/N$. From this (after some rearrangement), one obtains equation (9). Equation (10) now follows directly from (4). ■

A.6 Proof of Proposition 3

Proof of Proposition 3. The decomposition follows by substituting (9) into (10). The comparative statics of WL_1 and WL_4 follow because ψ increases with N whereas β_S is decreasing in N (as follows from Proposition 8). For the comparative statics of WL_2, note that $E[(\bar{v}_S - v_S)^2] = 1/\tau - \tau_\varepsilon/\tau^2$; this equality is a decreasing function of τ (which, in turn, decreases with N) for $1/\tau < 1/2\tau_\varepsilon$. Thus $\mathrm{WL}_2 = \frac{\psi^2 E[(v_S - \bar{v}_S)^2]}{2(w_S + w_L/N)}$ increases with N for $\mathrm{Var}(v_S|s_j, p)^{-1} = \tau_S + \tau_\varepsilon + \tau_\pi > 2\tau_\varepsilon$. Clearly, the last inequality holds if $\tau_\varepsilon < \tau_S$.

We now turn to the sufficient conditions for WL to be increasing in N. Since under merger/split comparative statics of WL and welfare is the same, we examine the welfare and derive conditions under which it decreases in N. First, we have

$$\begin{aligned}\mathrm{RTW} &= (v_S - v_L)\langle x_S\rangle - \frac{w_S}{2}\int_0^1 (x_j)^2\, dj - \frac{w_L}{2N}\langle x_S\rangle^2 \\ &= (v_S - v_L)\langle x_S\rangle - \frac{w_S}{2}\beta_S^2\tau_\epsilon^2 - \frac{w_S + w_L/N}{2}\langle x_S\rangle^2.\end{aligned}$$

We then substitute $\langle x_S\rangle = (\langle v_S\rangle - v_L)/(w_S + (w_L + \lambda)/N)$ and $\langle v_S\rangle = \tau_\pi/(\tau_S + \tau_\epsilon + \tau_\pi)\pi + \tau_\epsilon/(\tau_S + \tau_\epsilon + \tau_\pi)v_S + \tau_S/(\tau_S + \tau_\epsilon + \tau_\pi)\bar{v}_S$, and compute $E[\mathrm{RTW}]$.

Then we denote $w_L/w_S = Na$ and consider the asymptotics when a is small. We write $\delta = d_0 + d_1 a + O(a^2)$. We then substitute it in the equation (7) and match the coefficients for the same powers of a to find $d_0 = \phi$ and $d_1 = \frac{(N-1)\varkappa}{(N-2)(\theta+\phi^2)}$ is a solution, and that for such solution the SOC $\lambda > -w_L/2$ is satisfied (we use the same notation as in the proof of Proposition 5). It follows from Proposition 1 that such equilibrium is unique for large enough N.

Substituting the expression for δ to our closed-form expression $E[\text{RTW}]$ and then differentiating the resulting expression with respect to N yields

$$\frac{d}{dN}E[\text{RTW}] = -a\frac{\left(1-\rho^2\right)^2 \tau_\epsilon \left(\left(\rho^2-1\right)\tau_\epsilon + \rho\sqrt{\tau_L\tau_S} - \tau_S\right)}{(N-2)^2 w_S \left(\left(1-\rho^2\right)\tau_\epsilon + \tau_S\right)^3} + O\left(a^2\right),$$

which is negative iff $\left(\left(\rho^2-1\right)\tau_\epsilon + \rho\sqrt{\tau_L\tau_S} - \tau_S\right) > 0$. The statement follows.

■

A.7 Proof of Proposition 4

Proof of Proposition 4. The idea behind this proof is to consider the limiting equilibrium when $\tau_v \to 0$ and $\tau_\varepsilon \to 0$.

Let $x \equiv \sqrt{\tau_\pi}$ and write $x = x_0 + x_1\tau_\varepsilon + O(\tau_\varepsilon^2)$. Let $y \equiv \lambda\tau_\varepsilon$ and write $y = (Nw_S/\sqrt{\tau_L})(x\sqrt{1-\rho^2} - \rho\sqrt{\tau_S})(x^2 + \tau_S + \tau_\varepsilon) - w_L\tau_\varepsilon$. Substituting these expressions for x and y into (7) and then collecting zero- and first-order terms in τ_ε, we obtain two potential solutions for the coefficient x_0.

Case 1: $N\sqrt{1-\rho^2}\left(\tau_S + {x_0}^2\right) - 2\sqrt{\tau_L}x_0 = 0$. Solving this quadratic equation we get that in the limit as $\tau_v \to 0$ only one root is such that the SOC $\lambda > -w_L/2$ might hold. This is root is given by $x_0 = \frac{2}{Na_L\sqrt{\frac{1}{\tau_{u,L}a_L^2} + \frac{1}{\tau_{u,S}a_S^2}}} + O(\tau_v)$. Note that we substituted expressions $\tau_L = \frac{\tau_{u,L}\tau_v}{a_L^2\tau_{u,L}+\tau_v}$, $\tau_S = \frac{\tau_{u,S}\tau_v}{a_S^2\tau_{u,S}+\tau_v}$ and $\rho = \sqrt{\frac{\tau_{u,L}\tau_{u,S}}{\left(\tau_{u,L}+\frac{\tau_v}{a_L^2}\right)\left(\tau_{u,S}+\frac{\tau_v}{a_S^2}\right)}}$. We then substitute this solution into (7) and, matching first-order terms in τ_ϵ we find $x_1 = -\frac{N(2Nw_S(a_L-a_S)+w_La_S)}{4\left(w_Sa_S(Na_L-2a_S)\sqrt{\frac{\tau_{u,L}\tau_{u,S}}{\tau_{u,L}a_L^2+\tau_{u,S}a_S^2}}\right)} + O(\tau_v)$.

We then verify the SOCs, $y + w_L\tau_\epsilon/2 > 0$. We compute $y + w_L\tau_\epsilon = -\frac{4(\tau_{u,L}\tau_{u,S}w_Sa_S(Na_L-2a_S))}{N^2\left(\tau_{u,L}a_L^2+\tau_{u,S}a_S^2\right)} + O(\tau_\epsilon) + O(\tau_v)$. The leading term is positive if and only if

$$N < \bar{N} = 2a_S/a_L.$$

Next, we compute $d/d\tau_\epsilon(\tau_\pi + \tau_\epsilon) = 1 - \frac{2Nw_Sa_L - 2Nw_Sa_S + w_La_S}{Nw_Sa_L - 2w_Sa_S} + O(\tau_\epsilon) + O(\tau_v)$. The leading term is negative iff

$$\frac{w_L}{Nw_S} < \bar{h} = 2 - \frac{2}{N} - \frac{a_L}{a_S}.$$

The statements about $\mathcal{I}^{RPE}$ follow because $d/d\tau_\epsilon(\tau_\pi + \tau_\epsilon) < 0$ implies that $\mathcal{I}^{RPE}$ is decreasing in τ_ϵ.

Finally, we compute $d/d\tau_\epsilon(Var(v|s_j,p))$. We invoke Lemma 4 for the expression for $Var(v|s_j,p)$. Using this expression, we get

$$\tau_{u,L}^2\frac{dVar(v|s_j,p)}{d\tau_\epsilon} = \frac{N^4(Nw_S(a_L-2a_S) + a_S(w_L+2w_S))}{16w_S(Na_L-2a_S)} + O(\tau_{u,L}) + O(\tau_v) + O(\tau_\epsilon).$$

The leading term is positive if and only if $Nw_S(a_L - 2a_S) + a_S(w_L + 2w_S)) < 0$ which is equivalent to $\frac{w_L}{Nw_S} < \bar{h}$.

Case 2: $x_0 = \rho\sqrt{\tau_S}/\sqrt{1-\rho^2}$. Substituting x_0 to (7), we get a quadratic equation to solve for x_1. The discriminant of this equation, for small τ_v is negative if and only if

$$N^2 a_L^2 \left(2Nw_L w_S + (N-2)^2 w_S^2 + w_L^2\right) + w_L^2 a_S^2 < 2Nw_L a_L a_S((3N-2)w_S + w_L). \tag{26}$$

Thus, if the above condition holds, the equilibrium with $x_0 = \rho\sqrt{\tau_S}/\sqrt{1-\rho^2}$ does not exist.

■

Lemma 4. *For a given price, the conditional variance of fundamental value is*

$$\mathrm{Var}(v|p, s_j) = \tau_\eta^{-1} + \frac{\left(k_S + k_L\rho\sqrt{\tau_S/\tau_L} - k_L\sqrt{(1-\rho^2)\,\tau_\pi/\tau_L}\right)^2}{\tau_S + \tau_\epsilon + \tau_\pi}. \tag{27}$$

Moreover, for sufficiently small $h = \frac{w_L}{w_S N}$, $\partial \mathrm{Var}(v|p, s_j)/\partial\tau_\pi < 0$.

Proof of Lemma 4. Lemma 5 (to follow) allows us to write $v = k_S v_S + k_L v_L + \eta$, where $\eta \sim N(\bar{\eta}, \tau_\eta)^{-1}$ is independent of v_S and v_L. Lemma 3 allows us to write $v_L = A + Bv_S + C\zeta$, where $\zeta \sim N(0,1)$ is independent of v_S. Moreover, from Lemma 1 it follows that $\zeta = (\pi - v_s)\sqrt{\tau_\pi}$. Hence

$$\mathrm{Var}(v|p, s_j) = \mathrm{Var}(\eta + k_L A + v_S(k_S + k_L(B - C\sqrt{\tau_\pi})) + k_L C\sqrt{\tau_\pi}\pi|\pi, s_j).$$

Now note that η is independent of both v_S and v_L, hence it is also independent of their linear combination π. It is also independent of s_j, since s_j is a linear combination of v_S and ϵ_j and the latter is independent of everything else. Taking this into account, after some algebraic manipulation, the last displayed equation can be rearranged to yield (27).

For the last statement, we directly compute

$$\mathrm{sign}\left(\frac{\partial \mathrm{Var}(v|p, s_j)}{\partial\tau_\pi}\right) = -\mathrm{sign}\left(k_S + k_L\rho\sqrt{\tau_S/\tau_L} - k_L\sqrt{(1-\rho^2)\,\tau_\pi/\tau_L}\right),$$

and one can show that $k_S + k_L\rho\sqrt{\tau_S/\tau_L} - k_L\sqrt{(1-\rho^2)\,\tau_\pi/\tau_L} \to k_S > 0$ as $h \to 0$. The statement then follows. ■

Lemma 5. *Suppose that* $v_S = a_S v + u_S$ *and* $v_L = a_L v + u_L$, *where* $u_S \sim N(0, \tau_{u,S}^{-1})$, $u_L \sim N(0, \tau_{u,L}^{-1})$ *and* $v \sim N(\bar{v}, \tau_v^{-1})$ *are jointly normally distributed and independent. Then one can express* $v = k_S v_S + k_L v_L + \eta$, *where* $\eta \sim N\left(\bar{\eta}, \tau_\eta^{-1}\right)$ *is independent of* v_S *and* v_L *and the coefficients are given by* $k_S = \frac{a_S^2\tau_{u,S}}{\tau_V + a_S^2\tau_{u,S} + a_S^2\tau_{u,L}}$, $k_L = \frac{a_L^2\tau_{u,L}}{\tau_V + a_S^2\tau_{u,S} + a_S^2\tau_{u,L}}$, $\bar{\eta} = \frac{\tau_V\bar{v}}{\tau_V + a_S^2\tau_{u,S} + a_S^2\tau_{u,L}}$ *and* $\tau_\eta = \tau_V + a_S^2\tau_{u,S} + a_L^2\tau_{u,L}$.

Proof of Lemma 5. The idea is to project v on v_S and v_L. Compute

$$E[v|v_L, v_S] = \frac{\tau_V\bar{v}}{\tau_V + a_S^2\tau_{u,S} + a_S^2\tau_{u,L}} + \frac{a_S^2\tau_{u,S}v_S}{\tau_V + a_S^2\tau_{u,S} + a_S^2\tau_{u,L}} + \frac{a_L^2\tau_{u,L}v_L}{\tau_V + a_S^2\tau_{u,S} + a_S^2\tau_{u,L}}.$$

We also have that $\eta = v - E[v|v_L, v_S]$ is independent of v_S and v_L and

$$\begin{aligned}\mathrm{Var}\,(\eta) &= \mathrm{Var}\,(\eta|v_S, v_L) + \mathrm{Var}\,(E\,[\eta|v_S, v_L]) \\ &= \frac{1}{\tau_V + a_S^2\tau_{u,S} + a_L^2\tau_{u,L}}.\end{aligned}$$

The statements of the lemma then follow. ■

A.8 Proof of Proposition 5

Proof of Proposition 5. Let

$$\theta \equiv \frac{\tau_S + \tau_\varepsilon}{\tau_\varepsilon} > 1, \quad \xi \equiv \rho\sqrt{\frac{\tau_S}{\tau_L}}, \quad \varkappa \equiv \sqrt{\frac{\tau_L/\tau_\varepsilon}{1-\rho^2}} > 0,$$

$$\psi \equiv \frac{w_L}{Nw_S} > 0, \quad \phi \equiv \varkappa\xi = \frac{\rho}{\sqrt{1-\rho^2}}\sqrt{\frac{\tau_S}{\tau_\varepsilon}},$$

$$Q \equiv -4N\xi + 8\xi + 4\psi, \quad T \equiv 16N^2\xi\psi\left(\xi - \frac{2}{N}\right)(\psi+2),$$

$$l^\pm \equiv \frac{-G \pm \sqrt{G^2+F}}{2}, \quad G \equiv 1 + \frac{2(\theta-2)}{N} > 1, \quad F \equiv 2\psi + \psi^2 > 0.$$

Assume that the following inequalities hold,

$$Q < 0, \quad \xi < \frac{1}{N}, \quad Q^2 + T > 0, \quad \psi < 1, \quad N > 4. \tag{28}$$

Also, let $l \equiv \frac{2\lambda + w_L}{2Nw_S} > 0$. Then (25) can be rewritten as

$$\delta = \delta(l) \equiv \frac{2\varkappa}{N}\frac{l(l+\theta\xi+\psi/2)}{(l-l^+)(l-l^-)}, \tag{29}$$

and the equilibrium is the solution to the system consisting of (29) and

$$l = l(\delta) \equiv \frac{(\delta^2+\theta)(\delta-\phi)}{\varkappa} - \frac{\psi}{2}.$$

Consider all solutions to the equation

$$\delta(l) = \phi. \tag{30}$$

If the conditions (28) hold, then there exist two solutions to (30), which are given by

$$L^\pm = \frac{-Q \pm \sqrt{Q^2+T}}{8N(2/N-\xi)}.$$

Furthermore, both solutions $L^{\pm} > l^{+}$.[27] The existence of two solutions to (25) implies that the function $\delta(l)$ attains a local minimum in the region $l > l^{+}$ and that this minimum is less than ϕ.

Also, consider all solutions to

$$\delta(l) = \frac{\varkappa}{N}.$$

There are two solutions to this equation, as well—provided that (28) holds. Let L_m denote the maximal solution. Then

$$L_m = \frac{1}{2}(Q_m + \sqrt{Q_m^2 + T_m}) > L^{+},$$

where

$$Q_m \equiv \frac{2(\theta - 1)}{N} + 1 - \frac{2\theta\phi}{\varkappa} - \psi \quad \text{and} \quad T_m \equiv -(\psi^2 + 2\psi).$$

If

$$L_m < l\left(\frac{\varkappa}{N}\right) = \frac{(\varkappa^2 + \theta N^2)(\varkappa - N\phi)}{\varkappa N^3} - \frac{\psi}{2} \equiv l_m, \tag{31}$$

then there are at least three equilibria.

The condition $Q < 0$ is equivalent to

$$\xi > \frac{\psi}{N - 2}. \tag{32}$$

The condition $Q^2 + T > 0$ holds as long as

$$\xi > \frac{2\psi(N(\psi + 3) - 2)}{N(N(\psi + 1)^2 - 4) + 4} \quad \text{and} \quad N(N(\psi + 1)^2 - 4) + 4 > 0.^{28} \tag{33}$$

Given (28), the second inequality in (33) holds. Note that

$$\frac{2\psi(N(\psi + 3) - 2)}{N\left(N(\psi + 1)^2 - 4\right) + 4} < \frac{8\psi}{N - 4} > \frac{\psi}{N - 2}.$$

Therefore, both (32) and (33) hold if the following weaker condition also holds:

$$\xi > \underline{\xi}_1 \equiv \frac{8\psi}{N - 4}.$$

The preceding expression can be written as

$$\tau_L < \frac{\rho^2 \tau_S}{\underline{\xi}_1^2} \equiv \bar{\tau}_2. \tag{34}$$

[27] It is easy to see that both solutions are positive. However, $\delta(L) = \phi > 0$ is positive only if $L > l^{+}$.

[28] Indeed, $Q^2 + T = 16\xi^2(N(N(\psi + 1)^2 - 4) + 4) - 32\xi\psi(N(\psi + 3) - 2) + 16\psi^2$. Condition (33) ensures that the first two terms are positive.

Now suppose that

$$l_m - Q_m > 0.$$

Then (31) holds.[29] The inequality just displayed can be written as

$$\left(\frac{\varkappa^2}{N^2} - \theta\right)\left(\frac{1}{N} - \xi\right) > 1 - \frac{2}{N} - \frac{\psi}{2}.$$

Assume that

$$\xi < \frac{1}{2N}.$$

Then ξ is greater than $(\varkappa^2/N^2 - \theta)(1/2N)$, and the constraint holds, provided that

$$\frac{\varkappa^2}{N^2} - \theta > 2N - 4 - N\psi.$$

This inequality is equivalent to

$$\tau_L > (1-\rho^2)\tau_\varepsilon N^2(2N - 4 - N\psi + \theta).$$

The preceding expression holds if the following stricter inequality also holds:

$$\tau_L > (1-\rho^2)\tau_\varepsilon N^2(2N - 4 + \theta).$$

The constraint $\xi < 1/2N$ implies that

$$\tau_L > 4N^2\rho^2\tau_S.$$

In turn, those two constraints hold provided that

$$\tau_L > \underline{\tau}_2 \equiv \max\{4N^2\rho^2\tau_S, (1-\rho^2)\tau_\varepsilon N^2(2N-4+\theta)\}. \tag{35}$$

It is clear that

$$\underline{\tau}_2 > 4N^2\rho^2\tau_S > \bar{\tau}_1.$$

The final step is to derive the conditions under which $\underline{\tau}_2 < \bar{\tau}_2$. We have

$$\sqrt{\underline{\tau}_2} < \frac{\rho\sqrt{\tau_S}}{\underline{\xi}_1} = \frac{\rho\sqrt{\tau_S}}{8\psi}(N-4),$$

which is equivalent to

$$w_L < \bar{w} \equiv w_S\rho\frac{N(N-4)}{8}\sqrt{\frac{\tau_S}{\underline{\tau}_2}}. \tag{36}$$

■

[29] The expression (31) is equivalent to $Q_m^2 + T_m - (2l_m - Q_m)^2 = 2l_m(2Q_m - 2l_m) + T_m < 0$, which is true.

A.9 Proof of Proposition 6

Proof of Proposition 6. The proposition follows by noting that both $\mathcal{I}^{RPE} = \frac{\tau_S + \tau_\varepsilon(1+\delta^2)}{\tau_S}$ and λ (as given by (39)) are increasing functions of δ, which is a decreasing function of N (see the proof of Proposition 8). The statement about $\mathcal{I}^{FPE}$ follows because for small enough h, $\partial \text{Var}(v|p, s_j)/\partial \tau_\pi < 0$ (Lemma 4). ■

A.10 Proof of Proposition 7

We start with the following lemma.

Lemma 6. *The value function of large trader i at $t = 1/2$ is given by*

$$V_i\left(x_0^i, x_0^{-i}, Z\right) = x_0^i v + \frac{w\left(2k-1\right)\left(x_i^*\right)^2}{2} - \frac{w\left(x_0^i\right)^2}{2},$$

where $k = \frac{L-1}{L-2}$, $x_i^* = \frac{Zk + x_0^{-i} - (L-1)x_0^i}{kL}$, and $x_0^{-i} \equiv \sum_{j \neq i} x_0^j$.

Proof of Lemma 6. From the FOC and the fact that $\lambda = \frac{w_L}{L-2}$ it follows that

$$v - p = kwx_i^* + wx_0^i. \tag{37}$$

Therefore

$$\begin{aligned}(v-p)x_i^* - \frac{w\left(x_i^* + x_0^i\right)^2}{2} &= (kwx_i^* + wx_0^i)x_i^* - \frac{w\left(x_i^* + x_0^i\right)^2}{2} \\ &= \frac{w\left(2k-1\right)\left(x_i^*\right)^2}{2} - \frac{w\left(x_0^i\right)^2}{2}\end{aligned}$$

Summing up (37) across investors we get

$$v - p = \frac{1}{L}\left(kwZ + w\sum_i x_0^i\right).$$

It then follows

$$
\begin{aligned}
x_i^* &= \frac{1}{kw}\left(v - p - wx_0^i\right) \\
&= \frac{1}{kw}\left(w\frac{Zk + x_0^i + x_0^{-i}}{L} - wx_0^i\right) \\
&= \frac{1}{k}\left(\frac{Zk + x_0^i + x_0^{-i}}{L} - x_0^i\right) \\
&= \frac{Zk + x_0^{-i} - (L-1)x_0^i}{kL}.
\end{aligned}
$$

■

Having established equilibrium value function at $t = 1/2$, we proceed to $t = 0$. At t=0, HFT i solves

$$v(Z_0 + x) + E[V_i(x, x_0^{-i}(p), Z)|\eta] - px - \frac{w(Z_0 + x)^2}{2} \to \max_{x(p)}.$$

Note that post-trade allocations to other traders $x_0^{-i}(p)$ depend on market-clearing price. HFTs take this into account. The price at $t = 0$ is a noisy version of η, hence it is not useful in predicting Z. Thus, the conditional expectation above only includes η.

The key is that, as we show below, the FOC in the symmetric equilibrium can be written as

$$v - wZ_0 - w\frac{E[Z|\eta]}{L}\left(1 - \frac{1}{L-1}\right) - p - \lambda x - 2wx = 0. \tag{38}$$

To derive it, we write the $t = 0$ FOC as follows:

$$v - wZ_0 + \frac{\partial E[V_i|\eta]}{\partial x} + \frac{\partial E[V_i|\eta](x, x_0^{-i}(p))}{\partial x_0^{-i}} \overbrace{\underbrace{\frac{\partial x_0^{-i}(p)}{\partial p}}_{-(L-1)\gamma} \underbrace{\frac{\partial p}{\partial x}}_{=\lambda}}^{-1} - p - \lambda x - wx = 0.$$

Thus, the FOC can be written as

$$v - wZ_0 + \frac{\partial E[V_i|\eta]}{\partial x} - \frac{\partial E[V_i|\eta](x, x_0^{-i}(p))}{\partial x_0^{-i}} - p - \lambda x - wx = 0.$$

Using Lemma 6, we compute the first two terms in the equation above.

$$
\begin{aligned}
\frac{\partial E[V_i|\eta]}{\partial x} &= -w\left(2k - 1\right)\frac{E[Z|\eta]k + x_0^{-i}(p) - (L-1)x}{kL}\frac{L-1}{kL} - wx \\
&= \text{(in equilibrium)} \\
&= -w\frac{E[Z|\eta]}{L} - wx
\end{aligned}
$$

In the last equation above we have used the fact that in the symmetric equilibrium $(L-1)x^* = x_0^{-i}$. Similarly, for the second term,

$$\begin{aligned}
\frac{\partial E[V_i|\eta]}{\partial x_0^{-i}} &= w\,(2k-1)\,\frac{E[Z|\eta]k + x_0^{-i}(p) - (L-1)x}{kL}\frac{1}{kL} \\
&= \text{(in equilibrium)} \\
&= w\,(2k-1)\,\frac{E[Z|\eta]}{L}\frac{1}{kL} \\
&= \frac{wE[Z|\eta]}{L(L-1)}
\end{aligned}$$

From this we obtain (38).

We now check the second-order conditions. SOC:

$$\begin{aligned}
\frac{\partial^2 E[V_i|\eta]}{\partial x^2} &= -\frac{\partial}{\partial x}\left(w\,(2k-1)\,\frac{Zk + x_0^{-i}(p) - (L-1)x}{kL}\frac{L-1}{kL} + wx\right) \\
&= -\left(w\,(2k-1)\,\frac{-1-(L-1)}{kL}\frac{L-1}{kL} + w\right) \\
&= -w\left(-\,(2k-1)\,\frac{1}{k}\frac{L-1}{kL} + 1\right) \\
&= -w\,(1-1/k)
\end{aligned}$$

$$\begin{aligned}
-\frac{\partial^2 E[V_i|\eta](x, x_0^{-i}(p))}{\partial x \partial x_0^{-i}} &= -\frac{\partial}{\partial x}\left(w\,(2k-1)\,\frac{Zk + x_0^{-i}(p) - (L-1)x}{kL}\frac{1}{kL}\right) \\
&= -\left(w\,(2k-1)\,\frac{-1-(L-1)}{kL}\frac{1}{kL}\right) \\
&= -\left(w\,(2k-1)\,\frac{-L}{kL}\frac{1}{kL}\right) \\
&= \left(w\,(2k-1)\,\frac{1}{k}\frac{1}{kL}\right) \\
&= w\frac{1}{k\,(L-1)}
\end{aligned}$$

$$\text{second derivative} = -\left(w_L\,(1-1/k) - \frac{w_L}{k\,(L-1)} + 2\lambda + w_L\right)$$

It can be seen that if the SOC holds in the alternative economy (i.e., $w_l + 2\lambda > 0$), it also holds in the original economy.

We note that equivalence of FOCs in the economy from Section 3 implies only that marginal utilities are the same across two economies. The utilities, and thus the welfare, might differ. However, it is straightforward to show that they differ by a constant that is not affected by the comparative static exercises performed in the paper.

A.11 Proof of Proposition 8

Proposition 8. *In equilibrium, $\mathcal{I}^{RPE}$ is decreasing in N, whereas liquidity $\mathcal{L}$ is increasing in N. Suppose in addition that Assumption 1 holds. If $h \equiv \frac{w_L}{Nw_S}$ is sufficiently low, $\mathcal{I}^{FPE}$ is decreasing in N as well.*

Proof of Proposition 8. The equilibrium is a solution to the system (22)–(24), which can be written as follows:

$$\lambda = L(\delta; N) \equiv \frac{Nw_S}{\varkappa}(\delta - \phi)(\theta + \delta^2) - w_L; \tag{39}$$

$$\delta = D(\lambda; N) \equiv h\left(\frac{\lambda(w_L + Nw_S + \lambda)}{w_S(w_L + 2\lambda)}\right). \tag{40}$$

Here,

$$\varkappa \equiv \sqrt{\frac{\tau_L/\tau_\varepsilon}{1-\rho^2}}, \quad \phi \equiv \frac{\rho}{\sqrt{1-\rho^2}}\sqrt{\frac{\tau_S}{\tau_\varepsilon}}, \quad \theta \equiv \frac{\tau_S + \tau_\varepsilon}{\tau_\varepsilon},$$

and $\delta = h(x)$ is the inverse of $1 + \delta(\delta - \phi)$ on $\delta > \phi$.

Lemma 7 (to follow) implies that $\lambda > 0$ in equilibrium. Yet because that inequality is not possible when $\delta < \phi$, we may look for the curves ($L(\delta; N)$ and $D(\lambda; N)$) to intersect in the region where $\delta > \phi$ and $\lambda > 0$.

Since the function $1+\delta(\delta-\phi)$ is strictly increasing for $\delta > \phi$, it follows that the function $h(x)$ is both well-defined and strictly increasing. The equilibrium is therefore the intersection of the curves $\lambda = L(\delta; N)$ and $\delta = D(\lambda; N)$. Moreover, it is easy to see that $\frac{\partial L}{\partial \delta} > 0$ and $\frac{\partial D}{\partial \lambda} > 0$ for $\delta > \phi$, so both curves are strictly upward sloping for a given N. We next compute

$$\frac{\partial L}{\partial N} = \frac{w_S(\delta^2 + \theta)(\delta - \phi)}{\varkappa} - w_L'(N),$$

which is positive if w_L does not depend on N or if $w_L = w_1 N$ (in the second case, $\frac{\partial L}{\partial N} = \frac{\lambda}{N} > 0$).

Analogously, we compute

$$\frac{\partial D}{\partial N} = h'(\cdot) \times \begin{cases} \frac{\lambda}{2\lambda + w_L} & \text{if } w_L \text{ does not depend on } N \text{ and} \\ \frac{\lambda^2(w_1 + 2w_S)}{w_S(2\lambda + Nw_1)^2} & \text{if } w_L = w_1 N. \end{cases}$$

This expression is positive.

Hence, an infinitesimal increase in N shifts the curve $L(\delta; N)$ upward and the curve $D(\lambda; N)$

rightward. Their new intersection will therefore be below and to the left of the old one.[30] Thus we have

$$\frac{d\lambda}{dN} < 0 \quad \text{and} \quad \frac{d\delta}{dN} < 0.$$

Since $\mathcal{I}^{RPE} = \frac{\tau_S + \tau_\varepsilon(1+\delta^2)}{\tau_S}$ is increasing in δ and does not depend directly on N, and since $\mathcal{L}$ is inversely related to λ, it follows that

$$\frac{d\mathcal{I}^{RPE}}{dN} < 0 \quad \text{and} \quad \frac{d\mathcal{L}}{dN} > 0.$$

That $d\mathcal{I}^{FPE}/dN < 0$ follows because $\mathcal{I}^{FPE}$ depends on N only through τ_π, and by Lemma 4 (see section A.7), $\mathcal{I}^{FPE}$ increases in τ_π for sufficiently small h. ■

Lemma 7. *The equilibrium price impact λ is positive.*

Proof. Rewrite (7) as

$$\lambda = \frac{w_S(1 + \delta(\delta - \phi))(w_L + 2\lambda)}{w_L + Nw_S + \lambda}.$$

Then $\delta > \phi$, because otherwise $\lambda < -w_L$ and the second-order condition $2\lambda + w_L > 0$ would not hold. Therefore, $1 + \delta(\delta - \phi) > 0$. Other terms in the equality just displayed are positive, owing to the second-order condition $w_L + 2\lambda > 0$. ■

[30] The curve $\lambda = L(\delta; N)$ must intersect the curve $\delta = D(\lambda; N)$ from below because, for $\lambda = 0$, the curve $\lambda = L(\delta; N)$ is to the right of the curve $\delta = D(\lambda; N)$.

B Motivating evidence: Price informativeness and ESG scores

In this Appendix, we produce empirical evidence that is consistent with large ESG-conscious investors adding noise to prices. The empirical hypothesis is developed as follows. As we argued in Section 9.1, large asset managers around the world put increasingly more weight on firms' ESG performance in their investment decisions. It is thus plausible that stocks with higher ESG scores are owned proportionally more by ESG-conscious investors. As a result, these stocks would have lower price informativeness.

To carry out the empirical test, we combine the three quarterly price informativeness measures as constructed in Sammon (2021) with the annual KLD index data as constructed in Khan, Serafeim, and Yoon (2016). Sammon studies earnings announcements and argues that when the share of informed traders reduces or when they gather less information, 1) the pre-earnings turnover, 2) the pre-earnings drift, and 3) the share of volatility occuring on non-earnings-announcement days should decline.[31] The intuition is that in this case, informed traders would trade less, hence incorporating less information in prices before the announcements. This results in lower trading volume and weaker drift before announcement and more news, thus, more stocks returns volatility, happening on earnings-announcement day. MSCI KLD is one of the most widely used sustainability dataset in past studies and higher KLD index is considered as better ESG performance. We also use data from Compustat/CRSP as firm-level control variables. After dropping observations with missing firm-level data in Compustat/CRSP, the final sample ranges from 1996 to 2017, with 85,313 Firm-YearQuarter observations.

We find the hypothesized negative relationship between price informativeness and KLD index in a firm-level, cross-sectional regression analysis as reported in Table 1. For each of the three price informativeness measures, we have two main regression specifications. In the first specification (columns (1), (3) and (5) in the table), we use Year-quarter $\times$ Size decile fixed effects to essentially run cross-sectional regressions in a given quarter for firms in the same decile by market capitalization. We find negative coefficients on all three measures. In the second specification (columns (2), (4) and (6) in the table), we further add firm-level controls to absorb variations in price informativeness driven by factors other than ESG performance. As a result, the negative relationship becomes more economically and statistically significant.

[31] We use the data directly provided by the author and the measures are defined in the working paper version of Sammon (2021) dated on November, 2021.

Table 1

	Pre-earnings turnover		Pre-earnings drift		Non-earnings-day volatility	
	(1)	(2)	(3)	(4)	(5)	(6)
KLD_index	-0.0267*	-0.0406**	-0.0002	-0.0005***	-0.0036***	-0.0035***
	(0.0152)	(0.0173)	(0.0002)	(0.0001)	(0.0009)	(0.0009)
R^2	0.08376	0.09652	0.07527	0.11924	0.06407	0.08435
Observations	85,313	85,313	85,313	85,313	85,313	85,313
Year-quarter × Size decile F.E.	✓	✓	✓	✓	✓	✓
Firm-level controls		✓		✓		✓

Table 1: Firm-level cross-sectional regressions of Price Informativeness on ESG scores. *Note:* *p<0.1, **p<0.05, ***p<0.01. The dependent variables are the three measures of price informativeness defined in Sammon (2021), namely, pre-earnings turnover, pre-earnings drift, and the share of volatility occurring on non-earnings-announcement days. The independent variable is the KLD index as constructed in Khan et al. (2016). Firm-level controls include Size, Turnover, Analyst coverage, Advertising intensity, Institutional ownership, R&D, and Capital expenditure. Standard errors are clustered at firm level.

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Internet Appendix of "When Large Traders Create Noise"

IA.1 A Competitive Model with Price-Taking Large Traders

In this section we demonstrate the critical importance of large traders' market power and the associated strategic trading behavior in driving the main mechanism and the associated results of the paper, namely, trading complementarity, the unconventional adverse effects of competition on welfare and those of signal quality on informational efficiency, and fragility induced by market power. We show that if large traders take prices as given, none of the results continues to hold.

We first revisit the mechanism that underlies trading complementarity among investors and market fragility, which corresponds to results stated in Lemmas 1 and 2 and Theorem 1.

Proposition IA.1. *Consider the same model described in Section 3 but assume that large traders are price takers. Fix the parameters (α, β, γ) in large traders' demand schedules. When large traders trade more aggressively (i.e., when β increases), informational efficiency increases and small traders provide more liquidity (i.e., γ_S increases). Fix the parameters $(\alpha_S, \beta_S, \gamma_S)$ in small traders' demand schedules. When small traders provide more liquidity (i.e., when γ_S increases), the market becomes more liquid. However, larger traders' trading aggressiveness β does not change because they take price as given. As a result, the trading complementarity highlighted in Section 4 does not arise and the equilibrium is unique.*

Proof of Proposition IA.1. The proof of the first part of the proposition (partial equilibrium with (α, β, γ) fixed) is identical to the proof of Lemma 1. The optimal demand of large traders is $\frac{v_L - p}{w_L}$, implying that $\beta = \gamma = 1/w_L$. Thus, the aggressiveness β is a constant, and the second part of the proposition (partial equilibrium with $(\alpha_S, \beta_S, \gamma_S)$ fixed) follows. To find the overall equilibrium, we note that $(\alpha_S, \beta_S, \gamma_S)$ are given by the same functions of $\delta \equiv \sqrt{\tau_\pi/\tau_\epsilon}$ as in Theorem 1. For large traders, we have $\beta = \gamma = 1/w_L$, $\alpha_L = 0$. The δ is the unique positive solution to the cubic equation

$$\frac{\tau_\epsilon w_L}{N w_S \left(\left(\delta^2 + 1\right) \tau_\epsilon + \tau_S\right)} = \frac{\delta \sqrt{\tau_\epsilon - \rho^2 \tau_\epsilon}}{\sqrt{\tau_L}} - \rho \sqrt{\frac{\tau_S}{\tau_L}}. \tag{IA.1}$$

■

The proposition above highlights that strategic trading behavior of large traders (i.e., accounting for their price impact) is indispensable for the trading complementarity. Indeed, step (4) in Figure 1 is absent when large traders take prices as given. Consequently, the equilibrium is unique and the market fragility discussed in Section 7 no longer arises.

Next, we turn to the analysis of welfare and competition in Section 5. Propositions 8 and 3 show that increasing competition (N) via the breakup of large traders harms informational efficiency and might reduce welfare. Neither changes with N when large traders are price takers.

Proposition IA.2. *When large traders take prices as given, changes with N via mergers or splits affect neither informational efficiency nor welfare.*

Proof of Proposition IA.2. Examining equation (IA.1) we see that δ does not change with N when aggregate risk-bearing capacity N/w_L of large traders is a constant (unaffected by N). Hence, informational efficiency does not change with N. Following the steps as in Section 5 one can show that welfare loss is given by (8) with $\langle x_S \rangle = x_S^{FB} + b$, where $b = (\bar{v_S} - v_S)/(w_S + w_L/N)$. Once can then write welfare loss in terms of w_L/N and δ, neither of which changes with N. Since welfare in the first-best is unaffected by N, we have that the welfare does not change with N. ■

When larger traders are price takers, increasing N via splitting will not change their trading aggressiveness. Thus the amount of noise injected by them and informational efficiency remains unchanged. So does welfare.[32]

Finally, in Section 6 we show that improving the quality of private information can reduce informational efficiency (Proposition 4). This unconventional result no longer arises, and informational efficiency increases as in standard models when large traders are price takers.

Proposition IA.3. *When large traders take prices as given, an increase in the signal precision τ_ϵ enhances informational efficiency $\mathcal{I}^{RPE}$.*

Proof. Equation (IA.1) implies that equilibrium τ_π solves the following equation:

$$\frac{\tau_\epsilon w_L}{N w_S \left(\tau_\pi + \tau_\epsilon + \tau_S\right)} = \frac{\sqrt{\tau_\pi (1-\rho^2)}}{\sqrt{\tau_L}} - \rho \sqrt{\frac{\tau_S}{\tau_L}}.$$

Differentiating the above equation implicitly we get that τ_π in increases in τ_ϵ, from which the statement follows. ■

[32]Changing N via entry or exit will affect both welfare and informational efficiency because the aggregate risk-bearing capacity of large traders as a whole changes. These effects are well understood in the literature (Stein, 1987) and so we focus on merger/splits as the main comparative statics exercises.

IA.2 The Limiting Case of Large Correlation between v_L and v_S

In this section we investigate the case when the correlation between v_L and v_S is close to 1. This case is an interesting one for the following reason. Suppose that ρ is close to 1. Then, large traders would inject little noise in the price for small traders, weakening step (1) in the equilibrium mechanism in Figure 1. Then, one would expect that the complementarity highlighted in our paper would be weakened, so our mechanism would be less applicable to settings with little room for private values (as often assumed in finance). Below we show that the above argument is not correct. What it misses is that when ρ gets larger, small traders rely more on prices, reinforcing step (2) of the equilibrium mechanism. Thus, the complementarity will still be there even in the limit as $\rho \to 1$.

Theorem IA.1. *When $\rho \to 1$ the equilibrium parameter $\delta = \sqrt{\tau_\pi/\tau_\epsilon}$ satisfies*

$$\delta = \frac{x_0}{\sqrt{\tau_\varepsilon(1-\rho^2)}} + x_2\sqrt{\tau_\varepsilon(1-\rho^2)} + O(1-\rho^2).$$

The equilibria are characterized as follows.

1. *If $\sqrt{\frac{\tau_S}{\tau_L}}\frac{N}{2} > 1$ there is a unique equilibrium with $x_0 = \sqrt{\tau_S}$ and $x_2 = \frac{\lambda_0^* + w_L}{N\tau_S w_S}\sqrt{\tau_L} - \frac{\sqrt{\tau_S}}{2\tau_\epsilon}$, where λ_0^* is a greater root of quadratic equation (IA.5). In this equilibrium, $\mathcal{I}^{RPE}$ is increasing in τ_ϵ.*
2. *If $\sqrt{\frac{\tau_S}{\tau_L}}\frac{N}{2} < 1$:*
   - *There exists an equilibrium with $x_0 = \frac{2\sqrt{\tau_L}}{N}$ and $x_2 = \frac{N}{4}\left(\frac{2(2-N)w_S + w_L}{2\sqrt{\tau_L}w_S\left(1-\sqrt{\frac{\tau_S}{\tau_L}}\frac{N}{2}\right)} - \frac{2(\tau_\epsilon+\tau_S)}{\tau_\epsilon\sqrt{\tau_L}}\right)$. In this equilibrium, $\mathcal{I}^{RPE}$ is decreasing in τ_ϵ when $w_S\left(\sqrt{\frac{\tau_S}{\tau_L}}\frac{N}{2} - \frac{N}{2}\right) + w_L < 0$. Such equilibrium is unique if the two roots of quadratic equation (IA.5), $\lambda_0^\pm$ are such that $\lambda_0^\pm < -w_L/2$.*
   - *There might exist two additional equilibria with $x_0 = \sqrt{\tau_S}$ and $x_2 = \frac{\lambda_0^\pm + w_L}{N\tau_S w_S}\sqrt{\tau_L} - \frac{\sqrt{\tau_S}}{2\tau_\epsilon}$, where $\lambda_0^\pm$ are two roots of (IA.5). Such equilibria exist if both roots $\lambda_0^\pm > -w_L/2$.*

In all equilibria λ and $\mathcal{I}^{RPE}$ are increasing in ρ.

Proof of Theorem IA.1. Denote $a = \sqrt{\tau_\varepsilon(1-\rho^2)}$ and $x = \delta a$. Write

$$x = x_0 + x_1 a + x_2 a^2 + O(a^3). \tag{IA.2}$$

Denote

$$\lambda(x) = \frac{Nw_S}{\sqrt{\tau_L}}(x - \rho\sqrt{\tau_S})\left(x/a^2 + \frac{\tau_S + \tau_\varepsilon}{\tau_\varepsilon}\right) - w_L.$$

The equilibrium equation to pin down x is

$$\lambda(\lambda + Nw_S + w_L) - w_S(2\lambda + w_L)\left(\frac{x}{a}\left(\frac{x}{a} - \frac{\rho\sqrt{\tau_S}}{a}\right) + 1\right) = 0 \tag{IA.3}$$

Note that

$$\rho = \left(1 - a^2/\tau_\epsilon\right)^{1/2} = 1 - \frac{1}{2}a^2/\tau_\epsilon + O(a^3). \tag{IA.4}$$

Plugging (IA.2) and (IA.4) into (IA.3), multiplying that equation by a^4 and collecting the zero-order terms yields the following potential solutions for x_0 that might satisfy the SOC $\lambda > -w_L/2$:

$$x_0 = \sqrt{\tau_S},\ x_0 = \frac{2\sqrt{\tau_L}}{N}.$$

Case 1. $x_0 = x_0^1 \equiv \frac{2\sqrt{\tau_L}}{N}$. It will be a solution if and only if $\lambda(x_0) > -w_L/2$ for sufficiently small a. This will clearly be the case when $x_0 - \sqrt{\tau_S} > 0$ or $\sqrt{\frac{\tau_S}{\tau_L}}\frac{N}{2} < 1$. (In that case, $\lambda \to +\infty$ as $a \to 0$).

Matching first- and second-order coefficients, we find $x_1 = 0$ and $x_2^1 = \frac{N}{4}\left(\frac{2(2-N)w_S + w_L}{2\sqrt{\tau_L}w_S\left(1-\sqrt{\frac{\tau_S}{\tau_L}}\frac{N}{2}\right)} - \frac{2(\tau_\epsilon+\tau_S)}{\tau_\epsilon\sqrt{\tau_L}}\right)$.

Thus, we get our first solution,

$$\delta_1 = \frac{x_0^1}{\sqrt{\tau_\varepsilon(1-\rho^2)}} + x_2^1\sqrt{\tau_\varepsilon(1-\rho^2)} + O(1-\rho^2) \text{ if } \sqrt{\frac{\tau_S}{\tau_L}}\frac{N}{2} < 1.$$

Given this asymptotic, we can compute (recall $\delta = \sqrt{\tau_\pi/\tau_\epsilon}$)

$$\frac{d\tau_\pi}{d\tau_\epsilon} = 2\sqrt{\tau_\pi}\frac{d\sqrt{\tau_\pi}}{d\tau_\epsilon} = 2\sqrt{\tau_\pi}\left(\frac{N\left(4w_S\left(\sqrt{\frac{\tau_S}{\tau_L}}\frac{N}{2} - \frac{N}{2}\right) + w_L\right)}{8\sqrt{\tau_L}w_S\left(1 - \sqrt{\frac{\tau_S}{\tau_L}}\frac{N}{2}\right)} + O(a^2)\right)$$

It becomes negative infinity when $w_S\left(\sqrt{\frac{\tau_S}{\tau_L}}\frac{N}{2} - \frac{N}{2}\right) + w_L < 0$ and $\rho \to 0$. Thus, both τ_π and $\mathcal{I}^{RPE}$ will be decreasing in τ_ϵ for $\sqrt{\frac{\tau_S}{\tau_L}}\frac{N}{2} < 1$, ψ small enough and ρ sufficiently close to 1. It also follows that λ is increasing in ρ.

Case 2. $x_0 = x_0^2 \equiv \sqrt{\tau_S}$**.**

This is a more delicate case. One can find that $x_1 = 0$ by plugging into (IA.3) and collecting the terms of the same order in a. Then, the limiting λ is finite so one can write

$$\lambda(x) = \underbrace{\left(\frac{N\tau_S w_S \left(\frac{\sqrt{\tau_S}}{2\tau_\epsilon} + x_2 \right)}{\sqrt{\tau_L}} - w_L \right)}_{\equiv \lambda_0} + O\left(a^1\right).$$

Similarly,

$$\begin{aligned} \frac{x}{a}\left(\frac{x}{a} - \frac{\rho\sqrt{\tau_S}}{a}\right) &= \sqrt{\tau_S}\left(\frac{\sqrt{\tau_S}}{2\tau_\epsilon} + x_2\right) + O\left(a^1\right) \\ &= \frac{\lambda_0 + w_L}{N w_S}\sqrt{\frac{\tau_L}{\tau_S}} + O\left(a^1\right). \end{aligned}$$

Plugging this into (IA.3), and matching zero-order terms we get the quadratic equation on λ_0:

$$\lambda_0^2\left(1 - \frac{2}{N}\sqrt{\frac{\tau_L}{\tau_S}}\right) + \lambda_0\left(w_L\left(1 - \frac{3}{N}\sqrt{\frac{\tau_L}{\tau_S}}\right) + (N-2)\,w_S\right) - \frac{w_L^2}{N}\sqrt{\frac{\tau_L}{\tau_S}} - w_L w_S = 0 \quad \text{(IA.5)}$$

We are looking for the solutions such that $\lambda_0 > -w_L/2$.

Case 2a. $\sqrt{\frac{\tau_S}{\tau_L}}\frac{N}{2} > 1$. Examining (IA.5) we see that the quadratic is negative at $\lambda_0 = -w_L/2$ and is positive for λ_0 large enough. Thus, there exists unique λ_0 solving (IA.5) such that $\lambda_0 > -w_L/2$. Denote it λ_0^* (the greater root of (IA.5)) and note that λ_0^* does not depend on τ_ϵ.

Denote $x_2^2 = \frac{\lambda_0^* + w_L}{N\tau_S w_S}\sqrt{\tau_L} - \frac{\sqrt{\tau_S}}{2\tau_\epsilon}$. We have

$$\delta_2 = \frac{x_0^2}{\sqrt{\tau_\varepsilon(1-\rho^2)}} + x_2^2\sqrt{\tau_\varepsilon(1-\rho^2)} + O(1-\rho^2) \text{ if } \sqrt{\frac{\tau_S}{\tau_L}}\frac{N}{2} > 1.$$

One can show that in the case 2a τ_π always increases in τ_ϵ. Also, one can show that third-order terms for expansion of x in a are zero, from which it follows $\lambda a \to 0$ as $a \to 0$. Since a is decreasing in ρ it follows that λ is increasing in ρ.

Case 2b. $\sqrt{\frac{\tau_S}{\tau_L}}\frac{N}{2} < 1$. In this case, depending on parameters, we might either have 0 or 2 roots of (IA.5) such that $\lambda_0 > -w_L/2$. (One root is not possible because (IA.5) is negative at $\lambda_0 = -w_L/2$). One can compute the discriminant and the roots of (IA.5) in closed-form, which gives necessary and sufficient conditions for zero or two such roots to exist. We also note that w_L/w_S small enough is sufficient to guarantee existence of two such roots. ■

The theorem above demonstrates that the main consequences of complementarity are still there in the limit of $\rho \to 1$: multiple equilibria are possible (up to three) and the informational efficiency might decrease in τ_ϵ. We also derive the comparative statics with respect to ρ, when ρ is large: (i) informational efficiency is increasing in ρ (less noise when ρ is larger) and (ii) liquidity is decreasing in ρ (less gains from trade when ρ is larger).

IA.3 An Alternative Information Structure

In the main part of the paper we assume that small traders receive dispersed signals about *their* value, $s_j = v_S + \epsilon_j$. Here, we show that our results are unchanged if they receive dispersed information about the fundamental v. In particular, we suppose that $s_j = v + \epsilon_j$, where the assumptions about joint distribution of v and ϵ_j, $j \in [0, 1]$ is as in the main part of the paper. (Recall that in the main part of the paper, the signal $s_j = v_S + \epsilon_j$ bundles information about the private value u_S and the fundamental value v.) Here, we assume that small traders have some common source of information about u_S and so u_S has two components, $u_S = u_S^n + u_S^k$. The component u_S^k(resp., u_S^n) is known (resp., not known) to small traders and the two components are jointly normally distributed and independent from each other and all other random variables in the model.[33] Neither part of u_S is known to large traders.

We start by revisiting the equilibrium characterization. Unlike in the main model, we denote

$$\rho = corr(v, v_L) \text{ and } \tau_S = Var(v)^{-1}.$$

This is a convenient notation as it allows to keep the formulations of the propositions from main part of the paper almost unchanged. We now state the formulation of the Theorem 1 for this information structure.

Theorem 1.IA.3. *There exists a sufficient static π that is informationally equivalent to the price p, such that $\pi = v + \zeta/\sqrt{\tau_\pi}$, where $\zeta \sim N(0, 1)$ and is independent of v. There exists at least one equilibrium. All equilibrium variables can be expressed in closed form by way of an endogenous variable $\delta \equiv \sqrt{\tau_\pi/\tau_\varepsilon}$. In particular, price impact can be expressed as*

$$\lambda(\delta) = \frac{Nw_S}{\sqrt{\tau_L}}(\delta\sqrt{\tau_\varepsilon(1-\rho^2)} - \rho\sqrt{\tau_S})\left(\delta^2 + \frac{\tau_S + \tau_\varepsilon}{\tau_\varepsilon}\right) - w_L.$$

The equilibrium δ is the solution to the sixth-order polynomial equation

$$\lambda(\delta)(w_L + Nw_S + \lambda(\delta)) - w_S\left(1 + \delta\left(\delta - \frac{\rho}{\sqrt{1-\rho^2}}\sqrt{\frac{\tau_S}{\tau_\varepsilon}}\right)\right)(w_L + 2\lambda(\delta)) = 0 \qquad \text{(IA.6)}$$

such that $\lambda(\delta) > -w_L/2$.

Proof of Theorem 1.IA.3. First, note that we can always incorporate the known part of the small traders' private value to the $\bar{v}_S$. Thus, without loss of generality, we proceed to assume that $u_S^k = 0$. The first-order conditions from can be summarized as follows:

$$x_j = \frac{E[v_S|s_j, p] - p}{w_S} \quad \text{and} \quad x_i = \frac{v_L - p}{w_L + \lambda}.$$

The second-order condition for large traders, $\lambda > -w_L/2$, must also hold. Since neither the

[33] E.g., small traders might have a common signal s_u about u_S in which case $u_S^k = E[u_S|s_u]$ and $u_S^n = u_S - E[u_S|s_u]$. Note that settings where small traders know their value perfectly or do not know it at all are special cases of the one considered here.

price p nor the signal s_j contains information about u_S^n and we set $u_S^k = 0$, we have $E[v_S|s_j, p] = E[v|s_j, p]$. Thus, the first-order conditions would be the same as in the economy from the main part of the paper and $v_S = v$. The statements of the proposition then follow. ■

We now formulate our proposition about welfare.

Proposition 3.IA.3. *Suppose that* $(\rho^2 - 1)\,\tau_\epsilon + \rho\sqrt{\tau_L \tau_S} - \tau_S > 0$, *then for sufficiently small* $h = w_L/(N w_S)$, *there exists an equilibrium where* WL *increases in* N. *Such an equilibrium is unique for large enough* N.

Proof of Proposition 3.IA.3. First, as in the proof of Theorem 1.IA.3 we assume $u_S^k = 0$. We will compute the expectation of WL given u_S^k, so such normalization is without loss of generality. (The unconditional statements follow directly from the conditional ones.) Then, note that RTW $= (v_S + u_S^n - v_L)\langle x_S \rangle - \frac{w_S}{2}\beta_S^2 \tau_\epsilon^2 - \frac{w_S + w_L/N}{2}\langle x_S \rangle^2$. Taking expectation with respect to u_S^n, the expression becomes identical to that in the proof of Proposition 3. The rest of the proof then proceeds as in Proposition 3. ■

Next, we proceed to our result about informational efficiency.

Proposition 4.IA.3. *There exist* $\underline{\tau}_\varepsilon$, $\underline{\tau}_S$, *and* $\underline{h}$ *such that, for all* $\tau_\varepsilon < \underline{\tau}_\varepsilon$, $\tau_S < \underline{\tau}_S$ *and* $h \equiv \frac{w_L}{w_S} < \underline{h}$: *there exists a unique equilibrium in which informational efficiency* $\mathcal{I}^{FPE}$ *decreases as signal precision* τ_ε *increases for* $\tau_{u_S^k} = Var(u_S^k)^{-1}$ *large enough.*

Proof of Proposition 4.IA.3. First, we note that in this proposition we cannot incorporate u_S^k to $\bar{v}_S$, since an 'econometrician', from whose perspective the informational efficiency is defined, does not know it. From the perspective of an 'econometrician' who only knows s_j and p the price is informationally equivalent to $\beta_S v + N\beta v_L + \frac{1}{w_S}u_S^k$. After substituting v_L from Lemma 3 and undertaking some rearrangement, we obtain that the price is informationally equivalent to $\pi = v + \frac{u_S^k}{w_S \beta_S} + (1/\sqrt{\tau_\pi})\zeta$, where $\zeta = 1/C(v_L - A - Bv)$ (see Lemma 3).

Next, we use the law of total variance to write

$$\mathrm{Var}(v|p, s_j) = \mathrm{Var}\left(v|p, s_j, u_s^k\right) + \mathrm{Var}\left(E[v|p, s_j, u_s^k]|p, s_j\right).$$

The first can be computed invoking Lemma 4:

$$\mathrm{Var}(v|p, s_j) = \tau_\eta^{-1} + \frac{\left(k_S + k_L \rho\sqrt{\tau_S/\tau_L} - k_L\sqrt{(1-\rho^2)\,\tau_\pi/\tau_L}\right)^2}{\tau_S + \tau_\epsilon + \tau_\pi}.$$

The second term is given by

$$\mathrm{Var}\left(E[v|p, s_j, u_s^k]|p, s_j\right) = \left(\frac{\tau_\pi}{\tau_\pi + \tau_S + \tau_\epsilon}\right)^2 \frac{\tau_{u_S^k}^{-1}}{w_S^2 \beta_S^2}.$$

Note that $\tau_{u_S^k}$ does not affect the comparative statics of the first term. Thus, it will be a dominant term for large $\tau_{u_S^k}$. The result then follows from Proposition 4. ■

IA.4 A Model with Uninformed Large Traders

Here we consider a model that differs from the one in Section 3 only in that large traders do not know their values perfectly. Instead, a large trader i is endowed with a signal $s_i = v_L + n_i$, where the n_i are i.i.d. as $n_i \sim N(0, 1/\tau_n)$ and are independent of v_S and v_L.

We consider symmetric linear equilibria in which a large trader i and a small trader j have the following demand schedules

$$x_i = \alpha + \beta \cdot s_i - \gamma \cdot p \quad \text{and} \quad x_j = \alpha_S + \beta_S \cdot s_j - \gamma_S \cdot p, \tag{IA.7}$$

respectively. The coefficients (α, β, γ) and $(\alpha_S, \beta_S, \gamma_S)$ are identical for traders within the same group.

Since both groups of traders learn in the extended model, we introduce two measures of revelatory price efficiency, each one defined from the perspective of small and large traders as follows:

$$\mathcal{I}^S = \frac{\text{Var}(v_S)}{\text{Var}(v_S|s_j, p)}, \qquad \mathcal{I}^L = \frac{\text{Var}(v_L)}{\text{Var}(v_L|s_j, p)}.$$

The main results of this section are that (a) the complementarity described in Section 4 continues to hold in this extended setting and (b) an increase in the precision of small traders' signals can reduce informational efficiency both for large and small traders.

As in Section 4, we examine the mechanism's first part by fixing the demand parameters (α, β, γ) for large traders. Given these exogenously postulated demands for large traders, small traders rationally maximize their utilities. We then analyze (in Lemma 1.IA.4) how a change in β affects $\mathcal{I}^S$—and the amount of liquidity provided by small traders, γ_S—while keeping everything else fixed.

To examine the second part of the mechanism, we fix the demand parameters $(\alpha_S, \beta_S, \gamma_S)$ for small traders. Given these exogenously postulated demands for small traders, large traders rationally maximize their utilities. We then analyze (in Lemma 2.IA.4) how a change in γ_S affects liquidity ($\mathcal{L}$) and how aggressively large traders trade (β) while keeping everything else fixed. The full equilibrium is analyzed in Theorem 1.IA.4.

Lemma 1.IA.4. *The equilibrium price is informationally equivalent to a sufficient statistic $\pi \equiv v_S + (1/\sqrt{\tau_\pi})\zeta_u$, where $\zeta_u \sim N(0,1)$ is independent of v_S and where τ_π is the sufficient statistic's precision, as follows:*

$$\tau_\pi \equiv \text{Var}[\pi|v_S]^{-1} = \left(\left(\frac{\tau_L}{1-\rho^2}\left(\rho\sqrt{\frac{\tau_S}{\tau_L}} + \frac{\beta_S}{N\beta}\right)^2\right)^{-1} + \frac{N\beta^2}{\tau_n}\right)^{-1}.$$

The revelatory price efficiency for small traders can be written as

$$\mathcal{I}^S = \frac{\tau_S + \tau_\varepsilon + \tau_\pi}{\tau_S}. \tag{IA.8}$$

Small trader j's demand is given by $x_j = (E[v_j|s_j, p] - p)/w_S$, and her price sensitivity can be *written as*

$$\gamma_S = \underbrace{\frac{1}{w_S}}_{\textit{expenditure effect}} - \underbrace{\frac{1}{w_S}\frac{\partial E[v_S|s_j,p]}{\partial p}}_{>0,\ \textit{information effect}}.$$

Both τ_π and $\mathcal{I}^S$ are decreasing in β. The information effect, $\frac{\partial E[v_S|s_j,p]}{\partial p}$, is decreasing in β, whereas the expenditure effect, $1/w_S$, is independent of β; as a result, γ_S is increasing in β. Therefore, if large traders trade more aggressively, then the price is less informative for small traders and they provide more liquidity.

Proof of Lemma 1.IA.4. The price is informationally equivalent to $\beta_S v_S + N\beta v_L + \sum_{i=1}^N n_i$. After substituting v_L from Lemma 3 and then rearranging, we find that the price is informationally equivalent to $\pi \equiv v_S + (1/\sqrt{\tau_\pi})\zeta_u$, where

$$\tau_\pi \equiv \mathrm{Var}[\pi|v_S]^{-1} = \left(\left(\frac{\tau_L}{1-\rho^2}\left(\rho\sqrt{\frac{\tau_S}{\tau_L}} + \frac{\beta_S}{N\beta}\right)^2\right)^{-1} + \frac{N\beta^2}{\tau_n}\right)^{-1}. \tag{IA.9}$$

The formula for informational efficiency now follows directly from the projection theorem. We can see from the displayed formula that τ_π and hence $\mathcal{I}^S$ decrease as β increases.

The optimal demand of a small trader j can be written as $x_j = \frac{E[v_s|s_j,p]-p}{w_S}$. Then $\gamma_S = \frac{1}{w_S} - \frac{\partial E[v_s|s_j,p]}{\partial p}$. Now we write $E[v_s|s_j,p] = \frac{\tau_\pi}{\tau_S+\tau_\varepsilon+\tau_\pi}\pi + ...$; here, as before, "..." stands for terms that do not depend on p. One can write $\pi = \frac{\gamma_S+N\gamma}{\beta_S+N\beta\rho\sqrt{\tau_S/\tau_L}}p + ...$, from which (after some rearrangement) it follows that

$$\gamma_S = \frac{\frac{1}{w_S} - \frac{\tau_\pi}{\tau_S+\tau_\varepsilon+\tau_\pi}\frac{N\gamma}{\beta_S+N\beta\rho\sqrt{\tau_S/\tau_L}}}{1+\frac{\tau_\pi}{\tau_S+\tau_\varepsilon+\tau_\pi}\frac{1}{\beta_S+N\beta\rho\sqrt{\tau_S/\tau_L}}}.$$

It can be seen from this expression that γ_S decreases in β. ■

This lemma reveals that steps (1) and (2) of the equilibrium loop in Figure 1 continue to hold. The intuition for the first step is similar to that given in Section 4: Since large traders create noise in the price for small traders, it follows that large traders trade more aggressively, and inject more noise into the price for small traders, which makes it less informative to them. Step (2) in Figure 1 is also addressed by Lemma 1.IA.4: small traders provide more liquidity when the price is less informative to them. The information effect is weaker the less informative the price is, whereas the expenditure effect is unaffected by price informativeness, so when price is less informative, small traders provide more liquidity.

Lemma 2.IA.4. *Both liquidity $\mathcal{L}$ and aggressiveness β are increasing in γ_S, ceteris paribus. Therefore, if small traders provide more liquidity then the market becomes more liquid and large traders trade more aggressively.*

Proof of Lemma 2.IA.4. Let $x = \beta_S/\beta$ and $k = \beta_S(\rho\sqrt{\tau_L/\tau_S} + (N-1)/x)$. We can then write

$$\beta = \frac{\frac{\tau_n}{\tau_\iota + \tau_L + \tau_n}}{(\tau_\iota + \tau_L + \tau_n)(\frac{\tau_\iota}{k(\tau_\iota + \tau_L + \tau_n)} + \lambda + w_L)},$$

where τ_ι is the precision of the price from the perspective of large traders; this precision is independent of γ_S. Therefore β depends on γ_S only through λ. For the price sensitivity of large traders' demands, we can write

$$\gamma = \frac{1 - \frac{\tau_\iota}{\lambda k(\tau_\iota + \tau_L + \tau_n)}}{\frac{\tau_\iota}{k(\tau_\iota + \tau_L + \tau_n)} + \lambda + w_L}.$$

With $1/\lambda = (N-1)\gamma + \gamma_S$ the displayed equality implies that

$$1 - \gamma_S \lambda = (N-1)\frac{\lambda - \frac{\tau_\iota}{k(\tau_\iota + \tau_L + \tau_n)}}{\frac{\tau_\iota}{k(\tau_\iota + \tau_L + \tau_n)} + \lambda + w_L},$$

from which we can see that an increase in γ_S leads to a decrease in λ. ■

As small traders provide more liquidity, the overall liquidity of the market improves. This corresponds to step (3) in Figure 1. An improvement in liquidity reduces the price impact of large traders. Since large traders are strategic and take their own price impact into account, if that impact is lower, then they trade more aggressively. This behavior corresponds to step (4) in the figure. Thus the preceding two propositions confirm that complementarity is present also in the extended model.The full equilibrium is characterized in the following theorem.

Theorem 1.IA.4. *All equilibrium variables can be expressed in closed form through two endogenous variables: β_S/β and λ. The equilibrium is a solution to a system of two nonlinear algebraic equations presented in the proof below.*

Proof of Theorem 1.IA.4. Denote, only in this proof $x = \beta_S/\beta$. Following the steps of Lemmas 1.IA.4 and 2.IA.4, we can write

$$\tau_\pi = \left(\left(\frac{\tau_L}{1-\rho^2}\left(\rho\sqrt{\frac{\tau_S}{\tau_L}} + \frac{x}{N}\right)^2\right)^{-1} + \frac{Nx^2\beta_S^2}{\tau_n}\right)^{-1}$$

and

$$\beta_S = \frac{\tau_\varepsilon}{w_S(\tau_\pi + \tau_S + \tau_\varepsilon)}.$$

These two equalities allow one to express β_S through x in closed form. The elasticity can be written as

$$\gamma_S = \frac{1}{w_S}\left(1 - \frac{\tau_\pi}{\tau_S + \tau_\varepsilon + \tau_\pi}\frac{\gamma + 1/\lambda}{\beta_S(x)(1 + (N/x)\rho\sqrt{\tau_S/\tau_L})}\right),$$

which depends on x, λ, and γ. We now provide the following closed-form expression for γ as a function of λ and x:

$$\gamma = \frac{1 - \frac{\tau_\iota}{\lambda k(\tau_\iota + \tau_L + \tau_n)}}{\frac{\tau_\iota}{k(\tau_\iota + \tau_L + \tau_n)} + \lambda + w_L}.$$

Then β is given by

$$\beta = \frac{\frac{\tau_n}{\tau_\iota + \tau_L + \tau_n}}{(\tau_\iota + \tau_L + \tau_n)(\frac{\tau_\iota}{k(\tau_\iota + \tau_L + \tau_n)} + \lambda + w_L)}$$

and τ_ι can be expressed as

$$\tau_\iota = \left((N-1) \left(\frac{1}{N - 1 + x\rho\sqrt{\tau_L/\tau_S}} \right)^2 \frac{1}{\tau_n} + \frac{\beta_S(x)^2(1-\rho^2)}{\tau_S k^2} \right)^{-1}.$$

Finally, the equilibrium values of x and λ solve

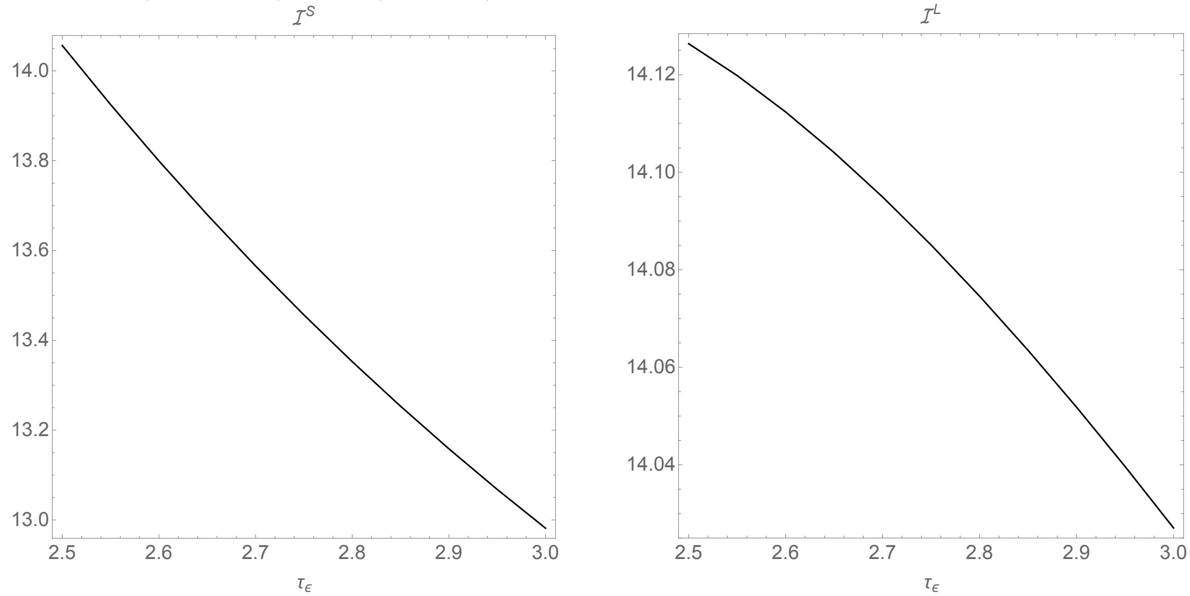
$$x = \frac{\beta_S}{\beta} \quad \text{and} \quad \frac{1}{\lambda} = \gamma_S + (N-1)\gamma,$$

respectively. ■

A central result in Section 6 is that price can be less informative for small traders (i.e., $\mathcal{I}^{RPE}$ can decrease) as the quality of their private information increases (i.e., as τ_ε increases). This outcome is possible because, with more informative signals, small traders provide more liquidity and thus make the market more liquid for large traders, who then trade more aggressively and thereby inject more noise into the price. Is it possible that price becomes less informative for large traders as well? The answer is yes. The reason is that when large traders trade more aggressively, they load more, not only on their value v_L but also on the noise n_i in their signals. Since there are few large traders, that noise does not vanish. This result is illustrated in Figure IA.1.

Figure IA.1: Effect of precision τ_ε on informational efficiency.

The graphs plot small investors' informational efficiency $\mathcal{I}^S$ (Panel (a)) and large investors' informational efficiency $\mathcal{I}^L$ (Panel (b)) as a function of τ_ε. Parameter values are $N = 13$, $\bar{v}_L = \bar{v}_S = 0$, $\tau_S = 1.5$, $\tau_\varepsilon = 1$, $\tau_L = 4$, and $\tau_n = 5$.



(a) (b)

IA.5 A Model with Heterogeneous Private Values

In the main part of the paper we assume that all large traders have the same value v_L and all small traders the same value v_S. Here, we show that our results are unchanged if large/small traders' ($k \in \{L, S\}$) private values are given by $a_L v + u_k^c + u_k^i$ with the common component $u_k^c \sim N(0, \tau_{u,k,c}^{-1})$ and an independent idiosyncratic component u_k^i i.i.d. $u_k^i \sim N(0, \tau_{u,k,i}^{-1})$. We assume large traders know all components of their value, but not that of other traders. We also assume that the only information small traders have about their value is given by $s_j = v + \epsilon_j$.

Unlike in the main model, we denote

$$\rho = corr\left(a_S v, a_L v + u_L + \frac{1}{N}\sum_i u_L^i\right), \tau_L = Var\left(a_L v + u_L + \frac{1}{N}\sum_i u_L^i\right)^{-1},$$

$$\tau_{u,L} = Var\left(u_L + \frac{1}{N}\sum_i u_L^i\right)^{-1}.$$

This is a convenient notation as it allows to keep the formulations of the propositions from main part of the paper almost unchanged. We now state formulation of the Theorem 1 for this setting.

Theorem 1.IA.5. *There exists a sufficient static π, that is infomationally equivalent to the price p, such that $\pi = v + \zeta/\sqrt{\tau_\pi}$, where $\zeta \sim N(0, 1)$ and is independent of v. There exists at* *least one equilibrium. All equilibrium variables can be expressed in closed form by way of an endogenous variable $\delta \equiv \sqrt{\tau_\pi/\tau_\varepsilon}$. In particular, price impact can be expressed as*

$$\lambda(\delta) = \frac{Nw_S}{\sqrt{\tau_L}}(\delta\sqrt{\tau_\varepsilon(1-\rho^2)} - \rho\sqrt{\tau_S})\left(\delta^2 + \frac{\tau_S+\tau_\varepsilon}{\tau_\varepsilon}\right) - w_L.$$

The equilibrium δ is the solution to the sixth-order polynomial equation

$$\lambda(\delta)(w_L + Nw_S + \lambda(\delta)) - w_S\left(1 + \delta\left(\delta - \frac{\rho}{\sqrt{1-\rho^2}}\sqrt{\frac{\tau_S}{\tau_\varepsilon}}\right)\right)(w_L + 2\lambda(\delta)) = 0 \qquad \text{(IA.10)}$$

such that $\lambda(\delta) > -w_L/2$.

Proof of Theorem 1.IA.5. The first-order conditions can be summarized as follows:

$$x_j = \frac{E[a_S v + u_S|s_j, p] - p}{w_S} + \frac{u_S^j}{w_S} \quad \text{and} \quad x_i = \frac{a_L v + u_L + u_L^i - p}{w_L + \lambda}.$$

The second-order condition for large traders, $\lambda > -w_L/2$, must also hold. Note that neither the price, nor the signal contain information about u_S, hence $E[u_S|s_j, p] = E[u_S] = 0$. The aggregate demand of small traders is $\int x_j dj = \int \frac{E[a_S v|s_j,p]-p}{w_S} dj$. Thus, the aggregate demand of small traders is as in the main model with small trades value equal to $a_S v$. Similarly, the aggregate demand of large traders is $\sum_i x_i = \sum_i \frac{a_L v + u_L + 1/N \sum_j u_L^j - p}{w_L + \lambda}$. Thus, the aggregate demand of large traders is as in the main model where all of them have value $a_L v + u_L + 1/N \sum_j u_L^j$. The equilibrium is then pinned down by the same conditions as in the main model with values $a_S v$ and $a_L v + u_L + 1/N \sum_j u_L^j$ for small and large traders, respectively. The proposition then follows. ■

Next we turn to the proposition concerning informational efficiency.

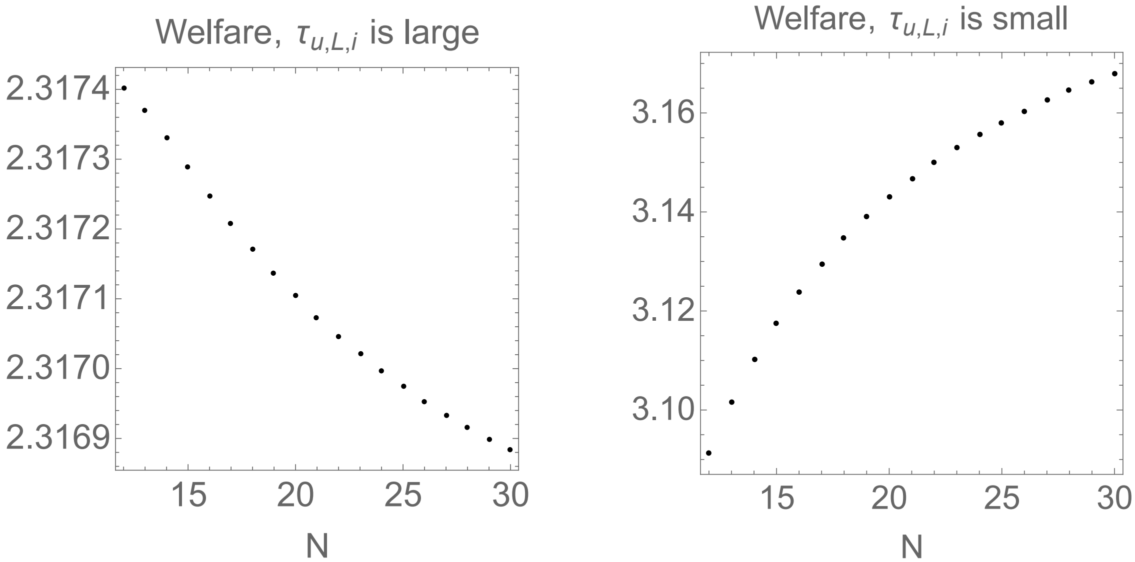
Proposition 4.IA.5. *Suppose Assumption 1 holds. Suppose that $2 \le N < \bar{N} \equiv 2a_S/a_L$ and $h \equiv \frac{w_L}{Nw_S} < \bar{h} \equiv 2 - \frac{2}{N} - \frac{a_L}{a_S}$. For small enough τ_ϵ, τ_v and $\tau_{u,L}$ there exists an equilibrium in which $\mathcal{I}^{FPE}$ and $\mathcal{I}^{RPE}$ both decrease as signal precision τ_ε increases. Such equilibrium is unique, provided that condition (26) in the Appendix holds.*

Proof of Proposition 4.IA.5. The proof is identical to that of Proposition 4 since the equilibrium conditions and expressions for informational efficiency are identical to that in the main model with values $a_S v$ and $a_L v + u_L + 1/N \sum_i u_L^i$ for small and large traders, respectively. ■

The model with heterogeneous private values is much less tractable when it comes to welfare. We are not able to prove analytically the analog of Proposition 3 in this case. We thus investigate the role of the two key new parameters, $\tau_{u,k,S}$ and $\tau_{u,k,L}$ numerically. Our simulations show that $\tau_{u,S}$ does not affect the comparative statics welfare in (N). This is because with a continuum of traders, u_S^i wash out in equilibrium and it can be shown that their contribution to welfare is given by $(\tau_{u,S,i})^{-1}/(2w_S)$, so u_S^i does not interact with N. The private values of large traders affect welfare through two novel channels. First, as N increases the

Figure IA.2: Effect of the extent of competition on welfare.

The graphs plot aggregate welfare as functions of N when $\tau_{u,S,i} = 10$ (Panel (a)) and $\tau_{u,S,i} = 1$ (Panel (b)). Other parameter values are $\bar{v}_L = \bar{v}_S = 0$, $a_S = 1$, $a_L = 0.4$ $var(v)^{-1} = 0.358$, $var(u_S^c)^{-1} = 0.139$, $var(u_L^c)^{-1} = 18.9$, $\tau_\epsilon = 0.1$, $w_L = N/2$, and $w_S = 1$.



(a) (b)

variance of $1/N \sum_i u_L^i$ decreases, implying less noise in the price. Second, each additional large trader has not only gains from trade with small traders (as in the main model) but also with large traders (new). Both of these channels tend to contribute to welfare increasing in N. We therefore find numerically that when $\tau_{u,L,i}$ is small enough (large enough heterogeneity in u_L^i), welfare increases in N. Figure IA.2 illustrates it: panel (a) has large $\tau_{u,L}$, and demonstrates the unconventional result; panel (b) has small $\tau_{u,L}$ and the conventional result prevails.

IA.6 A dynamic model

In this section we consider the following dynamic model. The trade happens at $t = 0$ and $t = 1/2$. The period $t = 0$ is as in the main model: small traders with value v_S trade with large traders with value v_L. We assume that large traders know both v_S and v_L. This implies that large traders have no uncertainty about the residual supply, which might result in the multiplicity of equilibria, as in Klemperer and Meyer (1989). To avoid such multiplicity we apply the robust Nash equilibrium selection criterion of Rostek and Weretka (2015a): We focus on large traders demands that are optimal even after adding full-support uncertain additive noise to their residual demand. At time $t = 1/2$ the values v_S and v_L are realized. Therefore, there is no informational friction at that time. In both periods, traders have quadratic inventory costs $w_k\, e_{k,t}^2/2$, $k \in \{L, S\}$, where $e_{k,t}$ denotes the inventory at the *end* of period t. We assume that at $t = 0$ small traders have information about the price at time $t = 1/2$, $s_j = p^*_{1/2} + \epsilon_j$. Thus, their information is short-term, rather than long-term.[34]

We will show that $t = 0$ demands of traders in the dynamic model coincide with that in the following static model, considered in the main part of the paper. Small traders with value $\hat{v}_S = m_S v_S + (1 - m_S)v_L$ trade with large traders with value $\hat{v}_L = m_L v_S + (1 - m_L)v_L$, where constants $m_S, m_L \in (0, 1)$. Small traders have dispersed information about v_S: $s_j = v_S + \epsilon_j$. Traders have quadratic inventory costs $\hat{w}_k\, e_k^2/2$, $k \in \{L, S\}$, where e_k denotes the inventory at the end of period 0. Such a model is a particular case of the model in the main part of the paper. Therefore, the equilibrium characterization and the proposition concerning informational efficiency follow immediately from those in that section. In the propositions that follow we refer to the two models just described simply as *dynamic* and *static*, respectively.

We now establish our main result, the isomorphism between the demands in the static and dynamic models.

Theorem IA.2. *Let $\lambda_{1/2}$ be the unique positive root of the quadratic equation $\frac{1}{\lambda_{1/2}} = \frac{1}{w_S} + \frac{N-1}{w_L+\lambda_{1/2}}$. The measures of informational efficiency and the demands of small and large traders in the dynamic model is the same as that in the static model with $\hat{v}_S = m_S v_S + (1 - m_S)v_L$, $\hat{v}_L = m_L v_S + (1-m_L)v_L$, $\hat{w}_S = w_S$ and $\hat{w}_L = 2w_L - \frac{(w_L+2\lambda_{1/2})w_L^2}{(w_L+\lambda_{1/2})^2} > w_L$, where $m_S = \frac{\frac{1}{w_S}}{\frac{1}{w_S}+\frac{N}{w_L+\lambda_{1/2}}}$ and $m_L = \frac{(w_L+2\lambda_{1/2})w_L}{(w_L+\lambda_{1/2})^2} m_S$. The equilibrium time-1/2 price is given by $p^* = \hat{v}_S$.*

Note that in a static model $\hat{v}_S$ loads positively on values of traders from other group, v_L, and similarly $\hat{v_L}$ loads positively on v_S. This is because in the dynamic model, at $t = 0$ traders care not only about the valuation of the asset but also about price next period, which is driven by both v_S and v_L. Suppose that v_S and v_L are independent. Then in a static model we have $corr(v_S, v_L) > 0$. The dynamic considerations increase the correlations of the values, reflecting the decreased gains from trade at $t = 0$ (part of gains from trade will be realized at $t = 1/2$).

Proof of Theorem IA.2.

[34] Our analysis complicates significantly if small traders have information about v_S or about v, as opposed to $p^*_{1/2}$, but the numerical analysis shows that the main takeaways still apply.

We proceed backwards and start from $t = 1/2$.

Lemma IA.1. *The time-1/2 value function of small traders is given by* $V_S^{j,*}\left(e_S^j, \left\langle e_S^j\right\rangle, \left\langle e_L^j\right\rangle\right) = E_i\left[\frac{w_S}{2}\left(y_S^{j,*}\right)^2 - \frac{w_S}{2}\left(e_S^j\right)^2\right]$, *that of a large trader is* $V_L^{i,*}\left(e_L^i, \left\langle e_S^j\right\rangle, \left\langle e_L^i\right\rangle\right) = \frac{w_L+2\lambda_{1/2}}{2}\left(y_L^{i,*}\right)^2 - \frac{w_L}{2}\left(e_L^i\right)^2$, *where* $\left\langle e_S^j\right\rangle \equiv \int e_S^j dj$, $\left\langle e_L^i\right\rangle = \frac{1}{N}\sum_i e_L^i$, *the equilibrium time-1/2 price is given by* $p_{1/2}^*\left(\left\langle e_S^i\right\rangle, \left\langle e_L^i\right\rangle\right) = \frac{\frac{v_S}{w_S}+\frac{Nv_L}{w_L+\lambda}-\left\langle e_S^i\right\rangle - N\left\langle e_L^i\right\rangle}{\frac{1}{w_S}+\frac{N}{w_L+\lambda_{1/2}}}$ *and time-1/2 price impact* $\lambda_{1/2}$ *is the unique positive root of quadrtic equation* $\frac{1}{\lambda_{1/2}} = \frac{1}{w_s} + \frac{N-1}{w_L+\lambda_{1/2}}$, *and allocations are given by* $y_S^{j,*} = \frac{v_S - w_S e_S^j - p_{1/2}^*}{w_S}$ *and* $y_L^{i,*} = \frac{v_L - w_L e_L^i - p_{1/2}^*}{w_L+\lambda_{1/2}}$

Proof of Lemma IA.1. We start with the FOCs. The small trader's problem can be formulated as

$$\max_y \left(v_S - p\right) y - \frac{w_S}{2}\left(e_S^j + y\right)^2 \implies \tag{IA.11}$$

$$y_S^{j,*} = \frac{v_S - w_S e_S^j - p}{w_S} \tag{IA.12}$$

Similarly, large traders solve, while accounting for their price impact (i.e., accounting for the fact that $p = const + \lambda_{1/2} y$)

$$\max_y \left(v_L - p\right) y - \frac{w_L}{2}\left(e_L^i + y\right)^2 \implies$$

$$y_L^{i,*} = \frac{v_L - w_L e_L^i - p_{1/2}^*}{w_L + \lambda_{1/2}}.$$

The price impact satisfies the consistency condition

$$\frac{1}{\lambda_{1/2}} = \frac{1}{w_S} + \frac{N-1}{w_L + \lambda_{1/2}}.$$

The unique positive root of the above quadratic equation is the only root that SOCs hold for large traders in equilibrium.

Substitiuting (IA.12) back to (IA.11), we get

$$V_S^{j,*} = E_i\left[\frac{w_S}{2}\left(y_S^{j,*}\right)^2 - \frac{w_S}{2}\left(e_S^j\right)^2\right].$$

For large traders, we get

$$V_L^{i,*} = \frac{w_L + 2\lambda_{1/2}}{2}\left(y_L^{i,*}\right)^2 - \frac{w_L}{2}\left(e_S^i\right)^2$$

■

We now look at the $t = 0$ problem.

Lemma IA.2. *The demand of large traders in the dynamic model is the same as in the main model with value* $\hat{v}_L = m_L v_S + (1 - m_L)v_L$*, and inventory cost parameter* $\hat{w}_L = 2w_L - \frac{(w_L+2\lambda_{1/2})w_L^2}{(w_L+\lambda_{1/2})^2} > w_L$*, where* $0 < m_L = \frac{(w_L+2\lambda_{1/2})w_L}{(w_L+\lambda_{1/2})^2} \frac{\frac{1}{w_S}}{\frac{1}{w_S}+\frac{N}{w_L+\lambda_{1/2}}} < 1$ *and* $\lambda_{1/2}$ *is given in Lemma IA.1.*

Proof of Lemma IA.2. Large traders maximize

$$\max_x (v_L - p)\, x - \frac{w_L}{2}x^2 + \frac{w_L + 2\lambda_{1/2}}{2} \left(y_L^{i,*}\right)^2 - \frac{w_L}{2} \left(e_S^i\right)^2$$

They do so accounting for their price impact (i.e. accounting for the fact that $p = const + \lambda_0 x$). The FOC is given by

$$v_L - p - \lambda_0 x - w_L x + \left(w_L + 2\lambda_{1/2}\right) y_L^{i,*} \frac{\partial y_L^{i,*}}{\partial x} - w_L x$$

$$\frac{\partial y_L^{i,*}}{\partial x} = \frac{-w_L - \partial p_{1/2}^* / \partial x}{w_L + \lambda_{1/2}}$$

Now note that

$$\frac{\partial p_{1/2}^*}{\partial x} = \frac{\gamma_S \lambda_0 - 1 + (N-1)\,\gamma_L \lambda_0}{\frac{1}{w_S} + \frac{N}{w_L+\lambda_{1/2}}} = 0$$

The last one is because $\frac{1}{\lambda_0} = \gamma_S + (N-1)\,\gamma_L$. The fact that $\frac{\partial p_{1/2}^*}{\partial x} = 0$ makes sense: The only way a large trader can affect the price $p_{1/2}^*$ is through the impact on total endowments, $\langle e_S^i \rangle + N \langle e_L^i \rangle$. But by market clearing, in equilibrium, $\langle e_S^i \rangle + N \langle e_L^i \rangle = 0$ (endowments are equal to time-0 trades).

Thus we have the FOC

$$v_L - p - (\lambda_0 + 2w_L)\, x - \left(w_L + 2\lambda_{1/2}\right) \frac{v_L - w_L x - p_{1/2}^*}{w_L + \lambda_{1/2}} \frac{w_L}{w_L + \lambda_{1/2}} = 0$$

Rearranging we get

$$v_L - p - \left(\lambda_0 + 2w_L - \frac{\left(w_L + 2\lambda_{1/2}\right) w_L^2}{(w_L + \lambda_{1/2})^2}\right) x - \left(w_L + 2\lambda_{1/2}\right) \frac{v_L - p_{1/2}^*}{(w_L + \lambda_{1/2})^2} w_L = 0$$

And in equilibrium $\langle e_S^i \rangle + N \langle e_L^i \rangle = 0$ and so $p_{1/2}^* = \frac{\frac{v_S}{w_S} + \frac{N v_L}{w_L + \lambda}}{\frac{1}{w_S} + \frac{N}{w_L+\lambda_{1/2}}}$. Thus,

$$v_L - p_{1/2}^* = v_L - \frac{\frac{v_S}{w_S} + \frac{N v_L}{w_L+\lambda}}{\frac{1}{w_S} + \frac{N}{w_L+\lambda_{1/2}}} = \frac{\frac{v_L - v_S}{w_S}}{\frac{1}{w_S} + \frac{N}{w_L+\lambda_{1/2}}}.$$

This FOC is the FOC of a large trader from the main model with value

$$\hat{v}_L = v_L - \underbrace{\frac{\left(w_L + 2\lambda_{1/2}\right) w_L}{\left(w_L + \lambda_{1/2}\right)^2} \frac{\frac{1}{w_S}}{\frac{1}{w_S} + \frac{N}{w_L + \lambda_{1/2}}}}_{<1} (v_L - v_S)$$

and the inventory cost parameter

$$\hat{w}_L = 2w_L - \frac{\left(w_L + 2\lambda_{1/2}\right) w_L^2}{(w_L + \lambda_{1/2})^2} > w_L.$$

It is straightforward to verify that the second-order conditions hold for $\lambda_0 > 0$, which will be true in equilibrium. ■

Lemma IA.3. *The demand of small traders in the dynamic model is the same as in the main model with value $\hat{v}_S = m_S v_S + (1 - m_S) v_L$, where $0 < m_L < m_S = \frac{\frac{1}{w_S}}{\frac{1}{w_S} + \frac{N}{w_L + \lambda_{1/2}}} < 1$ and $\lambda_{1/2}$ is given in the Lemma IA.1.*

Proof of Lemma IA.3. Small traders maximize

$$\max_x \left(E_j\left[v_S\right] - p\right) x - \frac{w_S}{2} x^2 + V_S^{j,*}$$

And the value function can be written as $V_S^{j,*} = E_j\left[x\left(p_{1/2}^* - v_S\right)\right] + \frac{1}{2w_S} E_j\left[\left(p_{1/2}^* - v_S\right)^2\right]$.

The FOC is then

$$E_j\left[v_S + (p_{1/2}^* - v_S)\right] - p - w_S x = 0.$$

This coincides with the FOC for a small trader from the main model with

$$\hat{v}_S = p_{1/2}^* = \frac{\frac{v_S}{w_S} + \frac{N v_L}{w_L + \lambda}}{\frac{1}{w_S} + \frac{N}{w_L + \lambda_{1/2}}}.$$

■ ■

Since we have reduced the dynamic model to a static one as in the paper, the main results about informational efficiency and multiple equilibria follow. Intuitively, in the dynamic model, the private values $\hat{v}_S$ and $\hat{v}_L$ become more correlated compared to v_S and v_L. Small traders now care about v_L and large traders care about v_S because the price at $t = 1/2$ is driven by both v_L and v_S. The increase in correlation between private values however does not invalidate our main results as we show in the Internet Appendix IA.2 that even in the limiting case when $\rho \to 1$, our results continue to hold. We note that the theorem above does not imply that the main welfare result follows from the static model because the value functions in static and dynamic models are different. In fact, they differ by a constant that depends on N. We verify numerically that there exist parameter values such that welfare in the dynamic model can decrease in N.